Analysis of the Least-Squares Adaptive Algorithms in Interference Cancellation Configuration

Silviu Ciochină, Constantin Paleologu, and Andrei Alexandru Enescu
Department of Telecommunications
University Politehnica of Bucharest
1-3 Iuliu Maniu Blvd., Bucharest, Romania
{silviu, pale, aenescu}@comm.pub.ro

***Abstract*— An analysis of the interference cancellation configuration working based on the least-squares (LS) adaptive algorithms is presented in this paper. The existence of a "residual leakage" phenomenon of the useful signal, to the output of the adaptive filter, through the error signal is demonstrated. As a consequence, the useful signal can be attenuated, up to complete cancellation. This process is important for low values of the weighting parameter λ and is practically absent for λ close to unit. The simulations performed in an echo cancellation configuration support the theoretical findings.**

Keywords-adaptive filters, least-squares (LS) algorithms, interference cancellation, echo cancellation

I. INTRODUCTION

A lot of applications in the telecommunications field require cancelling an unknown interference that corrupts a useful signal. Such a problem can be solved using an adaptive filter working in an interference cancellation configuration. The goal of this system is to produce an estimate of the interference that will be subtracted from the received signal [1], [2]. One of the most common applications of this configuration type is echo cancellation [3], [4]. In this case, the principle is to synthesize a replica of the echo and to subtract it from the returned signal.

Besides convergence rate, an important aspect of an echo canceller is its performance during "double-talk" (i.e., near-end speech) [3], [4]. In the case of the normalized least-mean-square (NLMS) algorithm [1], the presence of near-end signal considerably disturbs the adaptive process. To eliminate the divergence of echo cancellers the standard procedure is to inhibit the weight updating during the double-talk. The presence of double-talk is detected by a double-talk detector (DTD). The DTD acts with a delay since it requires a number of samples to detect the double-talk presence. However, this very small delay can be enough to generate a considerable perturbation of the echo estimate.

Therefore, it will be desirable to implement fast converging and double-talk robust adaptive algorithms [5] in future echo cancellers. Based on convergence performance alone, a least-squares (LS) algorithm [1] is clearly the algorithm of choice.

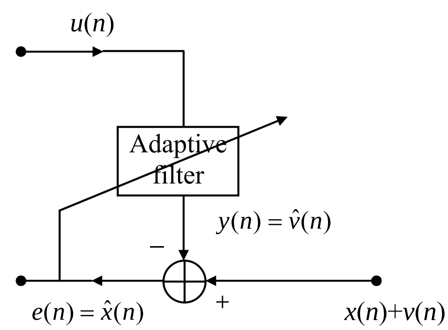

Figure 1. Interference cancellation configuration.

In this paper we will prove the existence of an interesting phenomenon that appears when an LS adaptive algorithm is used in an interference cancellation configuration. The phenomenon consists of a "residual leakage" of the useful signal, through the error signal into the output of the adaptive filter, even if it is uncorrelated with the input signal of the adaptive filter. The process depends on the algorithm weighting parameter λ. As a consequence, not only the perturbation (i.e., the echo), but also the useful signal can be severely attenuated, up to complete cancellation. Controlling this phenomenon in case of an echo canceller could solve the "double-talk" problem without any DTD system.

The paper is organized as follows. In Section II we briefly review the adaptive interference cancellation configuration, working in ideal conditions. In Section III, the real behavior of the configuration is considered, working based on an LS algorithm. Replacing the statistical averages by temporal estimators proves to lead to the "residual leakage" phenomenon, in the case of the low memory algorithms (i.e., using small λ). Simulations performed in the context of echo cancellation are presented in Section IV. Finally, Section V concludes this work.

II. IDEAL BEHAVIOR OF THE ADAPTIVE INTERFERENCE CANCELLATION CONFIGURATION

In the case of the interference cancellation applications (Fig. 1), an adaptive filter is used to cancel an unknown interference, $v(n)$, that corrupts a useful signal, $x(n)$.

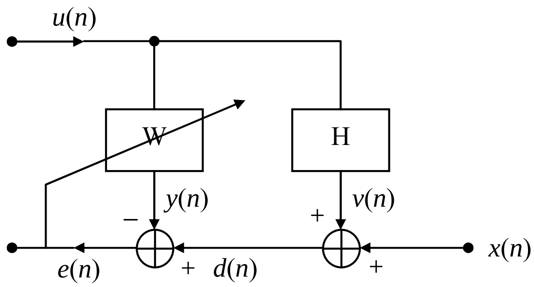

Figure 2. Echo cancellation configuration.

This scheme needs two inputs. A so-called primary signal consists of the corrupted signal, $x(n)$ + $v(n)$, and plays the role of the "desired signal" in the adaptive configuration. The second one, $u(n)$, is correlated with the perturbation $v(n)$ and uncorrelated with $x(n)$. It is applied to the input of the adaptive filter. In addition, $x(n)$ and $v(n)$ are mutually uncorrelated. The output of the adaptive filter $y(n)$ is expected to be an estimate of $v(n)$ and, consequently, the error signal $e(n)$ should be an estimate of $x(n)$.

A specific application of this configuration is echo cancellation [3], [4]. In this case (Fig. 2), $v(n)$ is the echo generated by a system characterized by the impulse response

$$\mathbf{h} = [h_0, h_1, \ldots, h_{N-1}]^H , \tag{1}$$

where the superscript H denotes Hermitian transposition (transposition and complex conjugation) and W is an adaptive filter, having the coefficients

$$\mathbf{w} = [w_0, w_1, \ldots, w_{N-1}]^H . \tag{2}$$

The sequence $u(n)$ is the far-end signal and $x(n)$ is the near-end. We suppose that $x(n)$ and $u(n)$ are uncorrelated, i.e.,

$$\mathrm{E}\left\{u(n)x^*(n-k)\right\} = 0 , \forall \quad k \in \mathrm{Z} , \tag{3}$$

where E is the expectation operator and superscript * denotes the complex conjugation.

Defining the vector:

$$\mathbf{u}(n) = [u(n), u(n-1), \ldots, u(n-N+1)]^T , \tag{4}$$

where superscript T denotes the transposition operation, we have:

$$v(n) = \mathbf{h}^H \mathbf{u}(n) , \tag{5}$$

and

$$e(n) = v(n) - y(n) + x(n) . \tag{6}$$

In case of the Wiener filter the optimal coefficients are given by normal equation [1]:

$$\mathbf{R}\mathbf{w}_{opt} = \mathbf{p} , \tag{7}$$

with

$$\mathbf{R} = E\left\{\mathbf{u}(n)\mathbf{u}^H(n)\right\} \tag{8}$$

and

$$\mathbf{p} = E\left\{\mathbf{u}(n)d^*(n)\right\} . \tag{9}$$

Taking into account the relations (3) and (6) we obtain:

$$\begin{aligned} \mathbf{p} &= E\left\{\mathbf{u}(n)d^*(n)\right\} = E\left\{\mathbf{u}(n)\left(x^*(n)+v^*(n)\right)\right\} = \\ &= E\left\{\mathbf{u}(n)x^*(n)\right\} + E\left\{\mathbf{u}(n)v^*(n)\right\} = \\ &= E\left\{\mathbf{u}(n)v^*(n)\right\} = E\left\{\mathbf{u}(n)\mathbf{u}^H(n)\mathbf{h}\right\} = \mathbf{R}\mathbf{h} \end{aligned} \tag{10}$$

so that, according to (7) and (10), the value of optimal coefficients results as

$$\mathbf{w}_{opt} = \mathbf{h} . \tag{11}$$

In this case,

$$y(n) = \mathbf{w}_{opt}^H \mathbf{u}(n) = \mathbf{h}^H \mathbf{u}(n) = v(n) \quad \Rightarrow \quad e(n) = x(n) , \tag{12}$$

so that the separation of signals $x(n)$ and $v(n)$ is correctly performed.

III. The Residual Leakage Phenomenon

Let us consider the real case of the classical recursive least-squares (RLS) adaptive algorithm [1]. In this situation, the statistical expectation is replaced by a weighted sum as follows:

$$E\{\bullet\} \rightarrow \sum_{i=1}^{n} \lambda^{n-i} \{\bullet\} , \tag{13}$$

where λ is the exponential weighting factor (also known as the "forgetting factor") of the RLS algorithm. Consequently, the normal equation from (7) becomes

$$\begin{aligned} \sum_{i=1}^{n} \lambda^{n-i}\mathbf{u}(i)\mathbf{u}^H(i)\mathbf{w}_{opt} &= \sum_{i=1}^{n} \lambda^{n-i}\mathbf{u}(i)(v^*(i)+x^*(i)) = \\ &= \sum_{i=1}^{n} \lambda^{n-i}\mathbf{u}(i)v^*(i) + \sum_{i=1}^{n} \lambda^{n-i}\mathbf{u}(i)x^*(i). \end{aligned} \tag{14}$$

For values of the exponential weighting factor λ very close to unit and for a value of n high enough we may write

 ISBN: 978-1-61208-127-4

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\lambda^{n-i}\mathbf{u}(i)x^*(i)\cong E\left\{\mathbf{u}(n)x^*(n)\right\}=\mathbf{0}\quad\Rightarrow$$

$$\sum_{i=1}^{n}\lambda^{n-i}\mathbf{u}(i)\mathbf{u}^H(i)\mathbf{w}_{opt}\cong\sum_{i=1}^{n}\lambda^{n-i}\mathbf{u}(i)v^*(i)=\sum_{i=1}^{n}\lambda^{n-i}\mathbf{u}(i)\mathbf{u}^H(i)\mathbf{h}$$

$$\Rightarrow\quad \mathbf{w}_{opt}\cong\mathbf{h} \tag{15}$$

so that $e(n) = x(n)$ like in the ideal case.

On the other hand, for a value of the exponential weighting factor small enough, so that $\lambda^k \ll 1$ for $k > n_0$, we may use the following approximation:

$$\sum_{i=1}^{n}\lambda^{n-i}\{\bullet\}\cong\sum_{i=n-n_0+1}^{n}\lambda^{n-i}\{\bullet\}. \tag{16}$$

According to the orthogonally principle, the normal equation becomes

$$\sum_{i=n-n_0+1}^{n}\lambda^{n-i}\mathbf{u}(i)e^*(i)=\mathbf{0}. \tag{17}$$

This is a homogeneous set of N equations with n_0 unknown parameters, $e(i)$. If $n_0 < N$ then the set of equations has the unique solution

$$e(i)=0 \quad \text{for} \quad i=n-n_0+1,\ldots,n \tag{18}$$

leading to

$$y(n)=\mathbf{w}^H(n)\mathbf{u}(n)=x(n)+v(n). \tag{19}$$

Consequently, there is a "residual leakage" of $x(n)$, through the error signal into the output of the adaptive filter. In this situation the useful signal, $x(n)$ is suppressed together with the perturbation $v(n)$. A small value of λ or a high value of N intensifies this phenomenon.

Concluding, in the real case of any LS adaptive algorithm used in an interference cancellation configuration two effects appear:

- $\mathbf{w}$ differs from $\mathbf{h}$ in a certain extent and this may be viewed as a divergence of the algorithm;
- $y(n)$ will contain a component proportional to $x(n)$, that will be subtracted from the total received signal; this phenomenon is in fact a leakage of the $x(n)$ in $y(n)$, through the error signal $e(n)$; the result consists of an unwanted attenuation of the desired signal.

If we consider an echo cancellation application where the echo path is significantly long, leading to a high value for the length of the adaptive filter, we have to use a value of λ very close to unit in order to reduce unwanted attenuation of the near-end signal.

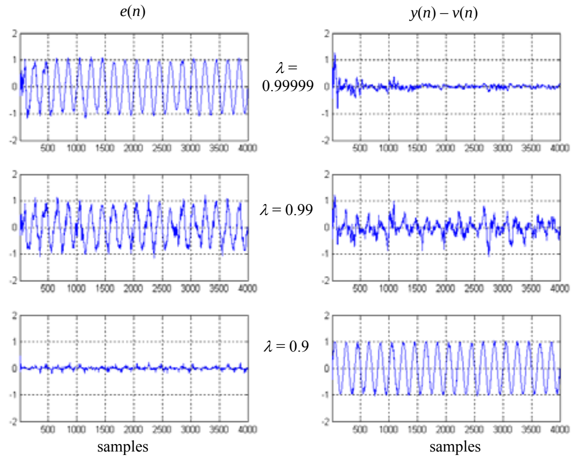

Figure 3. The filter length is $N = 32$ and different values of λ are used.

However, the requirements of an echo canceller are both rapid convergence and a low computational cost. Thus, a highly desirable algorithm is a "low cost" LS algorithm (in terms of the computational complexity), i.e., the fast LS algorithms [1].

IV. Simulation Results

For the first set of simulations we analyze a simple interference cancellation application. Using the configuration from Fig. 2 we choose $u(n)$ uniformly distributed in [–1;1] and $x(n)$ as a low frequency sine wave ($\omega_0 = 0.01\pi$). The adaptive filter has N coefficients. In the first experiment we fixed $N = 32$ and we take different values for the exponential weighting factor λ (Fig. 3).

In a second case we used a fixed weighting factor λ = 0.99 and different values for the length of the adaptive filter N (Fig. 4). In order to outline the "residual leakage" phenomenon we plot the signals $e(n)$ (the recovered sine wave) and the difference $y(n)-v(n)$ [the component from $x(n)$ leaked into the output of the adaptive filter].

We can notice that similar effects appear when the value of the exponential weighting factor λ decreases or the length of the adaptive filter N increases. The desired signal $x(n)$ leaks into the output of the adaptive filter, leading to an unwanted attenuation of the recovered signal $e(n)$.

In order to observe the phenomenon in the spectral domain we repeat the previous experiments using a higher frequency sine wave ($\omega_0 = 0.5\pi$) as desired signal. The corresponding spectral results are given in Fig. 5 and Fig. 6. The conclusions are practically the same. The recovered sine wave is strongly attenuated when the value of λ decreases. A similar effect appears when the length of the adaptive filter N increases.

In order to approach the context of echo cancellation, a second set of simulations was performed using speech sequences for both $u(n)$ and $x(n)$ signals.

Copyright (c) IARIA, 2011. ISBN: 978-1-61208-127-4

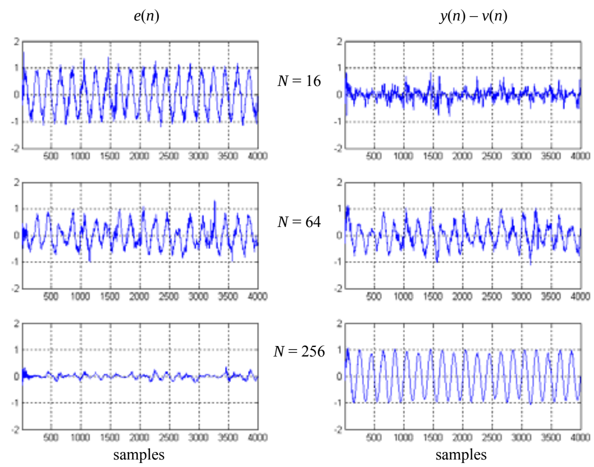

Figure 4. The exponential weighting factor is $\lambda = 0.99$ and different values of N are used.

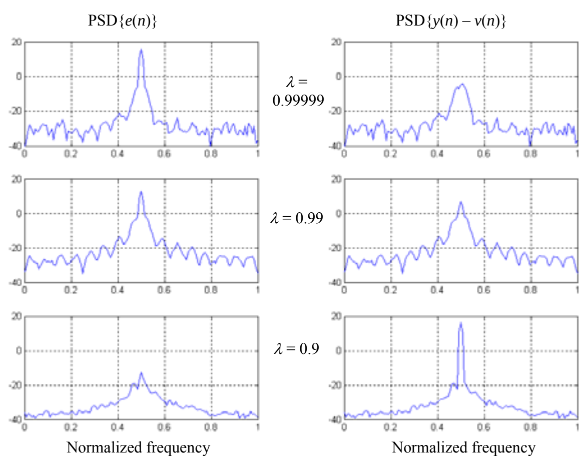

Figure 5. Power spectra [dB]; the filter length is $N = 32$ and different values of λ are used.

Moreover, the H filter is a particular echo path according with ITU-T G.168 Recommendation for digital echo cancellers [6]. It is convenient to subtract out the direct near-end component from the error signal $e(n)$ [7]. The residual error $r(n) = e(n) - x(n)$ cumulates the undesired attenuation of the near-end signal $x(n)$ and the imperfect rejection of the echo path response $v(n)$. In a real application such a subtraction can never be done because the signal $x(n)$ is not available.

In the first experiment we choose a value of the exponential weighting factor very close to unit ($\lambda = 0.99999$) and a 32 msec. echo path (corresponding to $N = 256$). One can see that the near-end signal $x(n)$ is recovered in $e(n)$ with slight distortions (Fig. 7). Next, in order to point out the leakage process, we decrease the value of λ (Fig. 8). Because of the lower value of λ the "residual leakage" phenomenon is significant. The adaptive filter rejects not only the far-end signal but also the near-end signal. The transmitted signal remains in the absence of the far-end signal (where there is no input signal for the adaptive filter).

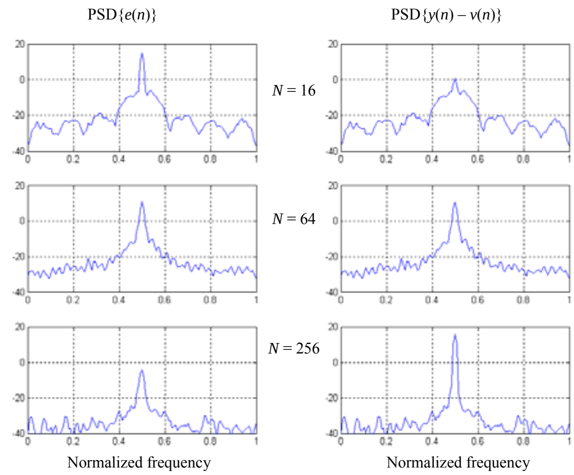

Figure 6. Power spectra [dB]; the exponential weighting factor is $\lambda = 0.99$ and different values of N are used.

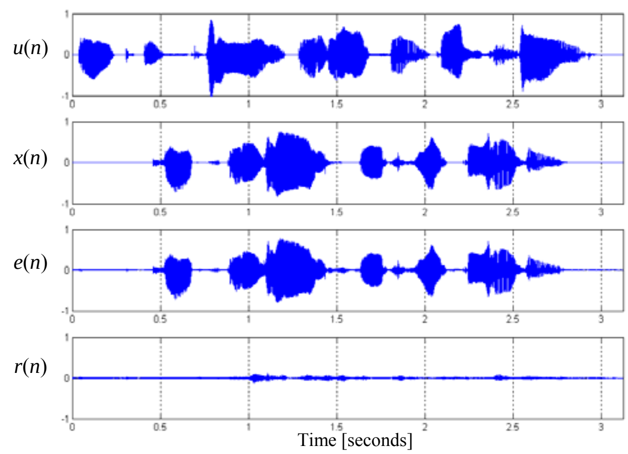

Figure 7. Double-talk situation. The exponential weighting factor is $\lambda = 0.99999$ and the filter length is $N = 256$.

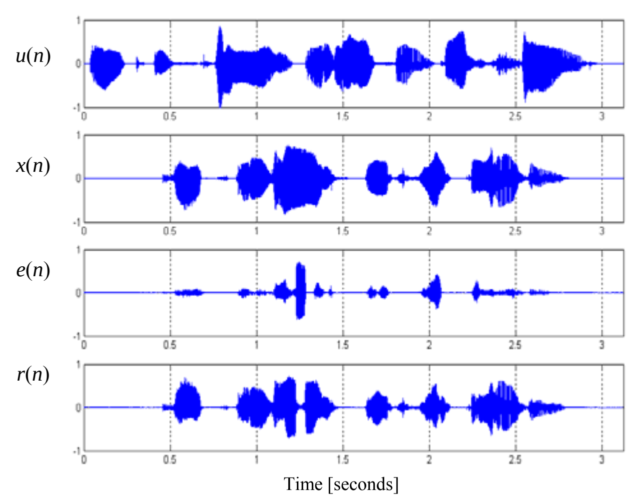

Figure 8. Double-talk situation. The exponential weighting factor is $\lambda = 0.99$ and the filter length is $N = 256$.

 ISBN: 978-1-61208-127-4

V. Conclusions

In this paper we have demonstrated the existence of a phenomenon that appears when a LS adaptive algorithm is used in an interference cancellation configuration. It consists of a “residual leakage” of the useful signal, through the error signal into the output of the adaptive filter that leads to an unwanted attenuation of the useful signal. This process strongly depends on the algorithm weighting parameter λ (it is important for low λ and is practically absent for $\lambda \cong 1$) and it is influenced by the length of the adaptive filter N (it is amplified by a high value of N).

Both the theoretical and experimental developments lead to the conclusion that the “residual leakage” phenomenon can be avoided for a memory of the algorithm significantly higher than the adaptive filter length.

Controlling this phenomenon in case of an echo canceller we can solve the “double-talk” problem without any DTD system. Taking into account that the length of an echo path is in general high enough, we have to use a value of λ very close to unit in order to reduce unwanted attenuation of the near-end signal.

Acknowledgment

This work was supported by the UEFISCSU Romania under Grant PN-II-RU-TE no. 7 / 05.08.2010 and Grant PN-II-RU-PD no. 39 / 12.08.2010.

References

[1] S. Haykin, *Adaptive Filter Theory*, Fourth Edition, Prentice Hall International, Inc., Upper Saddle River, New Jersey, 2002.

[2] J. Benesty and Y Huang, Eds., *Adaptive Signal Processing–Applications to Real-World Problems.* Berlin, Germany: Springer-Verlag, 2003.

[3] S. L. Gay and J. Benesty, Eds., *Acoustic Signal Processing for Telecommunication*. Boston, MA: Kluwer Academic Publisher, 2000.

[4] J. Benesty, T. Gaensler, D. R. Morgan, M. M. Sondhi, and S. L. Gay, *Advances in Network and Acoustic Echo Cancellation*. Berlin, Germany: Springer-Verlag, 2001.

[5] T. Gänsler, St. L. Gay, M. M. Sondhi, J. Benesty, “Double-Talk Robust Fast Converging Algorithms for Network Echo Cancellation”, *IEEE Trans. Speech and Audio Processing*, vol. 8, no. 6, pp. 656-663, Nov. 2000.

[6] *Digital Network Echo Cancellers*, ITU-T Rec. G.168, 2002.

[7] D.L. Duttweiler, “Proportionate Normalized Least-Mean-Squares Adaptation in Echo Cancelers,” *IEEE Trans. Speech and Audio Processing*, vol. 8, no. 5, pp. 508-518, Sept. 2000.

 ISBN: 978-1-61208-127-4