A Novel Tap Selection Design for Filters in Unequal-Passbands Scheme

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***Abstract*—A filter bank divides the input signal into *L* bands having different passbands called unequal-passbands-filter bank. This type of filter bank can be obtained in various ways, for example, from an equal-passbands-bank in which the signals of each band are combined to produce new unequal-passbands. Recent results showed that the unequal-passbands scheme has superior performance over the equal-passbands scheme. In this paper, a new method showing a decrease in the number of taps in the separation stage of the blind source separation system is presented. Decreasing number of taps is necessary to decrease the complexity cost. The simulation results prove that the proposed technique improves the convergence using filter bank in octaves with decomposition which was observed for colored input that has low-pass characteristics.**

Keywords- Filter bank; multiband; convergence; adaptive filters; taps.

I. INTRODUCTION

In recent years, some schemes for adaptive filtering were presented with the aim to accelerate the convergence to input signals correlated over time (color signals). In some cases, the aim was to reduce the computational cost, promoting the coefficients of the adaptive filters whose sampling rate is below that of the input signal. However, these schemes have an input-output delay and spectrum overlap between the various bands that should be reduced in advance promote adaptation of the filters [1]. Marelli and Minyue [2] proposed a scheme with maximum decimation able to make almost an exact modeling of the Finite Impulse Response (FIR) systems, through the insertion of cross filters and considering that there is spectrum overlapping between adjacent bands. In this case, both the input signal and the desired signal was decomposed into multiple bands, and the error generated in each band was used to update the respective adaptive filters (direct and crossed) related to the band.

Papoulis and Stathaki [3] proposed two schemes of non-maximally decimated ($F < L$) filter banks. As the effect of the overlapping spectrum is directly proportional to the decimation factor, the lower the value of F the smaller the minimum mean square error of the scheme. For fixed values of L and F, one can obtain optimum filter bank that minimizes the mean square error of the final scheme. The difference between the two proposed schemes is that the desired signal is decomposed into multiple bands, while the other one the final error of the scheme is decomposed.

Two other schemes have been proposed by Lian and Wei [4] and Brown [5]. Papoulis and Stathaki [3] use analysis bank without decimation, followed by adaptive filters of nonzero coefficients, whereas Brown's algorithm [4], which was derived from the first, uses a maximally decimated filter bank with perfect reconstruction and adaptive filters operate at reduced rate.

New research presented by McCloud and Etter [6], and Kim and Choi [7], showed that the error in the scheme is decreased in adaptive filters with *unequal-passbands* in analogy to the *equal-passbands*. The *unequal-passbands*-schemes presented in [6] employ noncritical decimation of the multi-band signals. In this work an *unequal-passbands*-scheme with maximally decimated random bands is proposed. The contribution of this paper is derivation of the unequal-passbands maximally decimated scheme from unequal-passbands scheme without decimation, which employs analysis bank and filters with nonzero-coefficients that are used to construct an equivalent FIR system. The rest of the paper is organized as follows: adaptive filter scheme without decimation is discussed in Section II, maximally decimated scheme with unequal-passbands and the extraction of the total number of taps used in the proposed scheme is presented in Section III, simulation results is discussed in Section IV and finally, the paper is concluded in Section V.

II. ADAPTIVE SCHEMES WITHOUT DECIMATION

The adaptive scheme is shown in Figure 1 and uses filters with nonzero coefficients that are capable of modeling only a particular class of FIR systems and cannot be generalized for all FIR systems because the length of analysis filters is greater than the number of adaptive coefficients. However, Apolibario and Alves [8] show that by a suitable selection of the filter, better parameters can be obtained to model the FIR system. We propose in Figure 1 a scheme that can model any FIR system but will include some delay.

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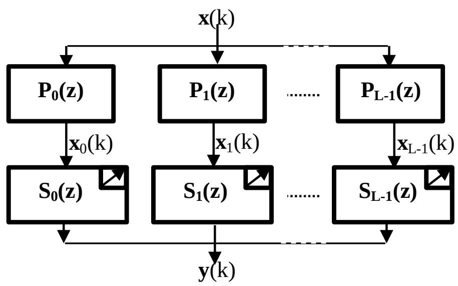

Figure 1. Scheme explains the use of adaptive filters.

Considering the analysis polyphase bank representation of the scheme in Figure 1, the polyphase matrix of dimension $L \times L$ is defined as [9]:

$$\mathbf{P}_m(z)=\begin{bmatrix} P_{0,0}(z) & P_{0,1}(z) & \cdots & P_{0,L-1}(z) \\ P_{1,0}(z) & P_{1,1}(z) & \cdots & P_{1,L-1}(z) \\ \vdots & \vdots & \ddots & \vdots \\ P_{L-1,0}(z) & P_{L-1,1}(z) & \cdots & P_{L-1,L-1}(z) \end{bmatrix} \tag{1}$$

where $P_{r,c}(z)$ are polyphase components of the r^{th} analysis filter $P_r(z)=\sum_{k=0}^{K_p-1} P_r(k)z^{-k}$, given by

$$P_{r,c}(z) = p_r(c)\sum_{k=1}^{K_p-1} p_r(kL+c)z^{-k} \tag{2}$$

where K_p is the length of the analysis filters.

Therefore, the system function used in Figure 1 can be expressed as:

$$P(z)=\left[S_0(z)S_1(z)\cdots S_{L-1}(z)\right]\mathbf{P}_m(z)\left(\begin{bmatrix}1 & z & \cdots & z^{L-1}\end{bmatrix}^T\right)^{-1}. \tag{3}$$

The taps of the filters of nonzero coefficients $S_r(z)$ are changed to give us the equivalent FIR scheme, which will be called $U(z)$. The decomposition of the polyphase transfer function of the unknown system is given by

$$U(z)=\left[U_0(z)U_1(z)\cdots U_{L-1}(z)\right]\left(\begin{bmatrix}1 & z & \cdots & z^{L-1}\end{bmatrix}^T\right)^{-1}. \tag{4}$$

From Equations (3) and (4), it can be seen that the scheme accurately models an unknown FIR system when

$$\left[S_0(z)S_1(z)\cdots S_{L-1}(z)\right]\mathbf{P}_m(z)=\left[U_0(z)U_1(z)\cdots U_{L-1}(z)\right]. \tag{5}$$

Equation (5) shows that the equality cannot be achieved as the length of the adaptive filters of nonzero coefficients is $L\kappa$ and the length of the analysis filters is K_p, while the product $S_r(z)P_r(z)$ has length $K_p + L_k - 1$, which is greater than the number of coefficients $L\kappa$ that was adapted. However, if

$$\left[S_0(z)S_1(z)\cdots S_{L-1}(z)\right]=\left[U_0(z)U_1(z)\cdots U_{L-1}(z)\right]\mathbf{Q}_m(z) \tag{6}$$

such that $\mathbf{Q}_m(z)\mathbf{P}_m(z) = \mathbf{I}$, where $\mathbf{I}$ is the unit matrix of dimension $L \times L$ with delay, the system function in Figure 1 will be

$$P(z)=U(z). \tag{7}$$

but with delayed $U(z)$. The matrices $\mathbf{P}_m(z)$ and $\mathbf{Q}_m(z)$ that satisfy the above conditions are, respectively, the polyphase matrix of the analysis and synthesis filter bank with perfect reconstruction. The synthesis polyphase bank matrix is defined as

$$\mathbf{Q}_m(z)=\begin{bmatrix} Q_{0,0}(z) & Q_{1,0}(z) & \cdots & Q_{L-1,0}(z) \\ Q_{0,1}(z) & Q_{1,1}(z) & \cdots & Q_{L-1,1}(z) \\ \vdots & \vdots & \ddots & \vdots \\ Q_{0,L-1}(z) & Q_{1,L-1}(z) & \cdots & Q_{L-1,L-1}(z) \end{bmatrix} \tag{8}$$

where $Q_{r,c}(z)$ are polyphase components of the r^{th} synthesis filter $Q_r(z)=\sum_{k=0}^{K_q-1} q_r(k)z^{-k}$ is given by

$$\mathbf{Q}_{r,c}(z)=q_r\left(L-(c+1)\right)\sum_{k=1}^{K_q-1} q_r\left(kL-c+L-1\right)z^{-k} \tag{9}$$

where K_q is the length of the synthesis filters.

Then, using an analysis filter bank, which allows perfect reconstruction and adaptive filters of nonzero coefficients with sufficient order to satisfy (6), the scheme of Figure 1, now can implement exactly the FIR system with transfer function given in equation (7). However, it should be emphasized that the delay introduced by the filter bank needs to be considered in the adaptation algorithm of the filters coefficients.

For lengths K_{un} and K_{pr} of the unknown prototype systems, respectively, the number of nonzero coefficients K must be at least:

$$\kappa = K_{un} + K_{pr} - 1. \tag{10}$$

III. Maximally Decimated Scheme with Unequal-Passbands

An analysis filter bank of unequal-passbands can be configured from the adaptive scheme of unequal-passbands shown in Figure 1, but employs analysis filters bank with unequal-passbands. This scheme is shown in Figure 2. The input signal $x(k)$, and $P_i(z)$ indicates the analysis of unequal-passbands with L-bands, the adaptive filters of nonzero coefficients will be denoted as $S_i(z)$, the required signal will be $\mathfrak{d}(k)$, where the error signal is $e(k)$.

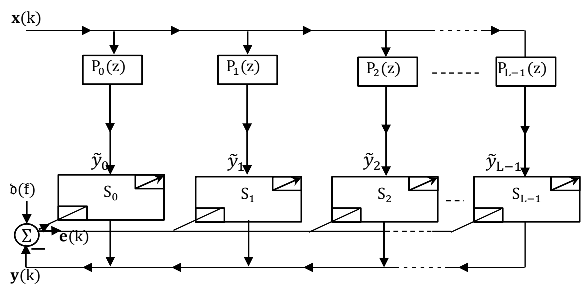

Figure 2. Adaptive Scheme of unequal-passbands without decimation.

The perfect reconstruction analysis filter bank of L-bands has orders

$$K_{P_0} = \sum_{c=0}^{L-1} 2^c K_{P^{0,c}} \; , \; K_{P_i} = \sum_{c=0}^{L-i} 2^c K_{P^{0,c}} + 2^{L-i-1} K_{P^{1,c}} \qquad (11)$$

Where $K_p^{0,c}$ are orders of $P^{0,c}(z)$ and $K_p^{1,c}$ are orders of $P^{1,c}(z)$.

The filters of unequal-passbands and with perfect reconstruction analysis $P_i(z)$ and synthesis filters $Q_i(z)$ are included after each of the sub-adaptive filter in Figure 2.

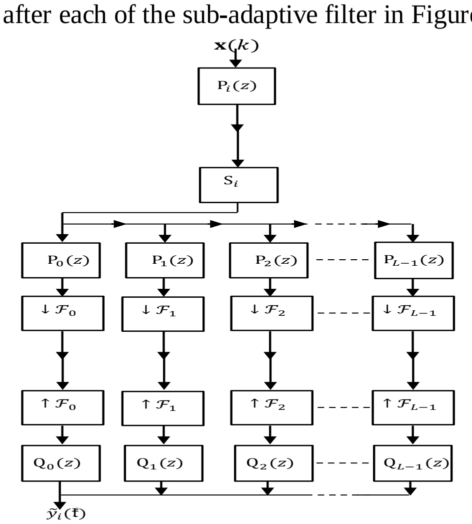

Figure 3. i^{th} band after implying the maximally sampled filter bank.

Figure 3 shows the i^{th} band of the resulting scheme, which allows the filters to operate at a lower sampling rate.

To obtain a scheme with less complexity we consider the analysis filters are sufficiently selective to accept spectrum interference only in frequency responses of neighboring bands. The i^{th} band that is shown in Figure 4, we see that $P_{r,c}(z) = P_r(z)P_c(z)$ are the filters of nonzero coefficients $S_i(z)$ shifted forward by F_i [10]. Looking at the i^{th} band of the simplified scheme shown in Figure 4, the sampling rate of the adaptive filters is F_i and F_{i+1} times less than the rate of the input signal.

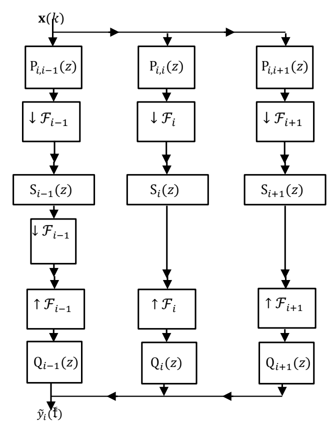

Figure 4. Adaptive filters work at lower rates.

The scheme can be further simplified by noting that $P_{r,c}(z) = P_{c,r}(z)$ and combining the signals in adjacent bands.

A. Taps Selection

As mentioned in section III, about a good design of analysis filters to avoid the spectrum overlap, the parameters of Figure 4 are similar to the parameters of Figure 2. From this hypothesis an equation will be extracted assuming the case of modeling a random FIR system.

The adaptive filters $S_i(z)$ of each band of Figure 2 are described by shifting $\chi_i = 1/F_i$ [11].

Defining $S_i(z)$ as follows:

$$\mathbf{S}_i(z) = \left[S_{i,0}(z) S_{i,1}(z) \cdots S_{i,\chi_{i-1}}(z)\right]^T \qquad (12)$$

and $S(z)$ is constructed by L filters $S_i(z)$, given by

$$\tilde{\mathbf{S}}(z) = \left[\tilde{S}_0(z)\tilde{S}_1(z)\cdots\tilde{S}_{\chi_0-1}(z)\right]^T = \left[S^T_{0}(z)S^T_{1}(z)\cdots S^T_{L-1}(z)\right]^T . \qquad (13)$$

$\mathbf{P}_m(z)$ is the matrix of dimension F_0 x F_0 contains analysis polyphase filters components - type I, given by

$$\mathbf{P_m}(z) = \left[\mathbf{P}^{\mathbf{T}}_{0}(z)\mathbf{P}^{\mathbf{T}}_{1}(z)\cdots\mathbf{P}^{\mathbf{T}}_{L-1}(z)\right]^T \qquad (14)$$

where $\mathbf{P}_i(z)$ is the matrix $\chi_i \mathbf{x} F_i$ with the r^{th} row (r=0,. χ_i).

The system function applied for the scheme of unequal-passbands in Figure 2 can be expressed as:

$$\mathscr{T}(z) = \breve{S}^T(z)\mathbf{P}_m(z)\left[1 \quad z^{-1} \quad \cdots \quad z^{-\mathcal{F}_0}\right]^T \tag{15}$$

To identify the unknown system, taps of $S_i(z)$ can be adjusted to match the required FIR scheme. The system function of the unknown system is denoted by $U(z)$ and written as

$$U(z) = \left[U_0(z) \;\; U_1(z) \cdots U_{L_0-1}(z)\right]\left[1 \quad z^{-1} \quad \cdots \quad z^{-\mathcal{F}_0}\right]^T \tag{16}$$

From equations (15) and (16), the multiband scheme accurately matches a FIR filter $U(z)$ at

$$U(z) = \left[U_0(z) \;\; U_1(z) \cdots U_{L_0-1}(z)\right]\left[1 \quad z^{-1} \quad \cdots \quad z^{-\mathcal{F}_0}\right]^T. \tag{17}$$

and

$$\tilde{\mathbf{S}}^T(z)\mathbf{P}_m(z) = \left[U_0(z) \;\; U_1(z) \quad \cdots \quad U_{\mathcal{F}_0}(z)\right]. \tag{18}$$

Multiplying both sides of equation (18) by the matrix $Q_m(z)$:

$$\mathbf{P}_m(z)\mathbf{Q}_m(z) = \mathbf{I}, \tag{19}$$

where I is the unit matrix with delays, and its dimension is F_0 x F_0, and

$$\tilde{\mathbf{S}}^T(z) = \left[U_0(z) \;\; U_1(z) \quad \cdots \quad U_{\mathcal{F}_0}(z)\right]\mathbf{Q}_m(z) \tag{20}$$

with delays. The matrix $Q_m(z)$ satisfying (19) corresponds to the synthesis polyphase filters matrix which results in a system with perfect reconstruction [12].

The matrix $\mathbf{Q}_m(z)$ is of dimension F_0 x F_0 containing components of the expanded synthesis polyphase filters, given by

$$\mathbf{Q}_m(z) = \left[\mathbf{Q}_0(z) \;\; \mathbf{Q}_1(z) \quad \cdots \quad \mathbf{Q}_{L-1}(z)\right] \tag{21}$$

where $\mathbf{Q}_i(z)$ is an $F_0 \mathbf{x} \chi_i$ matrix with the r^{th} column (r = 0, · · ·, χ_i).

The parameters of the i^{th} S-band filter $\breve{S}_i(z)$, assuming the existence of overlapping spectrum only between adjacent bands, are given by

$$\breve{S}_i(z) = \sum_{r=0}^{\chi_i} z^{-r} S_{i,r}(z) \tag{22}$$

where filters $S_{i,r}(z)$ are related to $\breve{S}_i(z)$ through equations (12) and (13).

According to equations (20) and (22) for a scheme of unequal-passbands with L-band synthesis filters with $K_{Q_i}(z)$, we can write

$$K_{S_i} = K_U + K_{Q_i} \tag{23}$$

where K_{Si} is the minimum number of taps for the filters and K_U is the required system order.

Then, using a filter bank, which allows perfect reconstruction of unequal-passbands and adaptive filters of nonzero coefficients with sufficient orders that satisfies equation (23). However, it should be emphasized that the delay introduced by the filter bank should be considered in the adaptation algorithm.

IV. SIMULATION RESULTS

A random signal with normal distribution was applied on an IIR filter with z = 0.73. A noise of 10^{-7} is used and a system of order K_U = 900 is considered. The decomposition was in octaves with L = 1, 2, 3, 4. Table I shows the parameter of the unequal-passbands scheme the downsampling parameters F_i , and the orders of the analysis filters χ_i , respectively and L = 4 bands. Figure 5 shows the frequency responses of the corresponding analysis filters.

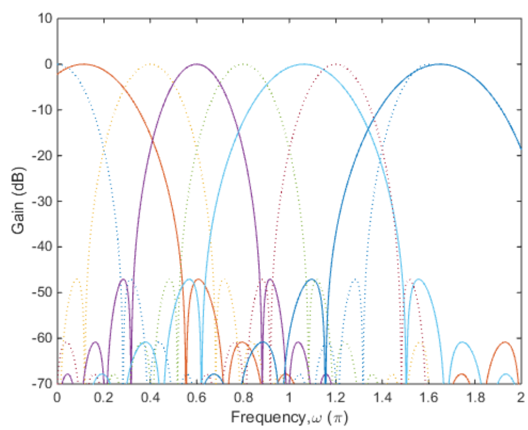

Figure 5. Frequency response of analysis filters

Figure 6 reflects the significant enhancement in the convergence rate of the proposed algorithm that can be obtained for colored input signals by increasing the number of bands in the multiband algorithm. In this research work, four bands for colored input were used that is enough to decorrelate their samples.

TABLE I. PARAMETERS OF THE UNEQUAL-PASSBANDS SCHEME

i	0	1	2
F_i	8	4	4
χ_i	1	2	2
K_{P_i}	332	332	155
K_{S_i}	149	149	256

Next, we compare the performance of mean square error considering different adaptive schemes with L = 4 bands. Figure 7 shows the performance of the mean square error for the proposed critical decimation scheme with unequal-passbands and the subsampled scheme with unequal-passbands. It can be seen that the suggested scheme with unequal-passbands offers faster convergence speed compared to the Ichikawa and Furukawa approach [13].

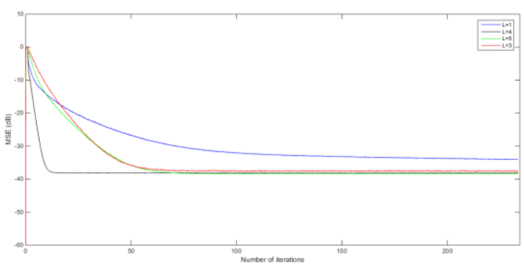

Figure 6. Performance of the mean square error of the scheme with unequal-passbands.

To decrease the problem of slow convergence in wider bands, we use a subsampled scheme.

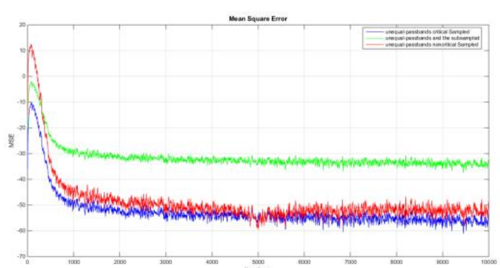

Figure 7. The mean square error with L = 4

The reason behind improving the convergence in the scheme with unequal-passbands in contrast to the one with equal-passbands is because of breaking down the input into narrower bands at the smaller frequencies that causes a lower rate between the largest and smallest powers [9].

V. CONCLUSION

In this paper, an unequal-passbands scheme was proposed and wider-band analysis filters were used. An equation is derived by modeling a random FIR system. This system is constructed from filters of nonzero coefficients that are used to design the equivalent FIR scheme. A perfect reconstruction is used by the help of the analysis filter bank. This bank allows us to obtain a scheme with less complexity by considering sufficiently selective analysis filters. By reducing the taps the computational cost is reduced. The results showed that the suggested method speeds up the convergence rate.

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