Low Complexity Recursive Least-Squares Algorithm for Adaptive Noise Cancellation

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***Abstract*—Adaptive Noise Cancellation (ANC) belongs to the interference cancellation class. It employs an adaptive filter to estimate a perturbation signal, which corrupts a primary acoustic source. In most of the corresponding applications, the goal is to imitate an original speech signal. This paper proposes the use of a low complexity recursive least-squares (RLS) adaptive algorithm for the ANC procedure. The combination between the RLS method and the dichotomous coordinate descent (DCD) iterations offers good performance with acceptable arithmetic costs. Simulation results are provided in order to demonstrate the validity of the ANC system based on the RLS-DCD adaptive algorithm.**

Keywords: adaptive noise cancellation; recursive least-squares; dichotomous coordinate descent.

I. INTRODUCTION

Modern technology allows the deployment of telecommunication networks in problematic environments, which frequently introduce strong acoustic interference. The high quality communication performed in extremely noisy surroundings, such as airplane cockpits or social gatherings, requires the real-time estimation of corrupted acoustic signals (usually speech sequences).

With the development of adaptive algorithms, the field of Adaptive Noise Cancellation (ANC) has also been the subject of intensive study [1][2]. The workhorse of signal processing systems employing adaptive methods is the Least Mean Squares (LMS) family [1]-[5]. Although the classical LMS adaptive algorithms were improved to a certain degree, their performances are limited when working with highly correlated signals. A new generation of efficiently implementable adaptive systems is required to increase the noise cancellation capabilities.

The standard recursive least-squares (RLS) adaptive methods have attractive convergence properties [1]-[5]. However, the classical solutions for directly solving the corresponding matrix inversion problem have high arithmetic complexities and require large amounts of computational resources. Moreover, the implementations employing the traditional RLS algorithms suffer from occasional numeric instability caused by higher order arithmetical operations, such as divisions. Although the Fast RLS (FRLS) [4] considerably reduces the arithmetic effort, it is not stable when working with nonstationary signals, such as speech.

In [6]-[8], the prohibitive nature of the RLS methods was approached using the combination with the dichotomous coordinate descent (DCD) iterations. The DCD portion of the algorithm replaces the classical matrix inversion problem with an auxiliary system of equations, which is solved using only additions and bit-shifts. The solution is based on the statistical properties of the input signals and reduces the overall arithmetic complexity to a value proportional to L, which is used to denote the adaptive filter's length. The resulting RLS-DCD algorithm is a numerically stable alternative, offering comparable results in terms of adaptation speed and precision, with a considerably reduced computational effort [6]-[10]. By comparison, the classical RLS method has a complexity of $O(L^3)$, which can be reduced using Woodburry's identity to $O(L^2)$ – both methods are considered prohibitive for practical applications [1][4].

The original RLS-DCD solution was rarely tested with colored signals, such as speech sequences [7][8]. It was later successfully applied for stereophonic acoustic echo cancelation (SAEC) setups requiring the estimation of multiple unknown systems [9]. This paper proposes the use of the RLS-DCD method for ANC systems employed in real-time recovery of speech signals. A theoretical model is presented and tested using different types of acoustic interference, with low Signal-to-Noise Ratio (SNR). Although the number of adaptive filter coefficients associated with ANC applications is lower than the case of acoustic echo cancellation (AEC) scenarios, the reduction in terms of computational workload (in comparison to the classical RLS) is valuable for mobile devices (i.e., headphones, mobile phones, etc.). As a consequence, the compromise between arithmetic complexity and performance is analyzed, and a comparison is performed with the classical RLS.

The paper is organized as follows. In Section II, the theoretical model of the ANC setup is defined. Section III describes a low complexity RLS-type adaptive algorithm which is suitable for the ANC procedure. The performances of the proposed adaptive method are demonstrated using simulations in Section IV. The classical RLS adaptive algorithm is employed as a reference. Finally, in Section V, a few conclusions are stated regarding the compromise

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between arithmetic complexity and the performance of the ANC system using a low complexity RLS method.

II. Theoretical Model

Figure 1 illustrates the ANC scheme. We denote by $\hat{\mathbf{h}}(n)$ the L x 1 vector comprising the adaptive filter's variable coefficients at time index n, i.e.:

$$\hat{\mathbf{h}}(n) = \left[h_0(n),\, h_1(n),\, \ldots, h_{L-1}(n)\right]^T, \tag{1}$$

where ^{T} is the transpose of a matrix. The desired signal $d(n)$ is the combination between the relevant signal $s(n)$ and the corrupting sequence $q(n)$ (also called the interference signal). The input of the adaptive algorithm $x(n)$ is a reference signal, which is linearly correlated with the interference $q(n)$. The theoretical model of the adaptive algorithm is completed with the L dimensional vector $\tilde{\mathbf{x}}(n)$ formed with the most recent L input samples:

$$\tilde{\mathbf{x}}(n) = \left[x(n),\, x(n-1),\, \ldots, x(n-L+1)\right]^T. \tag{2}$$

In literature, the relation between $x(n)$ and $q(n)$ is usually modelled through a finite impulse response (FIR) filter, which generates $q(n)$ using $x(n)$ as the input. In practical ANC applications, the samples corresponding to $x(n)$ and $d(n)$ are available through microphones [2]. The influence of the physical distance between the two acoustic sensors is represented in Figure 1 through the delay factor D, associated with the length of the mentioned FIR filter.

The purpose of the ANC system is to output an estimate $y(n)$ of $q(n)$ and subtract it from the desired signal. Consequently, the error signal $e(n)$ is an estimate of s(n), i.e. $e(n) \rightarrow s(n)$. The *error* of the adaptive algorithm is used to adjust the coefficients of the adaptive filter in order to minimize the noise interference. In an optimal situation, $e(n)$ is composed of the signal $s(n)$, free of the noise interference $q(n)$.

III. The RLS-DCD Adaptive Method

The core of the ANC system presented in Figure 1 is the adaptive algorithm. The usual methods employed for the update of $\hat{\mathbf{h}}(n)$ are the LMS-type adaptive algorithms, which have reduced performance when working with highly correlated input signals. In the ANC case, the samples of input signal $x(n)$ can be associated with speech, music, engine noise or other (usually highly correlated) acoustic signals. In such circumstances, the RLS-based systems can generate superior performance through their de-correlation properties. Despite the attractive features of the RLS algorithms, the classical versions use direct methods for computing the corresponding matrix inverse and solving the associated system of equations. Consequently, prohibitive workloads are imposed on signal processing chips, which usually handle multiple tasks.

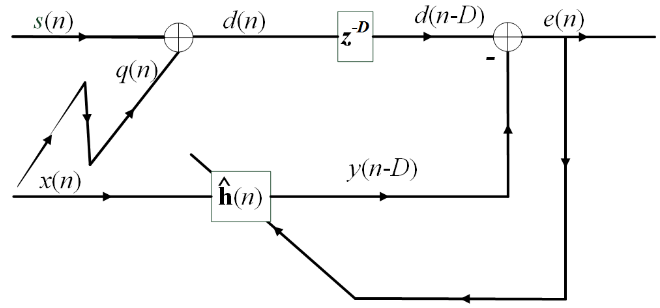

Figure 1. The ANC scheme

The RLS-DCD adaptive algorithm was proposed as a stable alternative for other low-complexity RLS versions (such as the FRLS). Initially, the method was mostly employed for processing weakly correlated signals and later for the identification of long unknown acoustic systems (e.g., the AEC/SAEC scenarios). We propose to use the method for real time retrieval of speech signals in ANC scenarios. Table 1 illustrates the RLS-DCD adaptive algorithm [6]-[9], where we denote by $\lambda\left(0 \ll \lambda < 1\right)$ the forgetting factor associated with the memory of the algorithm [1]. The L x L correlation matrix $\tilde{\mathbf{R}}_{\tilde{x}}(n)$ has the transpose property, i.e. $\tilde{\mathbf{R}}_{\tilde{x}}(n) = \tilde{\mathbf{R}}_{\tilde{x}}^T(n)$. It can be updated by copying the upper-left L-1 x L-1 block of $\tilde{\mathbf{R}}_{\tilde{x}}(n-1)$ to the lower-right L-1 x L-1 submatrix of $\tilde{\mathbf{R}}_{\tilde{x}}(n)$, and by computing only the first corresponding column [7][9]. The main diagonal of $\tilde{\mathbf{R}}_{\tilde{x}}(n)$ is initialized using the identity matrix $\mathbf{I}_L$ and the constant value δ, in order to avoid processing a singular matrix in the initial stages of the adaption course. The statistical properties of the matrix allow a significant reduction of complexity in step 1, to a value proportional to the adaptive filter's length.

The RLS-DCD method exploits the statistical properties of the input signals and solves an auxiliary system of equations using only additions and bit-shifts of the operands, therefore completely eliminating divisions. In steps 3 and 4, the DCD portion of the adaptive algorithm takes into account the results obtained at time index n-1 and generates, using a limited number of updates, the solution vector $\Delta\hat{\mathbf{h}}(n)$ (with values represented in the numerical interval [-H, H] using M_b bits). The updates are conditioned by the comparisons performed between the values comprising the residual vector $\mathbf{r}(n)$ and the values positioned on the main diagonal of $\tilde{\mathbf{R}}_{\tilde{x}}(n)$ [6]-[9]. It was demonstrated in [10] that the vector $\mathbf{r}(n)$ becomes almost null, as the adaptive filter reaches convergence state. Correspondingly, the vector values oscillate in a large dynamic range in the adaptation stages. The arithmetic complexity associated with step 4 is upper

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TABLE I. THE RLS-DCD ALGORITHM

Step	Computations
Init	$\hat{\mathbf{h}}(0)=\mathbf{0},\ \mathbf{r}(0)=0,\ \mathbf{R}_{\tilde{\mathbf{x}}}(0)=\delta\mathbf{I}_L$
For n = 1, 2, …	
1	$\hat{\mathbf{R}}^{(1)}_{\tilde{\mathbf{x}}}(n)=\lambda\hat{\mathbf{R}}^{(1)}_{\tilde{\mathbf{x}}}(n-1)+x(n)\mathbf{x}(n)$
2	$e(n)=d(n)-\hat{\mathbf{h}}^T(n-1)\mathbf{x}(n)$
3	$\mathbf{r}(n)=\lambda\mathbf{r}(n-1)+e(n)\mathbf{x}(n)$
4 {**DCD method**}	$\hat{\mathbf{R}}_{\tilde{\mathbf{x}}}(n)\Delta\hat{\mathbf{h}}(n)=\mathbf{r}(n)\Rightarrow\Delta\hat{\mathbf{h}}(n),\ \mathbf{r}(n)$
5	$\hat{\mathbf{h}}(n)=\hat{\mathbf{h}}(n-1)+\Delta\hat{\mathbf{h}}(n)$

limited by $2N_uL$ possible additions, where N_u is the number of *successful iterations* (or solution vector updates) performed by the DCD (usually N_u<10; one iteration uses only additions and bit-shifts) [7][9]. The value of N_u is usually low and represents a sufficient number of *successful* DCD iterations performed for the computation of $\Delta\hat{\mathbf{h}}(n)$ in order to achieve good RLS-DCD performance. The algorithm also updates $\hat{\mathbf{h}}(n-1)$ in step 5, through an addition to $\Delta\hat{\mathbf{h}}(n)$.

The overall complexity of the RLS-DCD can be reduced by choosing the forgetting factor as $\lambda=1-1/(KL)$, where K and the filter length L are powers of 2. Therefore, any multiplication with λ can be replaced by a bit-shift and one subtraction. The total amount of arithmetic operations corresponding to the algorithm described in Table I is represented by $3L$ multiplications and less than $6L+2N_uL$ additions for every time index n [8]. We notice that the value of M_b has no direct influence on the number of arithmetic operations (the parameter is relevant only for their complexity).

IV. SIMULATIONS

Simulations results are presented for the context illustrated in Figure 1, using the RLS-DCD and RLS adaptive algorithms. The performance of the ANC system is analyzed using spectrogram plots with 256 points Fourier Transforms for the generated error signals.

The acoustic test signals are sampled with a frequency of 8 kHz, using 16 bits/sample. The goal is to recover interference-free speech sequences available in the $s(n)$ waveforms [11]. The desired signal is generated by filtering the interference $x(n)$ with a Matlab *fir1* 12th order low-pass impulse response and adding the output $q(n)$ to $s(n)$.

The length of the adaptive filter is L=25 and the corresponding forgetting factor is set to $\lambda=1-1/(16L)$. Correspondingly, the L values comprising the RLS-DCD solution vector are represented in the numerical interval [-H, H]=[-1,1] using M_b bits. The parameter M_b directly influences the precision of the adaptive system and is varied in order to establish a compromise between the performance and complexity. Furthermore, $\Delta\hat{\mathbf{h}}(n)$ is updated for a maximum number of N_u=4 times per every time index n.

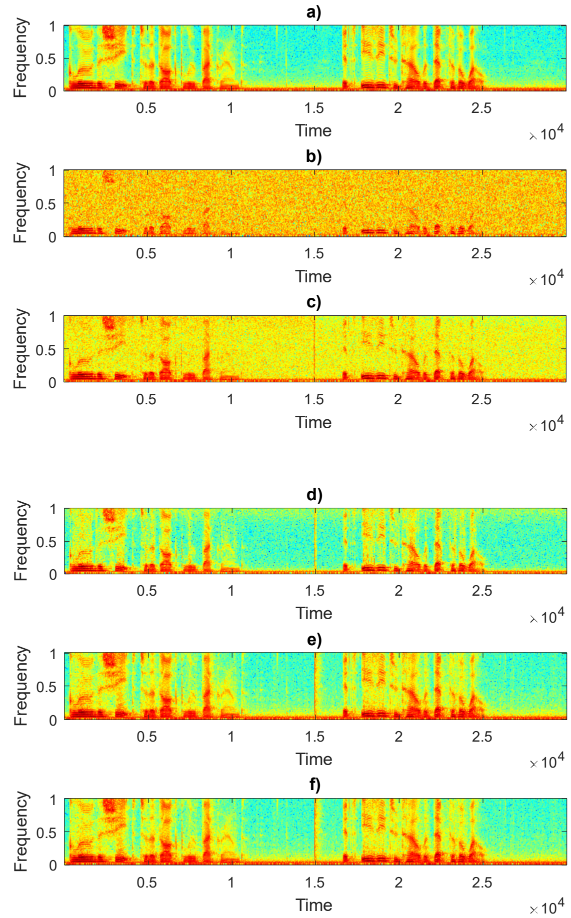

Figure 2. Spectrograms with 256 Fourier Transforms – the interference is Gaussian noise (SNR=0 dB): a) The speech sequence to be recovered; RLS-DCD error signal with b) M_b=3, c) M_b=6, d) M_b=8, e) M_b=16; f) RLS error signal

The first simulation compares the performance of the RLS-DCD and RLS algorithms using Gaussian noise as acoustic interference. The $s(n)$ and $q(n)$ signals have the same power (i.e., the corresponding SNR has the value 0 dB). It can be noticed in Figure 2 that increasing the number of bits used for the representation of the adaptive filter coefficients leads to better estimates of interference samples and a better reduction in noise level. Additionally, the comparison performed with the RLS spectrogram indicates that higher values of the parameter M_b provide similar performance from the RLS-DCD method, with lower arithmetic effort.

For the second simulation (Figure 3), the interference signal $x(n)$ is acoustic engine noise. The same value is used for the SNR (0 dB). In comparison to the previous scenario, it can be noticed that the settings M_b=8 and M_b=16 do not provide the same performance rating anymore. The properties of the second interference type require more

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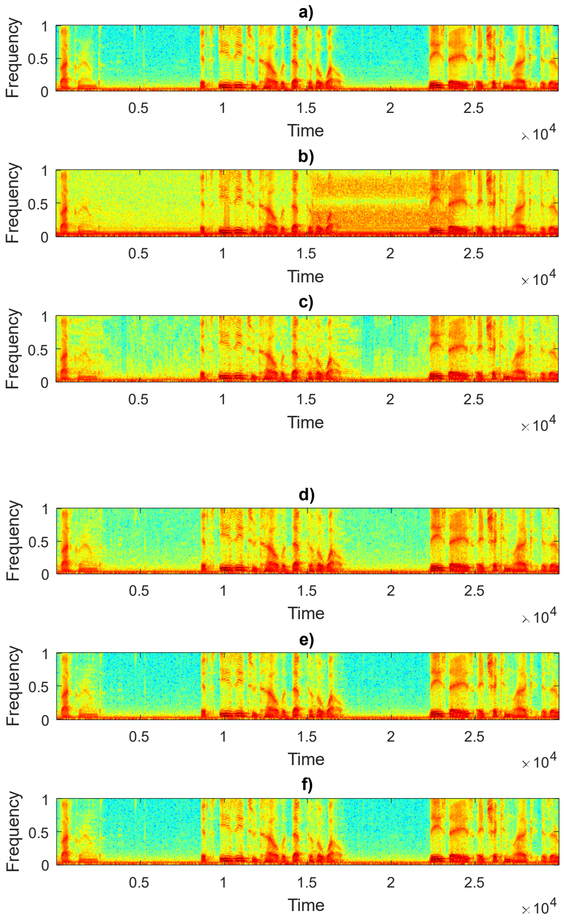

Figure 3. Spectrograms with 256 Fourier Transforms – the interference is engine noise (SNR=0 dB): a) The speech sequence to be recovered; RLS-DCD error signal with b) M_b=3, c) M_b=6, d) M_b=8, e) M_b=16; f) RLS error signal

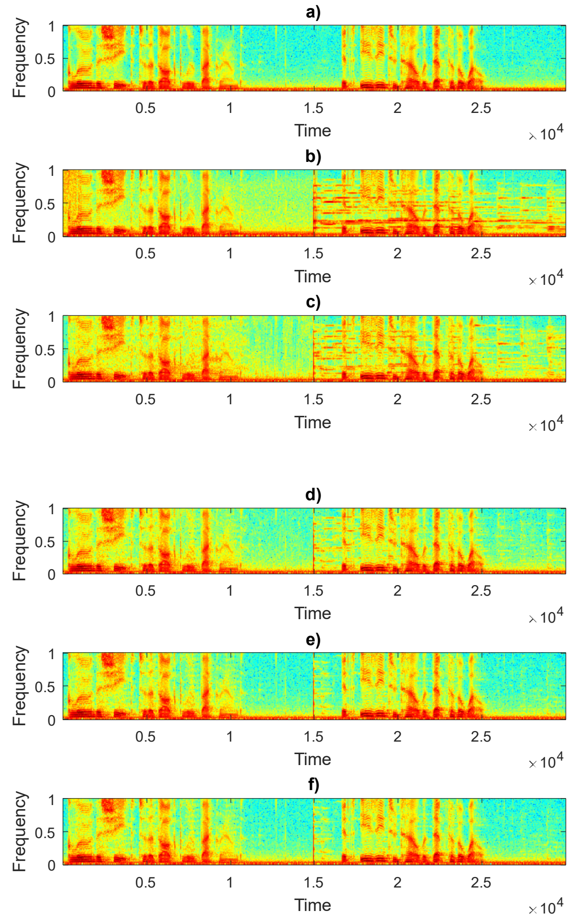

Figure 4. Spectrograms with 256 Fourier Transforms - the interference is engine noise, which changes to music at time index 15000 (SNR=-10 dB): a) The speech sequence to be recovered; RLS-DCD error signal with b) M_b=3, c) M_b=6, d) M_b=8, e) M_b=16; f) RLS error signal

precision in order to generate the similar results between the RLS-DCD and the RLS methods.

The spectrograms corresponding to a third experiment are illustrated in Figure 4. The speech $s(n)$ is corrupted for the first half of the simulation by engine sound, which is afterwards replaced by music. The SNR is set to -10 dB for the entire scenario. The change in interference produces a spike in each error spectrogram and the adaptive algorithms require an adaptation period. It can also be noticed that the music is harder to eliminate from the desired signal (the corresponding interference leaves easier noticeable traces in the error signal). As a consequence, the correlation properties of the interference signals have an important influence on the performance of the adaptive algorithms.

V. Conclusions

In this paper, the low-complexity RLS-DCD adaptive algorithm was employed for ANC scenarios with low SNR conditions. Simulations were performed in order to analyze the behavior of the proposed system, which indicated that the RLS-DCD has attractive performance, computational efficiency and is suitable for ANC hardware implementations.

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References

[1] S. Haykin, *Adaptive Filter Theory*. Fourth Edition, Upper Saddle River, NJ: Prentice-Hall, 2002.

[2] E. Hänsler and G. Schmidt, *Acoustic Echo and Noise Control – A Practical Approach*. John Wiley & Sons, Inc., Hoboken, New Jersey, 2004.

[3] A. H. Sayed, *Adaptive Filters*. New York, NY: Wiley, 2008.

[4] J. Benesty, C. Paleologu, T. Gänsler, and S. Ciochină, *A Perspective on Stereophonic Acoustic Echo Cancellation*, Springer-Verlag, Berlin, Germany, 2011.

[5] B. Farhang-Boroujeny, *Adaptive Filters - Theory and Applications*. Second Edition, John Wiley & Sons, Inc., Hoboken, New Jersey, 2013.

[6] Y. V. Zakharov and T. C. Tozer, “Multiplication-free iterative algorithm for LS problem,” *IEE Electronics Lett.*, vol. 40, pp. 567–569, Apr. 2004.

[7] Y. V. Zakharov, G. P. White, and J. Liu, “Low-complexity RLS algo-rithms using dichotomous coordinate descent iterations,” *IEEE Trans. Signal Processing*, vol. 56, pp. 3150–3161, July 2008.

[8] J. Liu, Y. V. Zakharov, and B. Weaver, “Architecture and FPGA design of dichotomous coordinate descent algorithms,” *IEEE Trans. Circuits and Systems I: Regular Papers*, vol. 56, pp. 2425–2438, Nov. 2009.

[9] C. Stanciu, J. Benesty, C. Paleologu, T. Gänsler, and S. Ciochină, “A widely linear model for stereophonic acoustic echo cancellation,” *Signal Processing*, vol. 93, pp. 511–516, Feb. 2013.

[10] C. Stanciu and C. Anghel, “Numerical of the DCD-RLS Algorithm for Stereo Acoustic Echo Cancellation,” in *Proc. COMM*, 2014, pp.65–68.

[11] The Open Speech Repository, http://www.voiptrouble shooter. com/open_speech/ [last accessed: 30.03.2017].