Codebook Subsampling and Rearrangement Method for Large Scale MIMO Systems

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***Abstract*—In large scale multiple-input multiple-output (MIMO) systems, the size of codebook increases greatly when transmitters and receivers are equipped with more antennas. Thus, there are demands to select subsets of the codebook for usage to reduce the huge feedback overhead. In this paper, we propose a novel codebook subsampling method using chordal distance of different codewords and deleting them to affordable payload of Physical Uplink Control Channel (PUCCH). Besides, we design a related codebook rearrangement algorithm to mitigate the system performance loss when there are bit errors in the feedback channel.**

Keywords-Large scale MIMO; codebook subsampling; codebook rearrangement; PMI feedback

I. INTRODUCTION

The explosive growth of wireless data service calls for new spectrum resource. Meanwhile, the available spectrum for further wireless communication systems is very limited and expensive. Since the capacity of a multiple-input multiple-output (MIMO) system greatly increases with the minimum number of antennas at the transmitter and receiver sides under rich scattering environments [1], the large scale MIMO [2] shown in Fig. 1 is one of the most important techniques to address the issue of exponential increasing in wireless data service by using spatial multiplexing and interference mitigating.

For the consideration of practical application, the number of antennas on the terminal side is restricted, and thus the number of multiplexing layers is limited though the number of antennas on the base station (BS) could be very large. As a result, we should explore the large scale MIMO system potentials by utilizing beamforming technologies. The performance of beamforming relies on the accuracy of precoding. However the size of codebook can be very large when antennas are increased, considering that the payload capacity of Physical Uplink Control Channel (PUCCH) is limited to 11 bits [3]. To decrease the overhead in Channel State Information (CSI) feedback, the choice of codebook subsampling for transmission is necessary [4][5].

Several subsampling methods have been proposed in Rel-10 in 2,4,8 Tx scenario. The subset selection in this case naturally corresponds to the reduction of granularity in direction and/or phase offset [6], such as uniform subsampling or staggered subsampling which keeps better granularity. However, the codebook design for large scale MIMO system may not be based on direction for each polarization and phase offset between polarizations; hence the application above for large scale MIMO is restricted.

In this case, we propose a novel codebook subsampling method which applies to all kinds of codebook design, in which we select the subset of codebook using chordal distance of different codewords and delete them to affordable payload of PUCCH. In addition, to further optimize the performance of Precoding Matrix Indication (PMI) when errors occur, we propose the codebook rearrangement method to decrease the impact of mismatch between PMI and the channel.

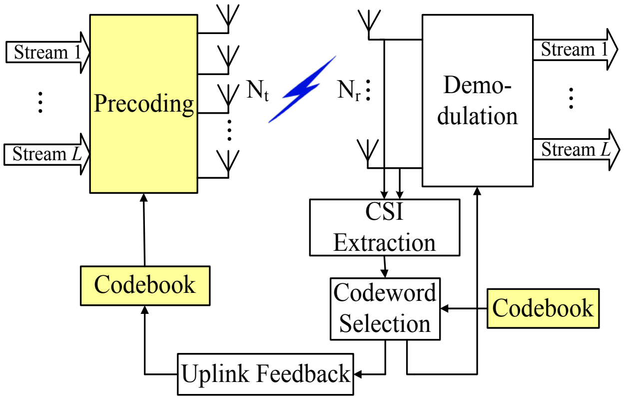

Figure.1 The close-loop MIMO system model

This paper is organized as follows: Section II introduces the model of the precoding system. Section III presents the codebook subsampling method and the codebook rearrangement method. Section IV shows the simulation results. And finally, Section V concludes the paper.

II. SYSTEM MODEL

A. System Model

In this paper, we discuss about a close-loop MIMO system with N_t transmit antennas and N_r receive antennas depicted in Fig. 1. For massive MIMO system N_t could be very large.

Firstly, the data vector *S* in the system is demultiplexed into *L* streams. *L* is limited by $1 \le L \le \min(N_t, N_r)$. When *L*=1 we call the transmission beamforming, while *L*>1 we call it multiplexing. After the data vector *s* is preprocessed by a $N_t \times L$ precoding matrix W_i, we get a $N_t \times 1$ signal vector *x* for the N_t antennas to transmit:

$$x = Ws \tag{1}$$

Thus, after *x* passing the channel and being added the noise, we will get the received signal *y*, which can be expressed as:

$$y = HW_i s + v \tag{2}$$

where *H* ($H \in C^{N_r \times N_t}$) denotes the fading channel matrix with its entry H_{ij} denoting the channel response from the j^{th} transmit antenna to the i^{th} receive antenna, and *v* denotes the white complex Gaussian noise vector with covariance matrix $N_0 I_{N_r}$.

The precoding matrix is selected from the predesigned codebook which is known to the transmitter and the receiver. Taking the downlink as an example, when UEs have received the pilots from the BS, the receiver can choose the optimal codeword after the channel estimation. Then the receiver reports the PMI with limited bits to BS [7] through the uplink channel. If the feedback is limited to *B* bits, the size of codebook satisfies $N = 2^B$. Thus the transmitter can retrieve the precoding matrix and perform the precoding.

B. Kerdock Codebook

The basic idea of the kerdock codebook design is utilizing the feature of Mutually Unbiased Bases (MUB) to construct precoding matrices. The main characteristic of the kerdock codebook is that all the elements of the matrix are ±1 or ±j. Hence, the kerdock codebook has some advantages, such as low requirement for storage, low computational complexity for codeword search, and the simple systematic construction.

The MUB property is described as follows:

$S = \{s_1, ..., s_{N_t}\}$, $U = \{u_1, ..., u_{N_t}\}$ are two orthonormal bases with size $N_t \times N_t$. If the column vectors drawn from S and U satisfy $\left|\langle s_i, u_j \rangle\right| = 1/\sqrt{N_t}$, we can say that they have the mutually unbiased property [8].

An MUB is the set $S = \{s_1, ..., s_{N_t}\}$ satisfying the mutually unbiased property. The Kerdock codebook has several construction methods such as Sylvester–Hadamard construction and power construction. In this paper, we use the Sylvester–Hadamard construction:

First, we construct the generating matrices D_n ($N_t \times N_t$ diagonal matrices with ±1, ±j elements) for $n=0,1,2\ldots N_t-1$ according to [9].

Then we construct the corresponding orthonormal matrix:

$$W_n = \frac{1}{\sqrt{N_t}} D_n \hat{H}_{N_t}, n = 0,1,...,N_t - 1 \tag{3}$$

where $\hat{H}_{N_t}$ is the $N_t \times N_t$ Sylvester–Hadamard matrix:

$$\hat{H}_{N_t} = \hat{H}_2 \otimes \hat{H}_2 .... \tag{4}$$

where $\hat{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

For the beamforming, we can construct the codebook by selecting each column of all the bases as the precoding vector:

$$C = \{f_1 = W_0^{\{1\}}, f_2 = W_0^{\{2\}}, ..., f_N = W_{N_t-1}^{\{N_t\}}\} \tag{5}$$

And for a *L*-layer spatial multiplexing codebook, the largest codebook is derived by taking all *L*-column combinations from each W_n.

C. Codeword Search

We can choose the optimal main codeword from $\mathrm{K} = \{K_1, K_2, ..., K_N\}$ through the estimate of the channel. The codebook is shared by the transmitter and receiver.

Codeword selection criteria: for 1-layer beamforming, the beamformer that minimizes the probability of symbol error for maximum ratio combining receiver is expressed as [10]:

$$\hat{f}[i] = \arg\max_{f \in C} \left\| H[i] f \right\|_2^2 \tag{6}$$

where f denotes a $N_t \times 1$ matrix. For spatial multiplexing with a zero forcing receiver, the minimum singular value selection criterion is expressed as:

$$\hat{F}[i] = \arg\max_{K \in C} \lambda_{\min}\{H[i]K\} \tag{7}$$

where $\lambda_{\min}$ denotes the minimum singular value of the argument. This selection criterion approximately maximizes the minimum sub-stream signal-to-noise ratio (SNR).

III. THE NOVEL CODEBOOK SUBSAMPLING AND REARRANGEMENT METHOD

A. Codebook Subsampling

To pursue the maximum SNR, we select the codeword with the smallest chordal distance from the transmission channel. The basic idea of the codebook subsampling method is to delete one codeword of the codeword pairs which have the smallest chordal distance. The chordal distance between two precoding vector is represented by

$$d_{chord}(f_i, f_j) = \sqrt{1 - \left(\frac{\langle f_i, f_j \rangle}{\|f_i\| \|f_j\|} \right)^2} \qquad (8)$$

with ||.|| being the norm of the vector. If the chord distance between two codewords is the smallest among the codewords pool, we may reserve only one of them and delete another. Therefore we could decrease the overhead as well as remain the performance of precoding at the utmost.

The process of subsampling is shown as follows:

(1) Suppose the codebook includes K codewords. Divide the codewords into g groups.

- a) Compute the chordal distance between any two codewords $d(f_i, f_j)$. Choose f_i and f_j as reference codewords if their distance is the largest.
- b) Compute the chordal distance between the rest $f_L(L \in (0, K], L != i \,\&\&\, L != j)$ and reference codeword. If $d(f_i, f_L) < d(f_L, f_j)$, put f_L in f_i 's group. Otherwise put f_L in f_j 's group.
- c) Repeat the procedure until the number of groups is g.

(2) Delete codewords and related PMI.

- a) Compute the chordal distance between any two codewords in $l^{th} (l = 1, 2, \cdots, g)$ group. Find the $\min d_l(f_i, f_j)$
- b) Choose the $\min\{\min d_l(f_i, f_j)\} (l = 1, 2, \cdots, g)$ as the codeword pair to deal with (suppose in the m^{th} group).
- c) Compute the chordal distance between the rest $f_L(L != i \,\&\&\, L != j)$ in m^{th} group and reference codeword. If $\sum d(f_i, f_L) < \sum d(f_L, f_j)$, delete f_i, otherwise delete f_j.
- d) Select the new $\min d_m(f_i, f_j)$. Back to a) until the number of codewords satisfy the requirement of PMI feedback.

The summary of the algorithm is given in Table I.

TABLE I. SIMULATED SUBSAMPLING ALGORITHM

```
//$K$: the total number of codewords
//$B$: cycling times
//$N=2^B$ : groups of codewords
//$n$: current number of groups
//dis[$i$][$j$]: matrix of chordal distance between codewords
 n=1;
  loop for B cycles
    loop for n cycles
      calculate any of two codewords $f_i$, $f_j$ with  dis[i][j]
      ($f_i$, $f_j$ ∈ the $n_k$ group)
      get the two codewords $f_i$, $f_j$ with the largest codewords distance
      in the $n_k$ group
     if dis(i,t)<dis(j,t) [t ∈ size($n_k$)， t≠i,j] then
        allocate the codeword to the group 2*($n_k$-1)
     else
        allocate to the group 2*$n_k$
      end if
    end loop
   n=n*2
  end loop
 loop for $N$ cycle
   calculate the $n_k$ group any of two codewords $f_i$, $f_j$  with dis{$n_k$}[i][j]
 end loop
 while($K$>payload)
   dis($n_k$)=min(dis{$n_k$}(:))
   $m$=argmin(dis($n_k$))
   In the $m$ group
   if ∑dis(i,t)<∑dis1(j,t) then
     delete the codeword $f_i$ in the m group
   else
     delete the codeword $f_j$ in the m group
   end if
   renew the m group with a minimum distance
 end while
```

B. Codebook Rearrangement

The error in PMI feedback could lead to severe mismatch of precoding vector and user's channel, thus greatly decreasing transmission gain and increasing unreliability. By decreasing the mismatch caused by PMI transmission error, we could compensate for the performance loss, even the precoding vector is not optimal.

Therefore we rearrange the PMI, reduce the Hamming distance of binary indexes of codewords with high correlation. Consider one bit error in PMI feedback. When the two indexes with one bit of Hamming distance are arranged to codewords with high correlation, even if the error occurs and the base station uses the wrong codeword, the wrong codeword could still perform well due to the high correlation with the correct one, thus ensuring the compatibility with the channel and decreasing the gain loss.

The process of subsampling is presented in Table II, and the description is given as follows:

(1) Divide PMI into B PMI groups based on binary code weight. f_i denotes the original codeword associated with PMI w_i, and U_i denotes the new one. $U_0 = f_0$.

(2) Select one $w_{b,i}$ in $b^{th} (b = 1,2,\cdots,B)$ PMI group. Find all the $w_{b-1,j}$ in $(b-1)^{th}$ PMI group that $d_{binary}(w_{b,i}, w_{b-1,j}) = 1$.

(3) In $l^{th} (l = 1,2,\cdots,g)$ codeword group, compute

$$\sum_j d(f_k, f_{b-1,j}), f_k \in l^{th}$$

If $\sum_j d(f_{k'}, f_{b-1,j}) = \min \sum_j d(f_k, f_{b-1,j})$

($f_k \in l^{th}$ codeword group), then $U_{b,i} = f_{k'}$.

(4) If all codewords in l^{th} codeword group are rearranged with new PMI, turn to $(l+1)^{th}$ codeword group.

(5) If all PMI in b^{th} PMI group are rearranged with new codeword, turn to $(b+1)^{th}$ PMI group.

TABLE II. SIMULATED REARRANGEMENT ALGORITHM

```
//$G$=log2(payload)
//PMI_B : groups of PMI based on code weight
loop for payload cycles
    restore each PMI in PMI_B{$i$} with code weight $i$;
end loop
$D_0=f_0$
for $i$=1:G
    for $j$=1:size(PMI_B{i})
        $P_{i_j}$=PMI_B{i}( j)
        $U_{j_{i-1}}=\{ f_{j_{i-1}}, d(P_{i-1},P_{i_j})=1 \}$
        $j_i'=\arg\min \sum d(f_m,U_{j_{i-1}})$ ($f_m \in$ the $n_k$ group)
        $D_{j_i}=f_{j_i'}$
        if the codeword in the $n_k$ group all have been
        allocated new PMI then
            go to next group
        else
            continue in this group
        end if
    end
end
```

IV. SIMULATION RESULTS

This section we present the simulation results under the configuration given in Table III. The simulation procedure follows the system model in Fig. 1.

TABLE III. SIMULATION PARAMETERS

Parameter	**Value**
Frequency	2.1 GHz
System bandwidth	10 MHz
Channel modelling	i.i.d, CN(0,1)
Number of BS antennas	32
Number of UE antennas	1
Channel estimation	Ideal
UE receiver	MMSE
SNR	10 dB

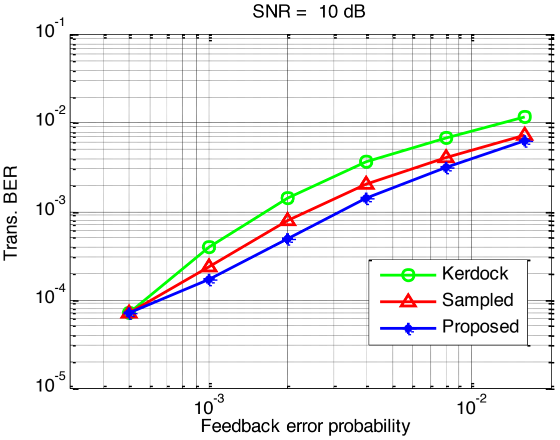

Figure.2 The BER performance of different codebooks

Fig. 2 shows the performance of downlink transmission BER vs. feedback error probability. The feedback error probability means the probability of each bit-error occurs in the PMI feedback, and the downlink transmission BER means the bit error rate of the downlink transmission. From the results, we can see the codebook after subsampling has significant BER performance gain compared with the original Kerdock codebook, because of the low probability of error occurrence due to the fewer bits for feedback. And the proposed codebook after rearrangement has the further BER performance gain since to configure high correlation codewords with reduced code distance, we decrease the performance loss of system when the mismatch of precoding vector and the channel occurs.

V. CONCLUSION

In this paper, we proposed a novel codebook subsampling method based on chordal distance as well as the related codebook rearrangement algorithm for codebook designs in large scale MIMO system. The codebook subsampling method can reduce the feedback overhead without impacting the system performance, and the rearrangement algorithm can significantly mitigate the system performance loss when errors in the feedback channel occur. Simulation results show that the Kerdock codebook after subsampling and rearrangement has significant performance gain under the non-ideal uplink feedback channel in large scale MIMO system.

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