Robust Optimization for Stochastic Wireless CDMA/TDMA Networks

Belarmino Núñez, Pablo Adasme, Ismael Soto
Departamento de Ingeniería Eléctrica,
Universidad de Santiago de Chile,
Avenida Ecuador 3519 Santiago, Chile
Emails: {belarmino.nunez,pablo.adasme,ismael.soto}@usach.cl

Abdel Lisser
Laboratoire de Recherche en Informatique,
Université Paris-Sud XI,
Bâtiment 650, Orsay Cedex, France
Email: abdel.lisser@lri.fr

***Abstract*—In this paper, we propose a distributionally robust formulation for packet transmission allocation in CDMA/TDMA networks. In particular, we adopt a utility-based framework where channel bit rates and packet experienced delays conditions are considered. Consequently, the total utility of the network subject to capacity and packet assignment constraints is maximized. For this purpose, we first formulate the problem as a (0-1) stochastic integer linear programming problem. Then, we transform the stochastic model into an equivalent deterministic formulation. Subsequently, we use the deterministic model to derive the distributionally robust counterpart. This is achieved while taking into account the set of all possible probability distributions for the input random parameters. Finally, we compare the optimal solutions of the stochastic and robust models. Our preliminary numerical results indicate that slight conservative solutions can be obtained when the instances dimensions increase.**

***Keywords*—Stochastic programming; distributionally robust optimization; code division and time division multiple access, wireless networks.**

I. INTRODUCTION

Code division and time division multiple access (resp. CDMA, TDMA) are two wireless multi-carrier transmission schemes currently embedded into modern technologies such as Wifi and Wimax [14]. By multiple access, we mean several users can send signals simultaneously over a single wireless communication channel. In particular, CDMA uses a spread spectrum technology combined with coding schemes to allow multiple users on the same physical channel. On the other side, TDMA has the property of scheduling users in time by assigning all bandwidth channel capacity to only one user at a given time slot in order to transmit signals. Although, these transmission schemes work differently, the underlying purpose in both of them is nearly the same, i.e., to make an efficient use of resources such as power and bandwidth channel capacity of the network. Hybrid transmission schemes do also exist such as CDMA/TDMA [14]. In this paper, we use a utility-based framework and formulate a stochastic resource allocation model for a CDMA/TDMA network. Thus, we associate for each packet transmitted by a base station, a utility function depending on the corresponding channel bit rates and experienced delays according to a CDMA/TDMA network. Consequently, we maximize the total utility of the network subject to capacity and packet assignment constraints. The capacity constraint appears as a technological constraint in these type of networks since a particular base station can not transmit packets in more than "M" frames within a given time slot [11]. We derive a distributionally robust formulation for this stochastic problem. For this purpose, we first formulate the problem as a (0-1) stochastic integer linear programming problem. Next, we transform the stochastic model into an equivalent deterministic formulation. In particular, we transform the probabilistic constraints using the approach proposed by [1]. Afterward, we use the deterministic formulation to derive the robust counterpart [16]. This is achieved while taking into account the set of all possible probability distributions for the input random parameters. Finally, we compare the optimal solutions of the stochastic and robust models. Preliminary numerical comparisons are given.

The paper is organized as follows. In section II, we provide a brief state of the art concerning resource allocation in wireless CDMA/TDMA networks, stochastic programming and robust optimization. Section III presents the stochastic CDMA/TDMA model under study and its equivalent deterministic formulation. Then, in section IV, we derive the distributionally robust formulation. In section V, we provide preliminary numerical comparisons. Finally, in section VI, we conclude the paper.

II. RELATED WORK

Resource allocation for wireless CDMA and TDMA networks has been studied separately (see e.g. [5], [12]) and jointly as well [11]. The latter approach, which we refer to as hybrid CDMA/TDMA transmission scheme, exploits delay tolerance of non-real time traffic in order to improve the use of bandwidth channel capacity of the network [17], [18]. This is possible since the base stations, which transmit signals to users, may have the possibility to choose different subchannels to transmit their signals. The resulting bit rate improvement is referred to as a multi-user diversity gain. Due to the inherent random nature of wireless channels in CDMA/TDMA networks, there have been proposed some few attempts while considering general stochastic approaches, e.g., [19]. However, to the best of our knowledge, none of them has yet considered stochastic programming as in [7] and/or distributionally robust optimization [16] approaches to deal with the inherent uncertainty of the problem.

Stochastic programming (SP) and robust optimization (RO) are two well known optimization techniques to cope with mathematical optimization problems involving uncertainty in

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the input parameters. In SP, one often assumes that the probability distributions are discrete and known [2]. Two well known scenario SP approaches are the recourse model and the probabilistic constrained approaches [7], [15]. Different from SP, the RO framework assumes that the input random parameters lie within a convex uncertainty set and that the robust solutions must remain feasible for all possible realizations of the input parameters. Thus, the optimization is performed over the worst case realization of the input parameters. In compensation, we obtain robust solutions, which are protected from undesired fluctuations of the input parameters. In practice, this means that the objective function provides more conservative solutions. We refer the reader to [3], [4] for a more general understanding on RO. In particular, the distributionally robust optimization approach bridges the gap between the conservatism level of robust optimization and the stochastic programming approach. The conservatism can be measured by the loss in optimality in exchange for a robust solution which is more protected against uncertainty [3]. This means, the less conservative the robust solutions are, the better the RO approach. Thus, the distributionally robust approach optimizes the worst-case objective over a family of possible distributions. This approach was pioneered in [6], [9]. In particular, Scarf [6] proposes an application for a news-vendor problem while in [8], Yue et al. minimizes the worst case absolute regret for all distributions with certain mean and variance. In a similar vein, ElGhaoui et al. [13] proposed worst-case value at risk bounds for a robust portfolio optimization problem using only bounds on the means of the assets and their covariance matrix. Similarly, Natarajan et al. [10] derived a distributionally robust model for portfolio optimization, where the investor maximizes his worst case expected utility over a set of ambiguous distributions described by the knowledge of the mean, covariance and support information.

In this paper, we propose a (0-1) stochastic integer linear programming problem for wireless CDMA/TDMA networks. Next, we transform the stochastic model into an equivalent deterministic formulation we use to derive a novel distributionally robust counterpart [16]. This is achieved while taking into account the set of all possible probability distributions for the input random parameters.

III. STOCHASTIC FORMULATION

We consider a wireless CDMA/TDMA network composed of a set of base stations $\mathcal{B} = \{1, .., B\}$ and a set of packets $\mathcal{N} = \{1, .., N\}$ waiting to be scheduled and transmitted by the different base stations in $\mathcal{B}$. We assume that the CDMA/TDMA network supports only non-realtime traffic. In practice, this means that the system can tolerate packet transmission delays. We also assume that the system is time slotted with each slot containing M frames. We consider the following (0-1) stochastic integer linear programming model for this problem and denote it hereby P_0 as

$$\max_{\{x,y\}} \quad E_\xi \left\{ \sum_{i=1}^{N} u_i(\xi) y_i + \sum_{i=1}^{N} \sum_{j=1}^{B} v_{ij}(\xi) x_{ij} \right\} \tag{1}$$

$$\text{s.t.:} \quad P_\xi \left\{ \sum_{i=1}^{N} m_{ij}(\xi) x_{ij} \leq M \right\} \geq 1 - \alpha; \forall j \in \mathcal{B} \tag{2}$$

$$y_i + \sum_{j=1}^{B} x_{ij} = 1; \forall i \in \mathcal{N} \tag{3}$$

$$y_i, x_{ij} \in \{0, 1\}, \forall i, j \tag{4}$$

where the term $E\{\cdot\}$ denotes mathematical expectation and $P\{\cdot\}$ represents a probability constraint which should be satisfied at least for $(1 - \alpha)\%$ of the cases where $\alpha \in (0, 0.5]$ represents the risk. The matrix $v = v(i, j)$ represents the utility gained by the system while transmitting packet $i \in \mathcal{N}$ through the base station $j \in \mathcal{B}$. Similarly, the vector $u = u(i)$ denotes the gained achieved by the system when packet $i \in \mathcal{N}$ is not transmitted during a particular time slot. This is possible as we assume non-realtime traffic in the network. Finally, let matrix $m = m(i, j)$ represent the number of required frames for transmission of packet $i \in \mathcal{N}$ by base station $j \in \mathcal{B}$ in a particular time slot. In P_0, $y \in \{0, 1\}^N$ and $x \in \{0, 1\}^{NB}$ are binary decision variables we define as follows. Variable y_i equals one if packet i is not scheduled to be transmitted in the current time slot and equals zero otherwise. Similarly, variable x_{ij} equals one if packet i is scheduled to be transmitted within the current time slot by the base station j and equals zero otherwise. The objective function (1) maximizes the total utility of the system while constraint (2) imposes a maximum number of M frames for each $j \in \mathcal{B}$. Finally, constraint (3) indicates that each packet $i \in \mathcal{N}$ must be transmitted or not by a unique base station in the system. Without loss of generality we assume that the matrix $v = v(\xi)$ and the vectors $u = u(\xi)$, and $m = m(\xi)$ are random variables distributed according to a discrete probability distribution Ω. As such, one may suppose that $v = v(\xi)$, $u = u(\xi)$ and $m = m(\xi)$ are concentrated on a finite set of scenarios as $v_{ij}(\xi) = \{v_{ij}^1, .., v_{ij}^K\}$, $u_i(\xi) = \{u_i^1, .., u_i^K\}$ and $m_{ij}(\xi) = \{m_{ij}^1, .., m_{ij}^K\}$ respectively, with probability vector $p^T = (p_1, .., p_K)$ such that $\sum_{k=1}^{K} p_k = 1$ and $p_k \geq 0$. This allows us to formulate a deterministic equivalent formulation for P_0 as follows [1]

$$\max_{\{x,y,z\}} \quad \sum_{k=1}^{K} p_k \left(\sum_{i=1}^{N} u_i^k y_i + \sum_{i=1}^{N} \sum_{j=1}^{B} v_{ij}^k x_{ij} \right) \tag{5}$$

$$\text{s.t.:} \quad \sum_{i=1}^{N} m_{ij}^k x_{ij} \leq M + z_{jk} L; \forall j \in \mathcal{B}; k = 1 : K \tag{6}$$

$$\sum_{k=1}^{K} p_k z_{jk} \leq \alpha; \forall j \in \mathcal{B} \tag{7}$$

$$y_i + \sum_{j=1}^{B} x_{ij} = 1; \forall i \in \mathcal{N} \tag{8}$$

$$y_i, x_{ij}, z_{jk} \in \{0, 1\}, \forall i, j, k \tag{9}$$

where constraints (6)-(7) are the deterministic constraints replacing the probabilistic constraint (2) in P_0. Hereafter, we denote by P_1 problem (5)-(9).

IV. ROBUST FORMULATION

In this section, we derive a distributionally robust model for P_1. To this purpose, we assume that the probability distribution of the random vector $p^T = (p_1, .., p_K)$ is not known and that it can be estimated by some statistical mean from some available historical data. Thus, we consider the maximum likelihood estimator of the probability vector p^T to be the observed frequency vector.

In order to formulate a robust model for P_1, we write its objective function as follows

$$\min_{\{x,y\}} \max_{\{\pi \in H_\beta\}} \quad \sum_{k=1}^{K} \pi_k \left(-\sum_{i=1}^{N} u_i^k y_i - \sum_{i=1}^{N}\sum_{j=1}^{B} v_{ij}^k x_{ij} \right) \quad (10)$$

and the left hand side of constraint (7) as the maximization problem

$$\max_{\{\pi \in H_\beta\}} \quad \sum_{k=1}^{K} \pi_k z_{jk}, \quad \forall j \quad (11)$$

where the set H_β is defined as

$$H_\beta = \left\{ \pi_k \geq 0, \forall k : \sum_{k=1}^{K} \pi_k = 1, \sum_{k=1}^{K} \frac{|\pi_k - p_k|}{\sqrt{p_k}} \leq \beta \right\} \quad (12)$$

and $\beta \in [0, \infty)$. Now, let $\delta_k = \pi_k - p_k$, then the inner max problem in (10) can be written as

$$\max_{\{\delta\}} \quad \sum_{k=1}^{K} (\delta_k + p_k) \left(-\sum_{i=1}^{N} u_i^k y_i - \sum_{i=1}^{N}\sum_{j=1}^{B} v_{ij}^k x_{ij} \right) \quad (13)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \frac{|\delta_k|}{\sqrt{p_k}} \leq \beta \quad (14)$$

$$\sum_{k=1}^{K} \delta_k = 0 \quad (15)$$

$$\delta_k \geq -p_k, \quad k = 1 : K \quad (16)$$

The associated dual problem is

$$\min_{\{w^1, \varphi^1, v^1\}} \sum_{k=1}^{K} p_k \left(-\sum_{i=1}^{N} u_i^k y_i - \sum_{i=1}^{N}\sum_{j=1}^{B} v_{ij}^k x_{ij} \right) + \sum_{k=1}^{K} p_k w_k^1 + \beta \varphi^1 \quad (17)$$

$$\text{s.t.} \quad \varphi^1 \geq \sqrt{p_k} \left(v^1 + w_k^1 - \sum_{i=1}^{N} u_i^k y_i - \sum_{i=1}^{N}\sum_{j=1}^{B} v_{ij}^k x_{ij} \right), \forall k \quad (18)$$

$$\varphi^1 \geq -\sqrt{p_k} \left(v^1 + w_k^1 - \sum_{i=1}^{N} u_i^k y_i - \sum_{i=1}^{N}\sum_{j=1}^{B} v_{ij}^k x_{ij} \right), \forall k \quad (19)$$

$$w_k^1 \geq 0, \quad \forall k \quad (20)$$

and φ^1, v^1, w^1 are Lagrangian multipliers for constraints (14)-(16), respectively. Similarly, we obtain a dual formulation for each j in (11) as follows

$$\min_{\{w^2, \varphi^2, v^2\}} \quad \sum_{k=1}^{K} p_k z_{jk} + \sum_{k=1}^{K} p_k w_k^2 + \beta \varphi^2 \quad (21)$$

$$\text{s.t.} \quad \varphi^2 \geq \sqrt{p_k} \left(v^2 + w_k^2 + z_{jk} \right), \forall k \quad (22)$$

$$\varphi^2 \geq -\sqrt{p_k} \left(v^2 + w_k^2 + z_{jk} \right), \forall k \quad (23)$$

$$w_k^2 \geq 0, \quad \forall k \quad (24)$$

where φ^2, v^2, w^2 are Lagrangian multipliers associated with its respective primal constraints. Now, replacing these dual problems in P_1 gives rise to the following distributionally robust formulation we denote by P_R

$$\max_{\{w^1, \varphi^1, v^1, w^2, \varphi^2, v^2, x, y, z\}} \sum_{k=1}^{K} p_k \left(\sum_{i=1}^{N} u_i^k y_i + \sum_{i=1}^{N}\sum_{j=1}^{B} v_{ij}^k x_{ij} \right) - \sum_{k=1}^{K} p_k w_k^1 - \beta \varphi^1 \quad (25)$$

$$\text{s.t.} \varphi^1 \geq \sqrt{p_k} \left(v^1 + w_k^1 - \sum_{i=1}^{N} u_i^k y_i - \sum_{i=1}^{N}\sum_{j=1}^{B} v_{ij}^k x_{ij} \right), \forall k \quad (26)$$

$$\varphi^1 \geq -\sqrt{p_k} \left(v^1 + w_k^1 - \sum_{i=1}^{N} u_i^k y_i - \sum_{i=1}^{N}\sum_{j=1}^{B} v_{ij}^k x_{ij} \right), \forall k \quad (27)$$

$$w_k^1 \geq 0, \quad \forall k \quad (28)$$

$$\sum_{i=1}^{N} m_{ij}^k x_{ij} \leq M + z_{jk} L; \forall j \in \mathcal{B}; k = 1 : K \quad (29)$$

$$z_{jk} \in \{0, 1\} \quad \forall j, k \quad (30)$$

$$\sum_{k=1}^{K} p_k z_{jk} + \sum_{k=1}^{K} p_k w_k^2 + \beta \varphi^2 \leq \alpha, \quad \forall j \in \mathcal{B} \quad (31)$$

$$\varphi^2 \geq \sqrt{p_k}(z_{jk} + v^2 + w_k^2), \quad \forall j, k \quad (32)$$

$$\varphi^2 \geq -\sqrt{p_k}(z_{jk} + v^2 + w_k^2), \quad \forall j, k \quad (33)$$

$$w_k^2 \geq 0, \quad \forall k \quad (34)$$

$$y_i + \sum_{j=1}^{B} x_{ij} = 1; \forall i \in \mathcal{N} \quad (35)$$

$$y_i, x_{ij}, z_{jk} \in \{0, 1\}, \forall i, j, k \quad (36)$$

In the next section, we provide numerical comparisons between P_1 and P_R. This allows measuring the conservatism level of P_R w.r.t. P_1.

V. Numerical Results

In this section, we present preliminary numerical results. A Matlab program is developed using Cplex 12.3 for solving P_1 and P_R. The numerical experiments have been carried out on a Pentium IV, 1.9 GHz with 2 GB of RAM under windows XP. The input data is generated as follows. The probability vectors p^T is uniformly distributed in [0,1] such that the sum is equal to one. The parameter α is set to 0.1. So far, each entry u_i^k, v_{ij}^k and $m_{ij}^k, \forall i, j, k$ is generated randomly and uniformly distributed in [0,10], [0,20] and [1,5], respectively.

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The value of M is computed as $M = \left(\sum_{i=1}^{N} m_{i1}^{1}\right) * 0.5$. So far, these random intervals are basically motivated on the range these parameters might be in realistic CDMA/TDMA systems. Nevertheless, more realistic input data will be used in a larger version of this paper.

In Table 1, the columns 1-3 give the size of the instances. Columns 4-5 provide the average optimal solutions over 25 different sample instances for P_1 and P_R, respectively. Finally, column 6 gives the average gaps we compute for each instance as $\frac{\mathrm{P}_1 - \mathrm{P}_R}{\mathrm{P}_1} * 100$ %. These results are calculated for different values of β. From Table 1, we mainly observe that the solutions tend to be less conservative when the instances dimensions increase. In particular, we observe slight conservative solutions when the number of packets to be sent by the different base stations $b \in \mathcal{B}$ increases. This is an interesting observation since the number of packets in real CDMA/TDMA networks is usually larger than the number of required base stations. Finally, we observe that by increasing the number of scenarios K in the robust model slightly affects the conservatism level. In order to see how the parameter β affects the optimal solutions given by P_R, in Figures 1, 2 and 3 we solve a small, a medium and a large size instance, respectively. These figures present the same information in their respective horizontal and vertical axes. In the horizontal axes, we give the value of beta while in vertical axes, we present the optimal solutions of P_1, P_R, and the gaps obtained in each of these figure. These gaps are computed as $\frac{\mathrm{P}_1 - \mathrm{P}_R}{\mathrm{P}_1} * 100$ %. From these figures, we first confirm the fact that less conservative gaps are obtained for larger instances. In figure 1, we obtain a gap between 10 and 15% while in figures 2 and 3, they are between 6 and 8 %, and 4 and 6 %, respectively. Secondly, that the increase of parameter β affects the optimal solutions of P_R only when $\beta \in [0, 30]$. In view of this observation, in figure 4 we solve a new small size instance for these values of β. This table presents the same information as for figures 1, 2 and 3. Here, we mainly observe that the major fluctuations of the optimal solutions of P_R are due to small values of $\beta \in [0, 5]$. While small fluctuations are observed for values of $\beta \in [5, 30]$. For

TABLE I. AVERAGE COMPARISONS OVER 25 INSTANCES.

Instance size			Avg. Opt. Sol.		Avg. Gap$_R$
N	B	K	P_1	P_R	
$\beta = 5$					
10	5	10	121.4334	104.9270	13.54 %
20	10	20	245.7756	221.4753	9.87 %
50	10	30	593.6139	554.9500	6.51 %
50	10	40	580.7174	544.7319	6.19 %
100	20	50	1175.4	1128.4	3.99 %
$\beta = 10$					
10	5	10	120.3395	102.9955	14.37 %
20	10	20	245.1590	215.7783	11.97 %
50	10	30	595.3698	550.1536	7.59 %
50	10	40	582.5017	534.8705	8.17 %
100	20	50	1175.9	1111.1	5.50 %
$\beta = 50$					
10	5	10	123.6391	106.1063	14.14 %
20	10	20	245.5830	218.0361	11.20 %
50	10	30	594.9709	549.7108	7.60 %
50	10	40	579.8950	530.8381	8.45 %
100	20	50	1174.7	1110.7	5.44 %
$\beta = 500$					
10	5	10	122.7709	105.1106	14.34 %
20	10	20	245.7271	217.3286	11.52 %
50	10	30	594.5845	545.0882	8.32 %
50	10	40	579.4147	529.8945	8.54 %
100	20	50	1173.8	1105.4	5.82 %

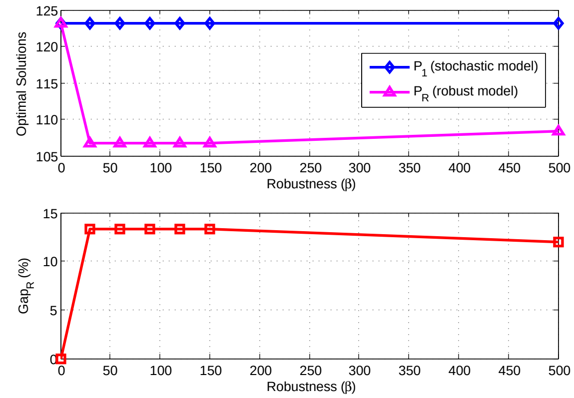

Fig. 1. INSTANCE # 1: N = 10, B = 5, K = 10.

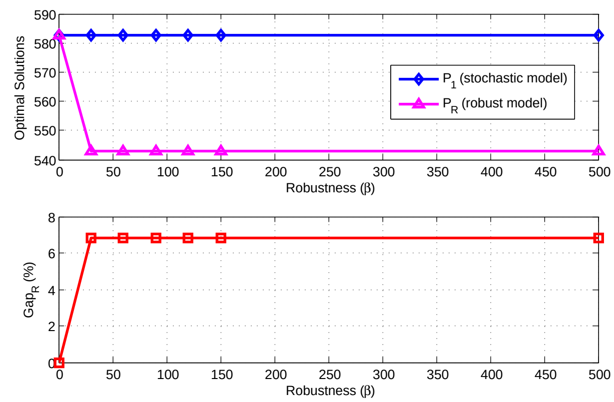

Fig. 2. INSTANCE # 2: N = 50, B = 10, K = 30.

example, when β goes from 0 to 5, we have a conservatism level gap increment of approximately 9% while for values of β that goes from 5 to 30, we obtain a gap increment of approximately 2,5%.

Finally, in figures 1, 2, 3 and 4 we observe similar gaps as in Table I. We recall that the gaps in these figures are obtained when using only one sample of the input data in P_R. This reflects, somehow, the accuracy of the gaps obtained with the proposed distributionally robust model.

VI. Conclusion and Future Work

In this paper, we proposed a distributionally robust formulation for packet transmission allocation in CDMA/TDMA networks where a utility function depending on channel bit rates and experienced delays is considered. Consequently, the total utility of the network subject to capacity and packet assignment constraints is maximized. To this purpose, we formulate the problem as a (0-1) stochastic integer linear programming problem which we transform into an equivalent deterministic model. Then, we use the deterministic model to derive the distributionally robust counterpart. This is achieved while taking into account the set of all possible probability distributions for the input random parameters. Then, we compared the optimal solutions of the stochastic and robust models. Preliminary numerical results indicate that the optimal solutions of the proposed robust model are much less conservative when

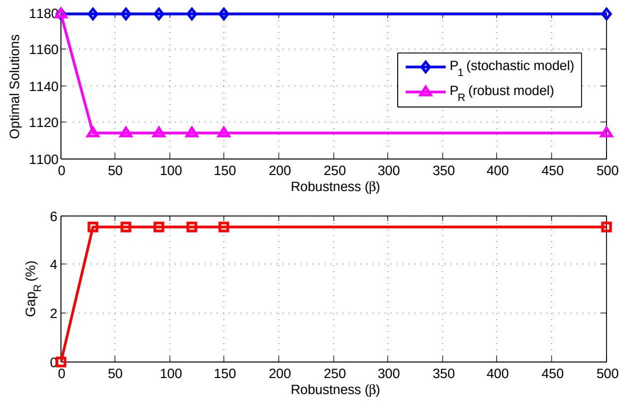

Fig. 3. INSTANCE # 3: N = 100, B = 20, K = 50.

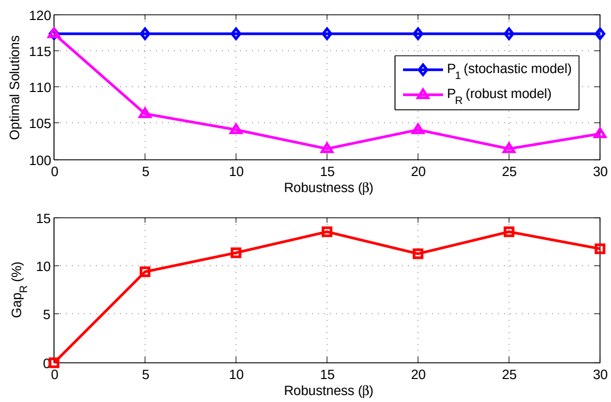

Fig. 4. INSTANCE # 4: N = 10, B = 5, K = 10.

the size of the instances increase. In particular, we emphasize the fact that slight conservative solutions are obtained when the number of packets to be sent by the different base stations increases w.r.t the number of bases stations. This is very important result in realistic CDMA/TDMA networks since the number of required base stations is often lower compared to the required bit rates throughput of the networks. Finally, we also highlight that by increasing the number of scenarios K in the robust model slightly affects the conservatism level. This is also another interesting result as one may use a much larger number of scenarios, which would allow approaching more realistic wireless networks.

As future research, we plan to consider other stochastic programming approaches while using continuous probability distributions and more realistic input data for CDMA/TDMA networks and also for different kinds of wireless networks such as orthogonal frequency division and space division multiple access transmissions schemes.

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