Time Diversity based Iterative Frequency Estimation with Weighted Average Scheme with Low SNR for Pulse-Doppler Radar

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***Abstract*— This paper proposes a time diversity based iterative frequency estimation scheme with low signal to noise ratio (SNR) for pulse Doppler radar based on weighted average. The conventional estimator can alleviate the effect of a diffused Doppler frequency. By applying a weighted average scheme to the received signal, the proposed algorithm improves the accuracy of the frequency estimation. The proposed estimator has better performance than the conventional estimators. Not only is the Cramer-Rao lower bound (CRLB) derived but also the performance of the proposed algorithm is analyzed and verified through Monte-Carlo simulations in an additive white Gaussian noise (AWGN).**

Keywords- time diversity; iterative frequency estimation; weighted average scheme; pulse Doppler radar.

I. INTRODUCTION

Doppler frequency estimation (DFE) algorithms are related to the estimation of sinusoidal waveforms in various applications such as radar and communication. Maximum likelihood (ML) based estimation [1], discrete Fourier transform (DFT) based algorithms [2][3], adaptive notch filters, the different least mean square error technique [4], and Kalman filters [5] are just some of the most important methods developed for frequency estimations of noisy sinusoids. Furthermore, the estimation of the frequency of a sinusoid is usually performed using a two-step method containing a coarse estimation and a fine search [6][7]. These fine search estimations have variances similar to that of the Cramer-Rao lower bound (CRLB) but nonetheless still show frequency-dependent performance. Iterative frequency estimation (IFE) [7] which is the state-of-the-art in estimation problem allows equal performance to be accomplished independent of the true signal frequency and the estimation accuracy to be improved.

In practical applications, for pulse Doppler radar signals with low signal to noise ratio (SNR), unavoidably, the Doppler frequency is quite diffused and very difficult to estimate. In order to solve the Doppler frequency problem with low SNR, we propose a weighted average for the iterative frequency estimation (WAIFE) which can alleviate the drawbacks to a certain extent.

This paper is organized as follows. The system model of the pulse-Doppler radar is presented in Section II. We describe the proposed WAIFE algorithm in Section III. To validate the proposed algorithm, Monte-Carlo simulations are provided in Section IV. Finally, a conclusion is drawn in Section V.

II. SYSTEM MODEL

We consider the pulse signals used for pulse-Doppler radar [8]. We consider these pulse signals, including the Doppler frequency, in order to extract the velocity of an object. For convenience of explaining the proposed concept, the radar signal is simply assumed to be composed of only N pulse repetition intervals (PRIs) during an M update time.

The radar signal, composed of only N PRI, is simply transmitted such that

$$s(t) = \left\{ A_T \cdot \sum_{j=0}^{M-1}\sum_{i=0}^{N-1} \exp\left[-2\pi \left(\frac{t - i \cdot T_{PRI} - j \cdot T_{UP}}{\tau_p} \right)^2 \right] \right\} \text{for} \quad 0 \le t \le (M-1)\cdot T_{UP} \tag{1}$$

where A_T is the amplitude of the transmitted signal, M is the total number of update times, N is the total number of PRIs, τ_p is a time-normalization factor, T_{UP} is the update time contained in the received signal with N PRI, and T_{PRI} is determined by means of $T_{PRI} = 1/f_{PRI}$.

At this point, the Doppler shift is estimated through an iterative method based on the DFE. The received signals $r_j(t)$ for the j-th update time are formulated in terms of the Doppler shift by

$$r_j(t) = \alpha_j s_j(t-\tau_j) e^{j(2\pi f_{d,j} t + \theta_j)} + \omega(t), \tag{2}$$

where α_j, $f_{d,j}$, τ_j and θ_j represent the complex amplitude, the Doppler frequency, the time delay, and the initial phase affected by moving objects during the j-th update time,

 ISBN: 978-1-61208-404-6

respectively. In addition, $\omega(t)$ is the complex additive white Gaussian noise (AWGN) with variance σ^2 .

Assuming that the received signals are sampled at a rate of $f_s = 1/T_s$, the received N ($T_{PRI}=T_s$) samples $r(nT_s)$, n=0,…,N-1 are changed into a sinusoid signal of the Doppler frequency. The received samples $r_j[n]$ are expressed as

$$r_j[n] = \alpha_j e^{j2\pi f_{dn,j} n} + \omega[n], \qquad (3)$$

where $f_{dn,j}$ is the Doppler frequency normalized during the j-th update time and $\omega[n]$ is the sampled value of $\omega(t)$ with $t=nT_s$. We also assume that the baseband signal is sampled at the peak point of $s(t)$. Here, the normalized Doppler frequency $f_{dn,j}$ is defined in terms of the Doppler frequency $f_{d,j}$ such that $f_{dn,j} = f_{d,j} / f_s = v/N$ during the j-th update time, where N represents the number of samples. The number v is composed of $k_{p,j}$ and δ, which are the integer part and the fractional part of the j-th update time, respectively.

III. Weighted Average for an Iterative Frequency Estimation (WAIFE)

In this section, we depict how the WAIFE is employed for the DFE of the received signal with time diversity and how the proposed estimator combines the IFE with a weighted-average scheme by considering the power of the received signal.

A. *Iterative Frequency Estimation*

From (3), the IFE is achieved in three steps during the j-th update time. The integer part $k_{max,j}$ is initially estimated from the index of the periodogram of the received signal; then, by conducting the first iterative scheme, the fractional part $\delta_{j,1}$ is estimated using the values of the periodogram at the appropriate frequencies. In the third step, the estimation of the fractional part $\delta_{j,2}$ is also iteratively achieved from the results of the periodogram using the previous integer $k_{max,j}$ and the fractional part $\delta_{j,1}$. The frequency corresponding to the maximum DFT amplitude coefficient is chosen as a frequency approximation of the ML estimate. We define DFT such that

$$y_j[k] = \sum_{n=0}^{N-1} r_j[n] e^{-j2\pi kn/N} \text{ for } k = 0,1,...,N-1, \qquad (4)$$

where the result of N point complex DFT operator, $\mathbf{y_j}$ =[$y_j[0]$, $y_j[1]$,…, $y_j[N-1]]^T$. Following Rife and Boorstyn [10], the first step of the frequency estimate, $\hat{f}_j$, can be obtained by means of

$$\hat{f}_j = \frac{k_{\max,j}}{N} f_s, \qquad (5a)$$

where

$$k_{\max,j} = \arg\max_k \{y_j[k]\}, \qquad (5b)$$

The power of the received complex amplitude $\hat{p}_j$ during the j-th update time is also estimated as

$$\hat{p}_j = \max\{|\mathbf{y}_j|^2\}, \qquad (6)$$

Through the first iterative scheme, the fractional part $\delta_{j,1}$ of the Doppler frequency can be estimated using the values of the periodogram. We define the modified DFT coefficients, X_p, using the equation below.

$$X_p = \sum_{n=0}^{N-1} r_j[n] e^{-2\pi n \frac{k_{\max,j}+p}{N}}, \; p = \pm 0.5., \qquad (7)$$

Based on the literature [7], we combine the input sinusoidal signal with X_p and carry out the necessary manipulations for the case of a noiseless condition. The yield is

$$\hat{\delta}_{j,q} = \frac{1}{2}\mathrm{Re}\left\{\frac{X_{0.5}+X_{-0.5}}{X_{0.5}-X_{-0.5}}\right\}, \qquad (8)$$

where the number of iterations $q=1$.

Last, to improve the estimation accuracy, we obtain the fractional part $\hat{\delta}_{j,2}$ using the second iterative scheme. Using the previous k_{max} and $\hat{\delta}_{j,1}$, the modified DFT with q=2 is achieved such that

$$X_p = \sum_{n=0}^{N-1} r_j[n] e^{-2\pi n \frac{k_{\max,j}+\hat{\delta}_{j,1}+p}{N}}, \; p = \pm 0.5, \qquad (9a)$$

and

$$\hat{\delta}_{j,2} = \hat{\delta}_{j,1} + \frac{1}{2}\mathrm{Re}\left\{\frac{X_{0.5}+X_{-0.5}}{X_{0.5}-X_{-0.5}}\right\}. \qquad (9b)$$

In order to obtain the fraction part $\hat{\delta}_{j,a}$ iteratively, we estimate the normalized Doppler frequency during the k-th update time using the equation

$$\hat{f}_{dn,j} = \frac{k_{\max,j}+\hat{\delta}_j}{N} f_s, \qquad (10)$$

where $\hat{\delta}_j = \hat{\delta}_{j,q}$ such that q=2.

B. *Weighted-Average Algorithm*

In practical applications with a low SNR, unavoidably, the Doppler frequency cannot be obtained accurately because it is diffused. To deal with this problem, the WAIFE considers the time diversity in the DFE. Through [9], when the Doppler frequency is slowly varied, the WAIFE precisely estimates the smoothed Doppler frequency. From (10), let $\hat{\delta}_j$ (j= 0, 1, . . . , M-1) and $\hat{p}_j$ denote the fractional part of the Doppler frequency and the received power during the j-th update time, respectively. Because the received signal with a good SNR is weighted, the WAIFE shows better performance.

To estimate a frequency that is contaminated with noise, the weighted-average Doppler frequency is expressed using

$$\hat{\delta}_{wa} = \sum_{j=0}^{M-1} \frac{\hat{p}_j \cdot \hat{\delta}_j}{\sum_{i=0}^{M-1} \hat{p}_i} \qquad (11)$$

Once the estimated $\hat{\delta}_{wa}$ is obtained, a new Doppler frequency can be constructed using

 ISBN: 978-1-61208-404-6

$$\hat{f}_{dn} = \frac{k_{\max} + \hat{\delta}_{wa}}{N} \tag{12}$$

IV. PERFORMANCE ANALYSIS

In this section, the root-mean-square error (RMSE) and the CRLB for the proposed WAIFE are derived. For an arbitrary $f_{dn,j}$ during the j-th update time, the RMSE of the iterative algorithm used for DFE was analyzed in earlier work [7] when the number of iterations is one in an AWGN channel. The RMSE of the proposed WAIFE is derived such that

$$\mathrm{var}\left[\hat{\delta}_{wa}\right] = \frac{\pi^2\left(\delta^2 - 0.25\right)^2\left(4\delta^2 + 1\right)}{4 \cdot M \cdot N \cdot SNR \cdot \cos^2(\pi\delta)}, \tag{13}$$

where the received power $\hat{p}_j$ (j=0,1,…,M-1) is constant at one in the j-th received signal and where SNR is the average signal-to-noise ratio.

According to the CRLB, the DFE is based on the frequency estimation for the transformed sinusoid of the Doppler frequency. For the Doppler frequency signal of (3) in the AWGN channel from earlier research [10], the CRLB for the DFE is derived such that

$$\delta_f^{\,2} \geq \frac{6 f_s^{\,2}}{2\pi \cdot SNR \cdot N \cdot M \cdot (N^2 - 1)}. \tag{14}$$

V. SIMULATION RESULTS

We present Monte-Carlo simulations to assess the Doppler frequency estimation performance of WAIFE. We define the RMSE as $\frac{1}{L}\sum_{n=1}^{L}\sqrt{(\hat{\delta}_{wa} - \delta)^2}$ with L=1,000. In the following simulations, we normally adopt a pulse-Doppler radar system as the setup with M=5, f_s=1, and N=1,024, and we assume that there is one moving source. Fig. 1 shows the RMSE values of various instances of δ in the proposed and in conventional estimators. Here, SNR = 0dB and N=1024 is used for practical situations of the proposed and conventional estimators. Compared to the ML estimator and the IFE with q=1, the proposed estimator has better RMSE performance than the conventional estimation such as [7] by more than about 40 times. Specifically, the proposed estimator shows performance similar to that of the CRLB at approximately -0.3 < δ < 0.3.

Fig. 2 presents the performance results for various estimators when δ =0.3. We compare our algorithm with the IFE and the ML methods. Fig. 2 shows that our algorithm is capable of much better Doppler frequency estimation performance and that the performance is very close to that of the CRLB. The proposed estimator shows that the analyzed and the simulated performances are in good agreement from approximately SNR=-13dB. Fig. 3 shows the RMSE of various estimators according to an increase in the FFT size N. When N increases, the RMSEs of the proposed estimator, the ML and the IFE is improved. In particular, with the proposed estimator, when N changes from 256 to 4,096, the RMSE characteristics improve by more than 60 times, with a change from 5.4e-5 to 8.6e-7. From Fig. 1 to Fig. 3, the proposed estimation improves the estimation performance through the strong evaluation compared with the conventional estimation.

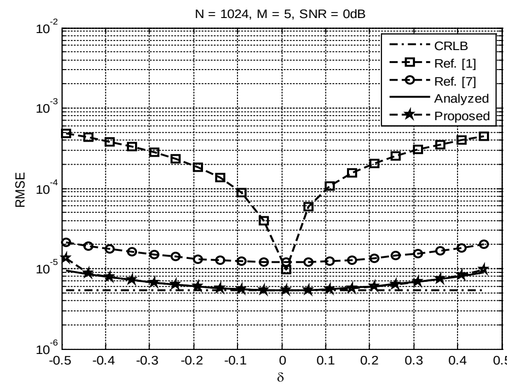

Figure 1. Performance of the WAIFE based on various δ with SNR = 0dB.

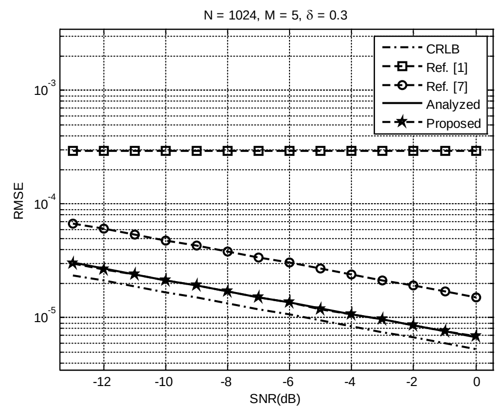

Figure 2. Performance of the WAIFE compared to that of the IFE and the ML methods with δ = 0.3.

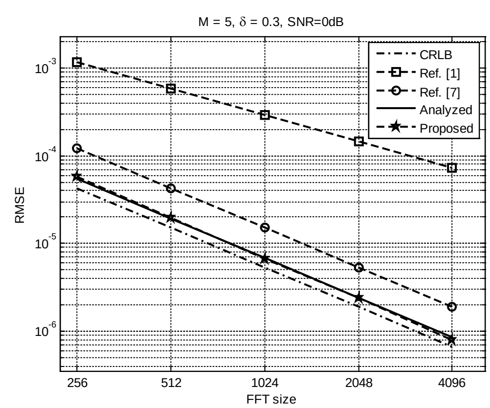

Figure 3. Performance of the WAIFE according to various FFT sizes with SNR = 0dB.

VI. CONCLUSIONS

In this paper, we proposed a WAIFE algorithm which improved the estimation performance of diffused Doppler frequencies in pulse-Doppler radar. Compared to conventional algorithms, our algorithm provided much better performance for the DFE and was capable of performance very close to that of the CRLB. The proposed algorithm worked well with other FFT sizes of N. At $-0.3 < \delta < 0.3$, the proposed estimator showed performance similar to that of the CRLB. In the future, the proposed estimator can be applied to various surveillance and safety systems based on pulse-Doppler radar that require good performance levels.

ACKNOWLEDGMENT

This work was supported by the DGIST R&D Program of the Ministry of Science, ICT and Future Planning, Korea (15-RS-01).

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