Analyzing the Optimum Switching Points for Adaptive FEC in Wireless Networks with Rayleigh Fading

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***Abstract*—Adaptive techniques have an important role in modern wireless communications networks, like cognitive radio networks. Adaptive modulation and adaptive Forward Error Correction (FEC) are two very important approaches used to improve the performance of the wireless networks. In an adaptive technique, a key factor to maximize the performance of the system is the optimum switching point between neighboring modulations, in an adaptive modulation system, or codes, in an adaptive FEC system. In this paper, we analyze the optimum switching points for adaptive FEC in a wireless network with Rayleigh fading. We compute the optimum switching points considering three criteria: maximum throughput, maximum packet error rate target and delay to transmit a correct packet. We show that the optimum switching points depend on several parameters, including the channel model and the selected criterion to define the switching points.**

Keywords—adaptive FEC; performance analysis; optimum switching points.

I. INTRODUCTION

A mandatory issue for the next generation of wireless networks is to improve the performance. Several technologies have been proposed to achieve this goal. One important technology is cognitive radio networks, in which the radio adjusts its parameters as a function of the radio environment in order to achieve the best performance [1].

Adaptive techniques (like adaptive modulation and adaptive error control) are fundamental to implement cognitive radio networks and also for other candidate technologies for the next-generation of the wireless networks. In particular, adaptive Forward Error Correction (FEC) schemes have been proposed to improve Quality of Service (QoS) for several technologies and techniques [2]-[7].

Adaptive FEC schemes vary the number of parity bits and, consequently, the error correction capacity of the error correcting code as a function of the quality in the wireless link.

A key point in implementing an adaptive FEC scheme is to define the optimum switching point from one error correcting code to its neighboring error correcting code. In our analysis, we consider two error correcting codes as neighboring if their error correction capacity differs by one bit. In other words, two error correcting codes are neighboring if one has an error correction capacity equal to t bits and the other one has an error correction capacity equal to $(t + 1)$ or $(t – 1)$ bits.

The optimum switching points for an adaptive FEC system have been analyzed by Brito and Bonatti in [8]. However, those analyses consider a memoryless channel, which may not be appropriate for some wireless links. In a more general case, the wireless channel has memory and the results presented in [8] are imprecise.

The goal of this paper is to analyze the optimum switching points for an adaptive FEC system considering a channel with Rayleigh fading. The criteria used to compute the optimum switching points are the same used in [8]: maximum throughput, for real time traffic, maximum Packet Error Rate (PER) target and, for non-real time traffic, the delay to transmit a correct packet.

To define the switching points it is necessary to adopt a model to calculate the PER in a Rayleigh channel. The model considered in our analysis has been presented by Sharma, Dholakia and Hassan in [9] and is summarized in Section II of this paper.

The remainder of this paper is organized as follows: in Section II, we summarize the mathematical model used to compute the PER in the wireless channel; in Section III, we analyze the optimum switching points that maximize the throughput in the wireless network; in Section IV, we consider the maximum PER target criterion; in Section V, we compute the optimum switching points considering the delay to transmit a correct packet as the QoS parameter; and finally, in Section VI, we present our conclusions.

II. MODEL TO COMPUTE THE PACKET ERROR RATE

In a Rayleigh channel, the errors tend to occur in bursts and the PER can not be computed using the Binomial distribution. Thus, it is necessary to use a model that considers the memory of the channel.

Several papers address the problem of calculating the PER in a channel with burst errors [9]-[12]. In this paper, we use the analytical model presented by Sharma, Dholakia and Hassan in [9]. In this model, the Rayleigh channel is represented by a Gilbert-Elliott Channel (GEC).

A GEC is represented by a discrete time Markov chain with two states: Good (G) and Bad (B). Figure 1 illustrates a GEC with transition probabilities α and β; each state is modeled as a Binary Symmetric Channel (BSC) with bit error probabilities p_g, in G state, and p_b in B state.

The steady state probabilities of the Markov chain illustrated in Figure 1 are given by

$$\pi_g = \frac{\alpha}{\alpha + \beta} \tag{1}$$

$$\pi_b = \frac{\beta}{\alpha + \beta} \tag{2}$$

where π_g and π_b are the steady state probabilities of the good and bad states, respectively.

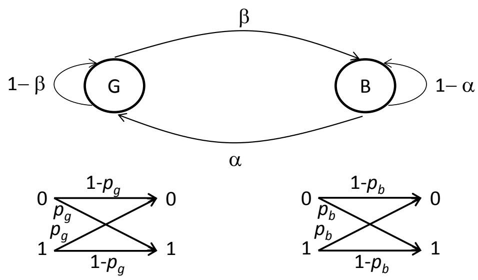

Figure 1. Gilbert-Elliott Channel.

Considering a slow fading channel, with respect to a bit interval, the probability density function of the Signal-to-Noise Ratio (SNR) is given by

$$f(\tau) = \frac{1}{\lambda} e^{\left(\frac{-\tau}{\lambda}\right)} \quad \tau > 0 \tag{3}$$

where λ is the average SNR and τ is the instantaneous SNR.

The status of the channel is defined by a given threshold (ψ): the channel is considered in good state if the SNR is above the threshold and in bad state if the SNR is below the threshold.

The transition probabilities of the GEC can be calculated by [9]:

$$\alpha = \frac{f_d T \sqrt{2\pi\Gamma}}{e^{\Gamma} - 1} \tag{4}$$

$$\beta = f_d T \sqrt{2\pi\Gamma} \tag{5}$$

where T is the symbol interval and f_d is the maximum Doppler speed. The parameter Γ is the ratio between the threshold (ψ) and the average SNR in the wireless channel. Following [9], in our analysis we set the parameter Γ equal to 0.1, meaning that a SNR 10 dB below the average SNR is the condition resulting in the transition from the good to the bad state.

The Bit Error Rate (BER) for each state of the channel is computed by (6) and (7) [9]:

$$p_g = \frac{\int_{\psi}^{\infty} BER(\tau) f(\tau) d\tau}{\int_{\psi}^{\infty} f(\tau) d\tau} \tag{6}$$

$$p_b = \frac{\int_{0}^{\psi} BER(\tau) f(\tau) d\tau}{\int_{0}^{\psi} f(\tau) d\tau} \tag{7}$$

where $BER(\tau)$ is the bit error rate for an Additive White Gaussian Noise (AWGN) channel with SNR equal to τ and $f(\tau)$ is the probability density function of the SNR for Rayleigh fading, given by (3).

After calculating the bit error rate by (6) and (7), we can compute the packet error rate in each state of the channel. In this paper, we are interested in analyzing the optimum switching points of an adaptive FEC system. Thus, we considered that an (*n*,*k*) error correcting block code is used in the wireless link, where *k* is the number of information bits and (*n* – *k*) is the number of parity bits in the block code. Defining the error correction capacity of the code as *t*, the PER in each state can be computed by [9]:

$$P(p_g) = 1 - \sum_{i=0}^{t} \binom{n}{i} (1 - p_g)^{n-i} p_g^{\,i} \tag{8}$$

$$P(p_b) = 1 - \sum_{i=0}^{t} \binom{n}{i} (1 - p_b)^{n-i} p_b^{\,i} \tag{9}$$

where p_g and p_b are the BER in good and bad states, given by (6) and (7), respectively.

Finally, the packet error rate in the Rayleigh channel is computed by

$$PER = \pi_g P(p_g) + \pi_b P(p_b) \tag{10}$$

where π_G and π_B are given by (1) and (2), respectively, and $P(p_g)$ and $P(p_B)$ are given by (8) and (9), respectively.

III. Optimum Switching Points Based on the Maximum Throughput Criterion

In this section, we analyze the switching points that maximize the throughput in the wireless link. To define the throughput we consider two factors: the overhead due to the error correcting code, expressed by the ratio between the number of information bits, *k*, and the total number of bits in a packet, *n*; and the percentage of packets received without error or containing only correctable errors, computed by 1 – PER. Thus, only packets received without error, after the error-correcting code action, are considered when computing the throughput (some authors refer to this as 'goodput'). As our definition of throughput does not consider the bandwidth of the channel and the modulation used in the wireless link, we refer to this throughput as a normalized throughput. Thus, the normalized throughput in the wireless link is given by:

$$T = \frac{k}{n} \cdot (1 - PER) \tag{11}$$

The parameter *n* depends on the current code used in the adaptive FEC system and *PER* is the packet error rate for this code, given by (10).

To compute the parameter *n* as a function of the error correction capacity, *t*, of the current code, it is necessary to define a particular family of codes or to use some known bound. In this paper, we opted to use the bound for the Bose, Chaudhuri and Hocquenghem (BCH) code. The BCH codes form a large class of powerful random error-correcting cyclic codes. The BCH bound can be summarized as: for any positive integers *m* ($m \geq 3$) and *t* ($t < 2^{m-1}$) exists a (*n*,*k*) binary *t*-error correcting BCH code with the following parameters: [13]

$$n = 2^m - 1$$
$$(n-k) \le mt \quad (12)$$
$$d_{\min} \ge 2t + 1$$

where d_{min} is the minimum distance of the code.

In order to compare our results with those presented in [8], we set k = 424 bits when computing the PER. For this value of k, a BCH code with suitable natural length or suitable number of information digits may not be obtained. However, by subtracting a proper number of bits from the natural code, a shortened BCH code can be implemented [13].

Thus, using the bounds given by (12) and considering k = 424 bits, we can compute the number of parity bits (and thus the value of n) as a function of the error correction capacity of the code, t, as:

$$(n-k) = 9t \ \text{ if } 1 \le t \le 9$$
$$(n-k) = 10t \ \text{ if } 10 \le t \le 59 \quad (13)$$

We are interested in computing the throughput as a function of the SNR in the channel. For this we follow the steps:

1. Define a given modulation (reference modulation) to the wireless channel. We use Quadrature Amplitude Modulation (QAM) with 256 points in its constellation (256-QAM modulation)
2. Compute the BER as a function of the Symbol Energy to Noise Density Ratio (E_S/N_0) in an AWGN channel, using classical formulas presented in the literature, like in [14].
3. Using (6) and (7), compute the BER in the good and bad states.
4. Define the error correction capacity of the current code and, using (13), the number of parity bits in the code and after the total number of bits, n.
5. Using (8), (9) and (10), compute the PER in the Rayleigh channel.
6. Finally, compute the normalized throughput for the current code using (11).

Figure 2 shows the normalized throughput as a function of E_S/N_0 considering codes with different error correction capacity, a 256-QAM in the wireless link and k = 424 bits. The optimum switching point between two neighboring codes is obtained by the cross point of the corresponding throughput curves.

Table I summarizes the optimum switching points presented in Figure 2 and, to permit comparisons, the optimum switching points previously published in [8].

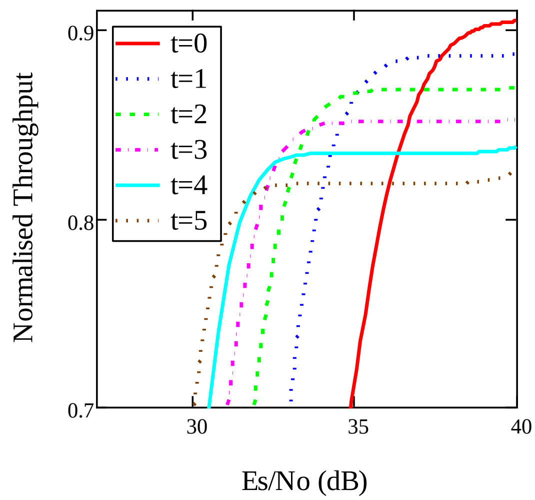

Figure 2. Normalized throughput as a function of Es/No for k = 424 bits and $0 \le t \le 5$.

TABLE I. OPTIMUM SWITCHING POINTS FOR MAXIMUM THROUGHPUT CRITERION, K = 424 BITS.

Switching		E_s/N_0 (dB) obtained from Fig. 2	E_s/N_0 (dB) obtained from [8]
From	to		
$t = 0$	$t = 1$	37.7	30.7
$t = 1$	$t = 2$	35.1	29.0
$t = 2$	$t = 3$	33.6	28.1
$t = 3$	$t = 4$	32.6	27.5
$t = 4$	$t = 5$	31.8	27.0

Based on the results presented on Table I, we can conclude that the optimum switching points depend on the channel model.

Another interesting point is to investigate the influence of the packet length, k, in the optimum switching points. For this, we compute again the normalized throughput considering now k = 848 bits (note that, with this new value of k, the Equation (13) needs to be modified based on the bounds presented on (12)). Figure 3 shows the new results obtained and Table II compares the optimum switching points for k = 424 bits and k = 848 bits.

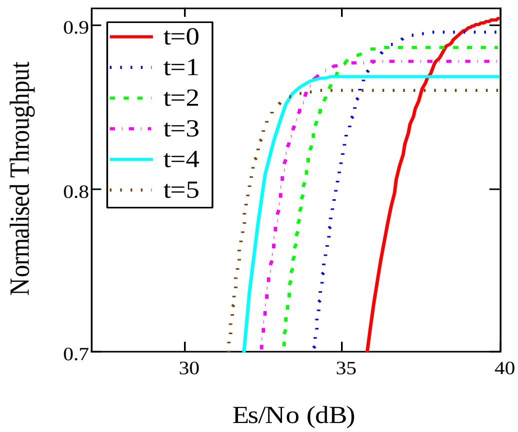

Figure 3. Normalized throughput as a function of *Es*/*No* for *k* = 848 bits and 0 ≤ t ≤ 5.

TABLE II. OPTIMUM SWITCHING POINTS FOR MAXIMUM THROUGHPUT CRITERION, K = 848 BITS.

Switching		E_s/N_0 (dB) k = 848 bits	E_s/N_0 (dB) k = 424 bits
from	to		
$t = 0$	$t = 1$	38.9	37.7
$t = 1$	$t = 2$	36.4	35.1
$t = 2$	$t = 3$	35.1	33.6
$t = 3$	$t = 4$	34.1	32.6
$t = 4$	$t = 5$	33.4	31.8

Based on the results summarized in Table II, we can conclude that the optimum switching points depend on the number of information bits in the packet, with the E_S/N_0 in the switching points increasing when *k* increases.

It is important to observe that the PER in the optimum switching points can be unacceptable for some applications. For example, considering *k* = 848 bits, the PER is about 0.11 in the switching points. This result leads us to the next criterion to define the optimum switching points: the maximum PER target.

IV. Optimum Switching Points Based on the Maximum PER Target

The criterion presented in the last section maximizes the throughput but does not consider any restriction about PER. If the application needs to maintain the PER below a given threshold, the maximum PER target is a better criterion. In this criterion we compute the PER for each error correcting code, as a function of the E_S/N_0. The optimum switching point in this case is the cross point between the PER curve and the PER threshold. To illustrate this approach, Figure 4 shows the switching points considering the maximum PER target equal to 0.15 (actually, this value is too high for most of the applications and has just been chosen to illustrate the concept). To plot Figure 4 we set *k* = 424 bits and we consider 256-QAM modulation. Table III summarizes the switching points showed in Figure 4.

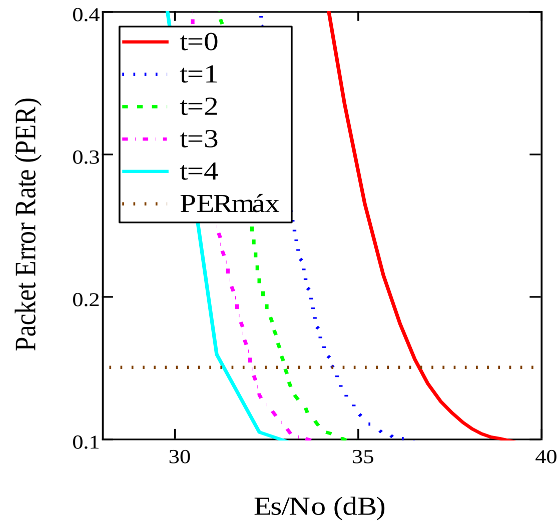

Figure 4. Switching points considering maximum PER criterion with target equal to 0.15 for 0 ≤ t ≤ 5 .

TABLE III. OPTIMUM SWITCHING POINTS FOR MAXIMUM PER TARGET.

Switching		E_s/N_0 (dB)
from	to	
$t = 0$	$t = 1$	36.7
$t = 1$	$t = 2$	34.3
$t = 2$	$t = 3$	32.9
$t = 3$	$t = 4$	32.0
$t = 4$	$t = 5$	31.3

Based on the results presented in Table III and comparing these results with those presented in Section II and those presented on reference [8], we can conclude that: the optimum switching points depend on the selected criterion; again, the optimum switching points depend on the channel model.

V. Optimum Switching Points Based on the Delay

In this section, we consider the mean delay to transmit a correct packet as the criterion to define the optimum switching points. This criterion is appropriate for non-real time applications where we can retransmit a packet in order to guarantee that all packets delivered to the receiver are correct.

To compute the delay it is necessary to define a reference scenario in terms of multiple access and the strategy to retransmit packets. In order to permit comparisons, we use here the same reference scenario used in [8]:

- A TDMA (Time Division Multiple Access) system with *X* time slots in a frame, whit *n* bits being transmitted in each time slot.
- Each packet is transmitted over *Z* frames (one packet needs *Z* time slots to be transmitted).
- Each packet is retransmitted until being correctly received (we consider that the number of retransmissions in unlimited).

- A slow fading channel, meaning that the duration of the fades is greater than de duration of the packet transmission.
- Each time slot is protected by an error correcting code, but the retransmissions occur at the packet level and not at the time slot level. In other words, the receiver requests retransmission of the whole packet.

With these assumptions, the mean delay to transmit a correct packet is given by [8]

$$E(T) = [(Z-1)X+1]\frac{n}{R}(1-PER) \quad (14)$$

where R is the transmission rate in the wireless link and the parameter PER is calculated at the packet level. As the FEC code is applied in each slot, we need to modify (8) and (9) to compute the PER associated with each state of the GMC. The new equations are:

$$P(p_g) = 1 - \left(\sum_{i=0}^{t} \binom{n}{i} (1-p_g)^{n-i} {p_g}^{i} \right)^{Z} \quad (15)$$

$$P(p_b) = 1 - \left(\sum_{i=0}^{t} \binom{n}{i} (1-p_b)^{n-i} {p_b}^{i} \right)^{Z} \quad (16)$$

With these new equations we can apply (10) to compute the PER and then we can calculate the delay using (14).

As we are only interested in comparing the delay for different error correcting codes and as the parameters Z, X and R are independent of the FEC code, we can define a normalized delay as:

$$E(T) = n(1-PER) \quad (17)$$

Figure 5 shows the delay for $0 \le t \le 5$, considering $k = 424$ bits and $Z = 5$. For each code, n is computed using (13) and PER is computed using (15), (16) and (10). Again, the optimum switching point between any two neighboring codes is obtained by the cross point of the corresponding curves illustrated in the figure.

Table IV summarizes the optimum switching points obtained from Figure 5. Comparing these results with those presented on previously sections, we can conclude that the optimum switching points depend on the selected criterion. Again, comparing these results with those presented in [8], we conclude that the optimum switching points depend on the channel model.

TABLE IV. OPTIMUM SWITCHING POINTS FOR DELAY CRITERION.

Switching		E_s/N_0 (dB)
from	to	
$t = 0$	$t = 1$	39.0
$t = 1$	$t = 2$	36.2
$t = 2$	$t = 3$	34.6
$t = 3$	$t = 4$	33.5
$t = 4$	$t = 5$	32.7

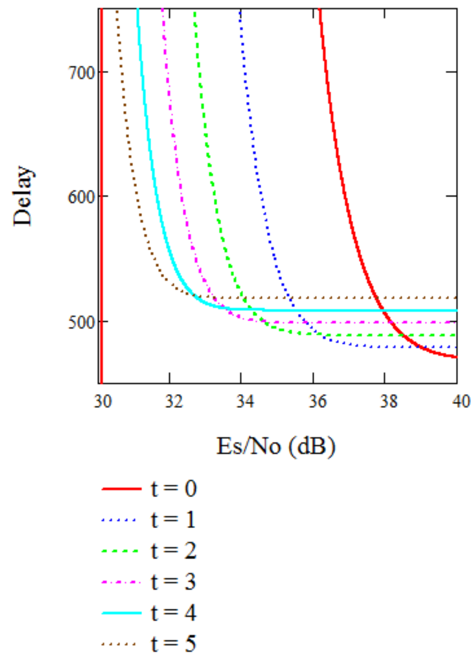

Figure 5. Switching points considering delay criterion for $0 \le t \le 5$.

VI. CONCLUSIONS

The switching point is a key factor to maximize the performance of systems using adaptive FEC. The optimum switching points for this kind of system have been analyzed in [8] considering three criteria: maximum throughput, maximum PER target and delay to transmit a correct packet. However, analyses performed in [8] consider an AWGN channel or that an interleaving process is used to randomize the burst errors in the wireless channel.

In this paper, we analyze the optimum switching points considering a wireless link modeled as a Rayleigh channel. We use the same three criteria proposed in [8]. We concluded that the optimum switching points depend on: the channel model, the selected criterion and the packet length.

In future works, we will extend the results presented here considering an adaptive hybrid system, in which we combine adaptive modulation and adaptive FEC in order to improve the performance of the system. Also, we will consider different fading models, like Rician model and Nakagami model.

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