Impulsive Control of Chua's Circuits Based on Rule-Wise Linear Computational Verb Systems

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***Abstract*—In this paper, an impulsive control approach is developed for controlling Chua's circuits that are generalized into rule-wise linear computational verb systems, which are used to approximately segment the dynamics of Chua's circuits into four qualitatively different patterns. Based on the new systems, several theorems are then presented to find conditions under which the chaotic Chua's circuits can be asymptotically controlled to the origin by impulsive control. One example is provided illustrate the effectiveness of the proposed methods.**

***Keywords*-Computational verb system; Chua's circuit; rule-wise linear system.**

I. INTRODUCTION

Since Yang presented computational verb concept in 1997 [1], [2]. computational verb theory has been successfully applied to many industrial products, such as visual card counters [3], visual flame detecting system [4], and so on. The building blocks of computational theory are computational verbs [5], [6], which are applicable to different kinds of control problems [7], [8].

A rule-wise linear computational verb system consists of a set of computational verbs of which the antecedents are conditions specified by using computational verbs and the consequences are linear dynamic systems [6]. In [9], the author presented the structure of verb proportional-integral-derivative (PID) controllers. Robust stability and stabilization of Takagi-Sugeno fuzzy systems that were presented in [10], were analyzed in [6]. Yang designed an asymptotically stable rule-wise computational verb controller for a class of high-order systems based on inverse solutions of Lyapunov equations in [11]. In [8], Tonelli and Yang used a computational verb controller to control the chaotic Chua's circuits based on rule-wise linearization and designed the controller through linear matrix inequalities. However, in their work, the closed-loop control system is constructed by a continuous input control method which is not available for the development of digital control devices.

This paper is devoted to providing an alternative and novel approach that combines computational verb control methodology with impulsive control for controlling a class of chaotic systems. The computational verb rule-wise linear models of Chua's circuits will be used to approximately segment the dynamics of Chua's circuits into those dynamics in the inner region, in the outer region and at boundaries of both regions. Instead of state feedback controllers for each subsystem, impulsive controllers are introduced and may offer a simple and efficient method to deal with systems based on the development of digital control devices which generate control impulses at discrete moments [12]. One numerical example is provided to show the effectiveness of the approach.

The rest of the paper is organized as follows. Section II gives some definitions of computational verb and verb similarity. Section III develops the rule-wise model for the Chua's circuit. Section IV presents the main results for controlled Chua's circuit under computational verb rules. In Section V, an example is provided to show the effectiveness of the main results. Conclusions appear in Section VI.

Throughout this paper, we use the following notations. $\mathbb{R}^n$ denotes the n-dimensional real space. $\mathbb{Z}$ represents the set of positive integer numbers. A^{T} and A^{-1} denote the transpose and inverse of matrix A, respectively. $\lambda_{\max}(A)$ denotes the maximum eigenvalue of the real symmetric matrix A.

II. THE DEFINITION OF COMPUTATIONAL VERB AND VERB SIMILARITY

A. Computational Verb

As stated [9], the definition of computational verb in [7] is too complex to be operational in the context of engineering applications. Here, a light working definition of computational verb from [6] is giving as follows.

Computational Verb: A computational verb $\mathbf{V}$ is defined by the following *evolving function*

$$\mathscr{E}_{\mathbf{V}} : \mathbb{T} \times \Omega \rightarrow \Omega, \tag{1}$$

where $\mathbb{T} \subseteq \mathbb{R}$ and $\Omega \subseteq \mathbb{R}^n$ are the time and the universe of discourse, respectively.

B. Verb Similarity

The similarity between verbs (verb similarity, for short) is of the essential importance to the inference of verb rules [7].

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Since there is no crisp definition of similarity between two dynamic systems, the verb similarity can be defined based on many different concerns as addressed in [7].

Verb Similarity: Given two computational verbs $\mathbf{V}_1$ and $\mathbf{V}_2$, the verb similarity $\mathcal{S}(\mathbf{V}_1, \mathbf{V}_2)$ should satisfy the followings.

1) $\mathcal{S}(\mathbf{V}_1, \mathbf{V}_2) \in [0, 1]$;

2) $\mathcal{S}(\mathbf{V}_1, \mathbf{V}_2) = \mathcal{S}(\mathbf{V}_2, \mathbf{V}_1)$;

3) $\mathcal{S}(\mathbf{V}_1, \mathbf{V}_2) = 1$ if $\mathbf{V}_1 = \mathbf{V}_2$ almost everywhere, where $\mathbf{V}_1 = \mathbf{V}_2$ means both computational verbs have the same evolving function.

III. THE RULE-WISE LINEAR SYSTEMS

In this section, we shall control chaotic dynamics of the well-known Chua's circuit [13] whose dynamical behavior is described by

$$\left\{\begin{array}{l} \frac{\mathrm{d}v_1}{\mathrm{d}t} = \frac{1}{C_1}[G(v_2 - v_1) - f(v_1)], \\ \frac{\mathrm{d}v_2}{\mathrm{d}t} = \frac{1}{C_2}[(v_1 - v_2) + i_L], \\ \frac{\mathrm{d}i_L}{\mathrm{d}t} = -\frac{1}{L}[v_2 + R_0 i_L], \end{array}\right. \tag{2}$$

where v_1, v_2 and i_L are the state variables, and $G = \frac{1}{R}$. The characteristic of the nonlinear resistor $f(v_1)$ is taken as the well known piecewise-linear characteristic

$$f(v_1) = G_b v_1 + \frac{1}{2}(G_a - G_b)(|v_1 - E| - |v_1 + E|),$$

where $G_a, G_b < 0$, $E > 0$ is the breakpoint voltage.

Assuming $v_1 \in [-d, d]$, $d \gg E > 0$, we obtain the following sector to bound $f(v_1)$:

$$f_1(v_1) = G_a v_1,$$

$$f_2(v_1) = \Big(G_b + \frac{(G_a - G_b)E}{d}\Big) v_1 \triangleq G_1 v_1.$$

Based on impulsive control, the state equations of the control Chua's circuit is given by

$$\left\{\begin{array}{rcl} \frac{\mathrm{d}v_1}{\mathrm{d}t} & = & \frac{1}{C_1}[G(v_2 - v_1) - f(v_1)],\ t \neq t_k, \\ \frac{\mathrm{d}v_2}{\mathrm{d}t} & = & \frac{1}{C_2}[(v_1 - v_2) + i_L],\ t \neq t_k, \\ \frac{\mathrm{d}i_L}{\mathrm{d}t} & = & -\frac{1}{L}[v_2 + R_0 i_L],\ t \neq t_k, \\ \Delta v_1(t) & = & b_{1k} v_1(t_k),\ t = t_k, \\ \Delta v_2(t) & = & b_{2k} v_2(t_k),\ t = t_k, \\ \Delta i_L(t) & = & b_{3k} i_L(t_k),\ t = t_k, \end{array}\right. \tag{3}$$

where $\Delta v(t)|_{t=t_k} = v(t_k^+) - v(t_k^-)$, $k \in \mathbb{Z}$. Here, $x(t_k^+) = \lim_{h \to 0^+} x(t_k + h)$, $x(t_k^-) = \lim_{h \to 0^+} x(t_k - h)$ with discontinuity instants $t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k \to \infty} t_k = \infty$, where $t_1 > t_0$. For convenience, let $t_0 = 0$ and $h > 0$ be sufficiently small. Without loss of generality, it is assumed that $x(t_k) = x(t_k^-) = \lim_{h \to 0^+} x(t_k - h)$. b_{1k}, b_{2k} and b_{3k} are the impulsive control coefficients.

Choose two membership functions:

$$\left\{\begin{array}{rcl} \mu_{\rm in}(t) & = & \max(0, 1 - |v_1/2|), \\ \mu_{\rm out}(t) & = & \min(1, |v_1/2|), \end{array}\right. \tag{4}$$

where $\mu_{\rm in}(t)$ and $\mu_{\rm out}(t)$ are for 'inner' region and 'outer' region, respectively. Then the state space of Chua's circuit is segmented into an 'inner' region, where $|v_1|$ is not bigger than E, and an 'outer' region, where $|v_1|$ is bigger than E. Denoting $x^{\rm T}(t) = [v_1,\ v_2,\ i_L]^{\rm T}$ as the state vector and following the similar design procedure presented in [8], the closed-loop control system based on computational verb rules can be transformed into the following region-wise systems.

*Rule 1: If $|v_1(t)|$ **stays** at the inner region, then*

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & A_1 x(t), t \neq t_k, \\ \Delta x(t) & = & B_{1k} x(t_k), t = t_k, \end{array}\right. \tag{5}$$

*Rule 2: If $|v_1(t)|$ **increases** from the inner region, then*

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & A_2 x(t), t \neq t_k, \\ \Delta x(t) & = & B_{2k} x(t_k), t = t_k, \end{array}\right. \tag{6}$$

*Rule 3: If $|v_1(t)|$ **decreases** from the outer region, then*

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & A_3 x(t), t \neq t_k, \\ \Delta x(t) & = & B_{3k} x(t_k), t = t_k, \end{array}\right. \tag{7}$$

*Rule 4: If $|v_1(t)|$ **stays** at the outer region, then*

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & A_4 x(t), t \neq t_k, \\ \Delta x(t) & = & B_{4k} x(t_k), t = t_k, \end{array}\right. \tag{8}$$

where

$$A_1 = A_2 = \begin{bmatrix} -\frac{G}{C_1} - \frac{G_a}{C_1} & \frac{G}{C_1} & 0 \\ \frac{G}{C_2} & -\frac{G}{C_2} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}, \tag{9}$$

$$A_3 = A_4 = \begin{bmatrix} -\frac{G}{C_1} - \frac{G_1}{C_1} & \frac{G}{C_1} & 0 \\ \frac{G}{C_2} & -\frac{G}{C_2} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}, \tag{10}$$

and B_{jk} $(j = 1, \cdots, 4)$ are diagonal impulsive control matrices to be designed.

Therefore, the overall expression of the rule-wise linear computational verb systems (5)-(8) is given by

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & \sum_{i=1}^{4} \mathcal{S}_i(t) A_i x(t) \Big/ \sum_{i=1}^{4} \mathcal{S}_i(t),\ t \neq t_k, \\ \Delta x(t) & = & \sum_{i=1}^{4} \mathcal{S}_i(t) B_{ik} x(t_k) \Big/ \sum_{i=1}^{4} \mathcal{S}_i(t),\ t = t_k, \end{array}\right. \tag{11}$$

where $\mathcal{S}_i(t)$ is the computational verb similarity between waveform $v_1(t)$ and the computational verb in the antecedent of the $i^{\rm th}$ computational verb rule.

To calculate the computational verb similarities $\mathcal{S}_i(t)$ $(i = 1, \cdots, 4)$, it follows from the methods presented in [9] that the canonical forms of verb **become**'s for verb rules in (5)-(8) are given by $\mathbf{become}_i, i = 1, 2, 3, 4,$

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respectively, that is,

$$\begin{cases} \textbf{become}_1 & \triangleq & \textbf{become}(\text{inner region}, \text{inner region}), \\ \textbf{become}_2 & \triangleq & \textbf{become}(\text{inner region}, \text{outer region}), \\ \textbf{become}_3 & \triangleq & \textbf{become}(\text{outer region}, \text{inner region}), \\ \textbf{become}_4 & \triangleq & \textbf{become}(\text{outer region}, \text{outer region}). \end{cases} \quad (12)$$

Let T_w be the length of the window and at moment t let us consider the history of dynamics during the period of $[t - T_w, t]$, then the evolving function of $\textbf{become}_i(\texttt{state 1}, \texttt{state 2})$ is given by:

If $\texttt{state 1} \neq \texttt{state 2}$, then

$$\mathscr{E}_{\textbf{become}_i}(\texttt{state 1}, \texttt{state 2})(\tau) = \begin{cases} 1 - \dfrac{\tau - (t - T_w)}{T_w}, \ \tau \in \left[t - T_w, t - \dfrac{T_w}{2}\right] \\ 0.5 + \dfrac{\tau - (t - T_w/2)}{T_w}, \ \tau \in \left[t - \dfrac{T_w}{2}, t\right]; \end{cases} \quad (13)$$

otherwise,

$$\mathscr{E}_{\textbf{become}_i}(\texttt{state 1}, \texttt{state 2})(\tau) \equiv 1, \ \ \tau \in [t - T_w, t]. \quad (14)$$

Now, denote $\mathscr{E}_{\textbf{become}}(t) \triangleq \mathscr{E}_{(\texttt{state 1}, \texttt{state 2})}(t)$, and the implementations for verb rules (5)-(8) are given by the following steps.

1) The first half window

$$\begin{aligned} a_1 &\triangleq \int_{\tau=t-T_w}^{t-T_w/2} \mathscr{E}_{\textbf{become}}(\tau) \wedge \mu_{\texttt{state 1}}(v_1(\tau))\mathrm{d}\tau, \\ b_1 &\triangleq \int_{\tau=t-T_w}^{t-T_w/2} \mathscr{E}_{\textbf{become}}(\tau) \vee \mu_{\texttt{state 1}}(v_1(\tau))\mathrm{d}\tau. \end{aligned} \quad (15)$$

2) The second half window

$$\begin{aligned} a_2 &\triangleq \int_{\tau=t-T_w/2}^{t} \mathscr{E}_{\textbf{become}}(\tau) \wedge \mu_{\texttt{state 2}}(v_1(\tau))\mathrm{d}\tau, \\ b_2 &\triangleq \int_{\tau=t-T_w/2}^{t} \mathscr{E}_{\textbf{become}}(\tau) \vee \mu_{\texttt{state 2}}(v_1(\tau))\mathrm{d}\tau. \end{aligned} \quad (16)$$

3) The balance factor ϖ

$$\varpi = 2 \min\left(\frac{a_1}{b_1 + b_2}, \frac{a_2}{b_1 + b_2}\right). \quad (17)$$

4) The entire window

$$\mathcal{S}(\textbf{become}, v_1(t)) = \frac{a_1 + a_2}{b_1 + b_2}\varpi. \quad (18)$$

Consequently, $\mathcal{S}_i(t)$, $i = 1, 2, 3, 4$, are calculated by

$$\mathcal{S}_i(t) = \mathcal{S}(\textbf{become}_i, v_1(t))\delta_i(v_1(t)), i = 1, 2, 3, 4, \quad (19)$$

where

$$\begin{aligned} \delta_1(v_1(t)) = \delta_2(v_1(t)) &= \begin{cases} 1, & \text{if } |\mathrm{v_1(t)}| \leq \mathrm{E}, \\ 0, & \text{otherwise.} \end{cases} \\ \delta_3(v_1(t)) = \delta_4(v_1(t)) &= \begin{cases} 1, & \text{if } |\mathrm{v_1(t)}| > \mathrm{E}, \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (20)$$

Note that $\mathcal{S}_i(t)$, $i = 1, 2, 3, 4$, are also computational verb similarities.

IV. Main Results

Now we are arriving at presenting our main theorems for guaranteeing the stability of the impulsively controlled Chua's circuit under computational verb rules. Based on the Lyapunov function $V(t) = x^{\mathrm{T}}(t)Px(t)$ and impulsive control theory, we obtain the following criteria.

Theorem 1: Let $n \times n$ matrix P be symmetric and positive definite, and $Q = PA_i + A_i^{\mathrm{T}}P$, $i = 1, 2, 3, 4$, and λ_1 is the largest eigenvalue of $P^{-1}Q$, λ_2 is the largest eigenvalue of the matrix $P^{-1}(I + B_{ik})^{\mathrm{T}}P(I + B_{ik})$, where $i = 1, 2, 3, 4$, $k \in \mathbb{Z}$, then the origin of impulsive control system (11) is asymptotically stable if there exists a $\xi > 1$ and a differentiable at $t \neq t_k$ and nonincreasing function $K(t) \geq m > 0$ which satisfies

$$-\frac{D^+K(t)}{K(t)} \leq \lambda_1 \leq \frac{1}{(1+\varepsilon)\delta_2} \ln \frac{K(\tau_{2k}^+)K(\tau_{2k-1}^+)}{K(\tau_{2k+1})K(\tau_{2k})\xi\lambda_2^2} \quad (21)$$

or

$$-\frac{D^+K(t)}{K(t)} \leq \lambda_1 \leq \frac{1}{\max\{\delta_1, \delta_2\}} \ln \frac{K(\tau_k^+)}{K(\tau_{k+1})\xi\lambda_2}, \quad (22)$$

where $\delta_1 = \sup_k\{t_{2k+1} - t_{2k}\} < \infty$, $\delta_2 = \sup_k\{t_{2k} - t_{2k-1}\} < \infty$, and for a given constant $\varepsilon > 0$, $t_{2k+1} - t_{2k} \leq \varepsilon(t_{2k} - t_{2k-1})$, $\forall k \in \mathbb{Z}$.

Theorem 2: If there exist a symmetric positive definite matrix P, and positive scalars α_k, β_k, such that the following conditions are satisfied.

$$\mu_M\left(A_i^{\mathrm{T}}P + PA_i - \frac{\alpha_k}{\delta_k}P\right) < 0,$$

$$\mu_M\left(B_i^{\mathrm{T}}P + PB_i - \beta_kP + B_i^{\mathrm{T}}PB_j\right) < 0,$$

$$\alpha_k + \beta_k \leq 0, \ \beta_k \geq -1,$$

where $i, j = 1, 2, 3, 4$, $\delta_k = t_k - t_{k-1}$, $k \in \mathbb{Z}$, then the origin of impulsive control system (11) is asymptotically stable.

V. Numerical simulations

In this section, we shall provide simulation results to illustrate the proposed method. For simplicity, here we only illustrate the effectiveness of the criterion in Theorem 1. As in [8], the parameters for Chua's circuit (2) are chosen as $R = \frac{10}{7}$, $R_0 = 0$, $C_1 = 0.1$, $C_1 = 2$, $L = \frac{1}{7}$, $G_a = -4$, $G_b = -0.1$, and $E = 1$. With these parameters, Chua's circuit exhibits chaotic dynamics under initial condition $[v_1(0), v_2(0), i_L(0)] = [-2.6, -3.2, 1.1]$ and state $v_1(t)$ will be bounded in the interval $[-15, 15]$; therefore, let us take $d = 15$. To control the Chua's circuit, according to Theorem 1, we set $P = I, K(t) \equiv 1$, $B_{1k} = -0.53I$, $B_{2k} = -0.61I$, $B_{3k} = -0.55I$, $B_{4k} = -0.62I$, and $\xi = 1.1$, which imply that $\lambda_1 = 66.8078$, and $\lambda_2 = 0.2209$, In the simulation, let

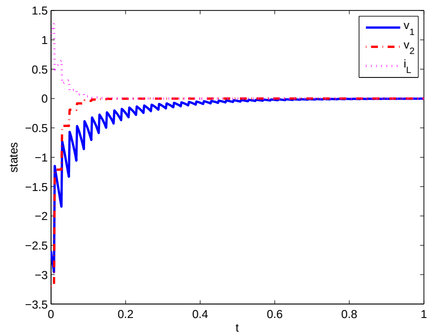

Figure 1. The state trajectories v_1, v_2, i_L of the controlled Chua's circuit

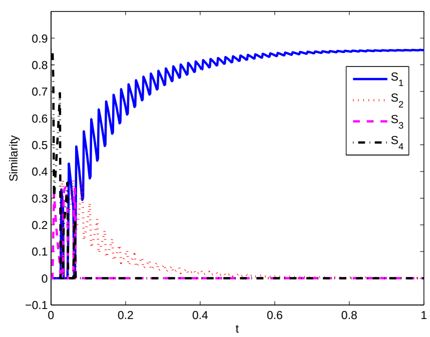

Figure 2. The waveforms of verb similarities $\mathcal{S}_i(t)$ $i = 1, 2, 3, 4$

the impulses be equidistant from each other, that is, $\varepsilon = 1$, thus from (21) we have $0 \leq \delta_1 = \delta_2 \leq 0.0219$. Figure 1 shows the state trajectories of chaotic system (2) under the designed impulsive control and the computational verb controllers for $t_k - t_{k-1} = 0.02$. Clearly, the time series of all the variables of the system converge to zero. Figure 2 shows the waveforms of computational verb similarity $\mathcal{S}_i(t)$ of the i^{th} computational verb control rule for $i = 1, 2, 3, 4$. Note that the similarity $\mathcal{S}_1(t)$ approaches 0.8571 asymptotically while the similarity $\mathcal{S}_2(t)$ approaches zero at the meantime. The third and fourth similarities only had non-zero values at the very beginning and dropped to zero rapidly.

VI. Conclusions

In this paper, an impulsive framework for controlling chaotic Chua system has been proposed through combining impulsive control technique with intelligent rule-wise computational verb methodology. Based on rule-wise linearization, the dynamics of Chua's circuits are approximately segmented into four regions. Two criteria have been proposed to guarantee the global asymptotic stability of the computational verb rule-wise linear systems. The applicability and validity of the proposed control scheme have been illustrated through numerical simulations.

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