A Topology Aggregation-based Approach for the Unsplittable Shortest Path Routing Problem

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***Abstract*—The Unsplittable Shortest Path Routing (USPR) problem is one of the optimization problems that has been well studied in the field of traffic engineering for IP networks due to its importance for improving the network's Quality of Service (QoS). Given a directed graph representing the IP network and a set of commodities depicting the demands to be sent between its nodes, the USPR problem consists in identifying a set of routing paths and the associated administrative weights such that each commodity is routed along the unique shortest path between its origin and its destination following these weights. Due to the NP-hardness of this problem, we propose a topology aggregation-based approach to solve it, which consists in efficiently aggregating the network's graph to make the solving process more scalable. The experimental results show the efficiency of this proposal in terms of scalability and network's maximum load reduction compared to other methods from the state-of-the-art.**

***Keywords*—Traffic engineering; Mixed Integer Linear Programming; Topology Aggregation; Algorithms.**

I. INTRODUCTION

With the continuous growth of traffic and the strong requirement for optimizing the use of networks' resources, Traffic Engineering (TE) becomes a major requirement of telecommunication networks. The goal of Traffic Engineering is to fit traffic flows with networks' constraints and operators' requirements. For instance, an objective can be to minimize the load of some network links to prevent the network from overload.

Unlike in Multiprotocol Label Switching (MPLS) networks, which embed some suitable centralized or distributed mechanisms for TE, intra-domain TE in Open Shortest Path First (OSPF) and IS-IS networks using Interior Gateway Protocol (IGP) based on Shortest Path First (SPF) algorithms is a very challenging task. This is particularly true if networks managers want to manage fine-grained internet traffic dynamically with short time slots.

The routing mechanism implemented in such IGP networks is based on the computation of the shortest path between nodes, using as lengths the weights assigned to each link in the network by the routing protocol. In each router, the next link on all shortest paths to all possible destinations is stored in a table. A flow arriving at the router is sent to its destination by splitting the flow between the links that are on the shortest path to the destination.

Solving the problem of finding the best and possibly unique set of weights assigned to each link to guarantee the correct flow of traffic and to fit some traffic constraints is NP-hard. It cannot be solved exactly in an acceptable delay when the network size exceeds some dozen of nodes. When it is required that each flow should be routed on a unique shortest path between its origin and its destination, it is called the Unsplittable Shortest Path Routing (USPR) problem.

The problem is even more difficult when the network is composed of thousands of nodes, without mentioning the fact that we can expect to adapt to short delay traffic variations. An empirical approach proposed by CISCO is to assign weights inversely proportional to the links capacities. This way provides a practical traffic engineering solution but it also provides results far from the optimal, without uniqueness guarantee for paths flows, and without adaptation to traffic variations in time.

The framework of telecommunications' networks is commonly characterized as Scale Free because the growing of the number of edges is sub-linear to the growing of the number of nodes. They are very often composed with some nodes, designed as hub, strongly connected with their neighboring nodes, and connected together with some edges. Telecommunications' networks can also be characterized as sparse networks because the number of edges is far lower than the maximum number of links within the network. In the end, sometimes some nodes or edges can be characterized as separators because when removed, they split the network in two parts.

These common features occurring frequently in telecommunication networks suggest that a heuristic to solve some NP-hard problems like the USPR problem applied to networks graphs could be designed based on their usage. The principle would be to simplify the topology by using the structure of the network and trying to keep some useful properties of the graph to guarantee that the algorithm will provide acceptable solutions, in acceptable delays.

In this work, we propose a topology aggregation-based (TA) approach to solve the USPR problem. The idea is to aggregate some parts of the network graph to tailor the new graph to the solver performances applied on a Mixed Integer Linear Programming (MILP) modelling of the problem. This process could possibly be done in an iterative way. We conducted some experimentation on a set of various networks, and we achieved some promising results. To the best of our knowledge, using

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topology aggregation to tackle the USPR problem has not been considered in previous studies.

The rest of the paper is organized as follows. In Section II, we present state-of-the-art of topology aggregations' techniques in telecommunications' networks and of MILP formulations of the USPR problem. Section III provides the detailed description of the problem and our proposal. In Section IV, we present a set of experimental results. Finally, Section V summarizes the achievements of this paper and describes future works.

II. Related Work

The shortest path routing problem (splittable and unsplittable cases) has been widely studied in the literature [1].

In [2], the authors studied the problem of computing a set of unique weights enforcing a given set of routing paths. They then proposed linear programs to solve this problem and boundaries for heuristics based resolution methods. The authors in [3] studied the Internet traffic optimization through OSPF weights setting. They proposed a local search heuristic which provided good solutions close to the optimal for their particular network. In [4], a MILP formulation of the problem's splittable version was developed using a family of valid inequalities. The MILP model is solved with a branch and cut algorithm capable of providing solutions in an acceptable delay for small networks with a controlled optimality gap. A local search heuristic is used to initialize the branch and cut algorithm.

Bley proposed in [5] an integer programming algorithm for the minimum congestion unsplittable shortest path routing problem. A general decomposition approach is adopted to provide a master and a client sub problems, which results in a significant improvement for small and middle size problems solving. The authors in [6] studied the delay constrained unsplittable shortest path routing problem. They extended the results presented in [5] by adding new valid inequalities, strengthening the linear relaxation of the problem. Then, they used this new formulation as a building block to perform a dynamic programming approach resolution.

Approximating dense graphs by sparse ones or preserving some useful properties like cuts or spectral representations of the graph are explored in some papers (for instance in [7]). The idea is that some algorithms can run faster on the sparse graph approximation to find good solutions in a reasonable time. However, there are no reasons that these techniques are suitable for each problem to be solved on graphs.

Topology Aggregation (TA) for routing purposes in networks is a well studied subject in the literature [8]. In [9] and [10], a TA tutorial for hierarchical routing in ATM networks containing the basics and conventional methods for topology aggregation along with performance evaluation of several aggregation schemes is provided. The authors in [11] analyzed the impact of TA on different QoS routing algorithms. Multiple TA methods were used with different networks settings. In [12], a novel QoS model for topology aggregation in delay-bandwidth sensitive networks was presented. Later on, several works [13]–[15] used TA dealing with problems like Virtual Network Embedding, Service Function Chain Orchestration and Virtual Network Function placement for scalability or security issues.

In this paper, we propose a TA-based approach adapted for solving the Unsplittable Shortest Path Routing Problem, extending the work presented in [6].

III. Problem statement and proposal

In this section, we present a formal description of our problem followed by an in-depth look of the proposed solution.

A. Problem description

Let $G = (V, E)$ be a directed graph representing an IP network where $V = \{v_1, \ldots, v_M\}$ depicts the nodes of the network and $E \subset V \times V$ is the set of links connecting them. Each node $v \in V$ represents a router and each edge $e \in E$ corresponds to a logical link between adjacent nodes u and v with an associated capacity (bandwidth) denoted by c_{uv}. We denote by K a set of commodities representing the traffic demands to be routed between nodes of the graph G. Every commodity $k \in K$ is defined by a pair (s^k, t^k) where s^k and t^k being respectively the origin and destination of k. A traffic volume $D^k \geq 0$ to be routed from s^k to t^k is affected to each $k \in K$.

Let's recall that the USPR problem consists in finding a set of weights to assign to the graph's edges that will be used to generate a set of routing paths such that (i) there is a unique shortest path for each commodity and (ii) the network congestion is minimum. The load of an edge $uv \in E$ given a routing configuration $R \in \mathcal{R}(G, K)$, where $\mathcal{R}(G, K)$ depicts the set of all possible routing configurations of K in G, is the ratio between the total flow that goes through the edge and its capacity and it is defined as:

$$load(R, u, v) = \frac{1}{c_{uv}} \sum_{p^k \in R : uv \in p} D^k,$$

where a partial routing path for a commodity $k \in K$ in a graph G is a pair (p, k) denoted by p^k (p being a path).

The congestion $cong(R)$ of a routing configuration $R \in \mathcal{R}(G, K)$ is depicted as:

$$cong(R) = \max_{uv \in E} load(R, u, v).$$

It represents the maximum load over all arcs. The goal is then to minimize $cong(R)$, i.e., minimize the load of the most loaded link (denoted L).

B. MILP formulation

We define the binary variable x_e^k that takes the value 1 if commodity k is routed along a path containing edge e and 0 otherwise. Let u_e^t be a binary variable that takes 1 if e belongs to a shortest path towards destination t and takes 0 if not. We further depict w_{uv} as the weight assigned to the edge uv and r_v^u as the potential of node u, which is the distance between

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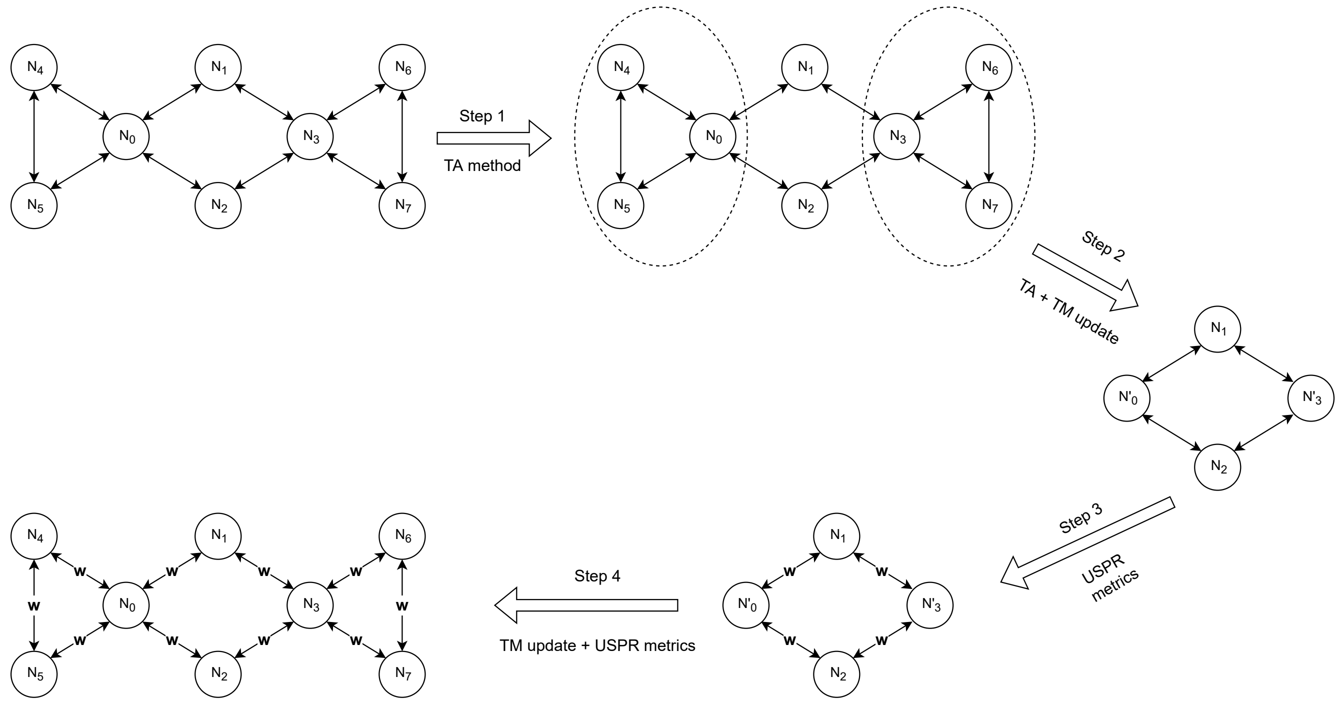

Fig. 1. Topology aggregation method for USPR.

nodes u and v. The USPR problem can then be described with the following MILP formulation [6]:

$$\text{minimize } L \tag{1}$$

$$\text{s.t.} \sum_{e\in\delta^+(v)} x_e^k - \sum_{e\in\delta^-(v)} x_e^k = \begin{cases} 1 & \text{if } v = s^k, \\ -1 & \text{if } v = t^k, \\ 0 & \text{otherwise,} \\ & {\scriptstyle \forall v\in V, \forall k\in K.} \end{cases} \tag{2}$$

$$\sum_{k\in K} D^k x_e^k \le c_{uv} L, \forall e \in E, \tag{3}$$

$$\sum_{e\in\delta^+(v)} u_e^t \le 1, \forall v \in V, \forall t \in T, \tag{4}$$

$$x_e^k \le u_e^{t^k}, \forall e \in E, \forall k \in K, \tag{5}$$

$$u_e^t \le \sum_{k\in K, t^k = t} x_e^k, \forall e \in E, \forall t \in T, \tag{6}$$

$$w_{uv} - r_u^t + r_v^t \ge 1 - u_{uv}^t, \forall uv \in E, \forall t \in T, \tag{7}$$

$$w_{uv} - r_u^t + r_v^t \le M(1 - u_{uv}^t), \forall uv \in E, \forall t \in T. \tag{8}$$

The objective (1) is to minimize the load of the most loaded link, denoted L. Inequality (2) ensures that a unique path is associated to each commodity k and (3) expresses the load over an edge e. Inequalities (4) and (5)-(6) are anti-arborescence and linking constraints, respectively. In particular, inequality (4) ensures that there is at most one path traversing any node v towards a given destination $t \in T$, which is necessarily implied by Bellman property. Constraints (7) and (8) guarantee that the weight of any edge used by a shortest path towards a destination t corresponds to the difference of potentials between the end nodes of this edge and larger otherwise (a more detailed formulation with additional valid inequalities and constraints is presented in [6]).

C. Solving USPR using TA

1) Topology aggregation-based approach: As mentioned earlier, the USPR problem is NP-hard and using the previously described formulation to obtain an optimal solution is not practical in terms of running time and scalability. In this paper, we propose a topology aggregation-based approach to be used combined with the MILP formulation to solve the USPR problem and generate near-optimal solutions efficiently. Topology aggregation is defined as a set of techniques that abstract or summarize the state information about the network topology to be exchanged, processed, and maintained by network nodes for routing purposes [8]. TA is generally applied for mainly two reasons: security concerns (hiding the network's internal topology) and scalability issues, which represents in this work our main concern. In our case, we want to use TA in order to compress the topology information and reduce its complexity to enable the application of the USPR solver and provide solutions in reasonable delays.

An overview of the approach proposed is depicted in Figure 1. The starting point is the network's topology to be studied, which is represented by the graph G and a traffic matrix K containing the set of demands to be routed between nodes. For sake of simplicity, G in Figure 1 is represented by a bi-directed graph, but in real networks, the upload and download links between nodes are separated and can have different weights. The first step consists in using an aggregation method (which will be detailed in the next section)

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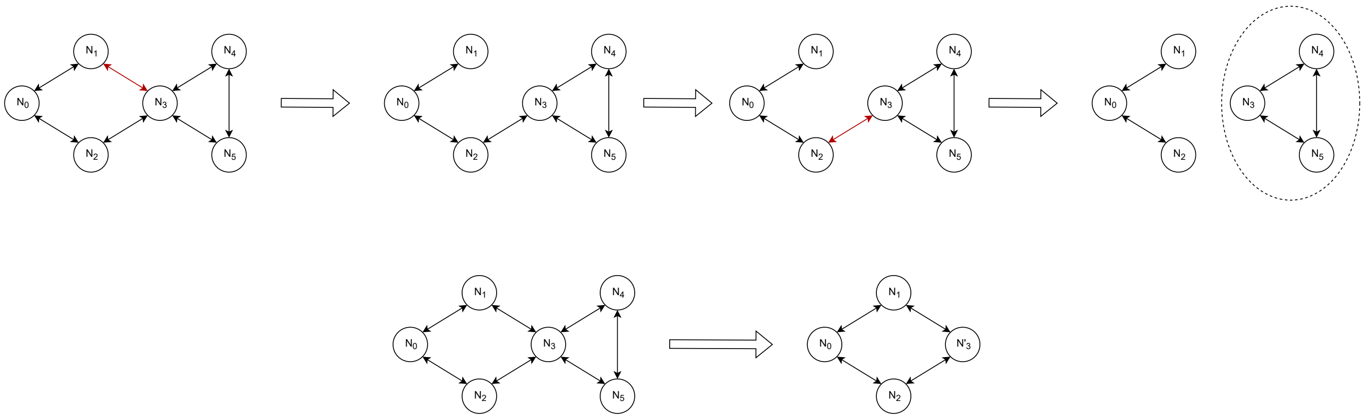

Fig. 2. GNC algorithm.

to select the nodes or the sub-graphs to be merged. Then, a new graph is generated according to the merged entities created along with a new traffic matrix where commodities will also be merged based on the aggregated graph. More specifically, the source (respectively the destination) of each commodity will be replaced by the aggregated node if it belongs to one. The next step now consists on applying the USPR solver on the newly generated graph to compute the set of different weights to be assigned to the links. In the next and final step, we adapt the initial traffic matrix (by keeping in each commodity only the sub-flows that have not been treated yet) in order to be able to apply the USPR solver on the aggregated sub-graphs. At the end, we have a full configuration of weights on all the links.

Algorithm 1 Aggregation algorithm

procedure GNC(Graph G)
  $List\_Aggregations \leftarrow \{\}$
  while $E \neq \emptyset$ **do**
    Calculate betweenness scores for all links in the graph
    Remove from the graph the link with the highest score
    if Disconnected sub-graph G' forms a clique **then**
      $List\_Aggregations.insert(G')$
    end if
  end while
  return List_Aggregations
end procedure

2) Topology aggregation method: As we have seen previously, our goal is to select relevant sub-graphs from the initial topology to be aggregated. This choice will heavily impact the generated links' weights and thus, the network's maximum load that we want to minimize. In this paper, we propose a topology-aggregation method (see Algorithm 1) based on the Girvan-Newman (G-N) algorithm [16]. The Girvan–Newman algorithm detects communities by progressively removing edges from the original graph. The algorithm removes the "most valuable" edge, traditionally the edge with the highest betweenness centrality, at each step. The betweenness of a particular link is determined by computing the shortest paths for each couple of vertices in the graph representing the network and counting how many times each link appears on those shortest paths [17].

During each step of the G-N process and after computing all the links' betweenness values, the link having the largest value is removed. These steps will be repeated until a stopping condition is reached. Otherwise, the process will result in the removal of all the links, which reduces the network to its nodes. A stopping condition can be for instance the number of communities of a given size. In [16], the authors use the modularity metric as a quality indicator for the clustering process. This metric evaluates the partitioning of a graph by computing the ratio of intra-communities edges to the number of inter-communities edges (see [16] for details on the modularity metric's formula). In our process and at each link removal step, if an emerging community represented by a disconnected sub-graph forms a clique, it is aggregated (see Figure 2). Let's recall that a clique of a graph G is a sub-graph that is complete where each node is directly connected to all the others. In case where no cliques are found, we select sub-graphs having a maximum diameter of 2, *i.e*, the length of the shortest path between the most distanced nodes in the sub-graph.

In the proposed aggregation process, using the G-N algorithm to form communities will allow us to avoid aggregating the most valuable edges by removing them progressively from the network. These links are most likely to carry the maximum loads and as a result, their abstraction can lead the USPR solver to increase the overall maximum load. Conversely, the communities to be aggregated are set up with the less valuable edges.

IV. Results

We present in this section the experimental results related to our proposed TA-based approach for solving the USPR problem.

TABLE I. NETWORKS TOPOLOGIES CHARACTERISTICS.

Topology	$\lvert V \rvert$	$\lvert E \rvert$	$\lvert K \rvert$	Traffic pattern	Traffic volume	Links capacities	Avg node degree
Abilene	12	30	132	Uniform	Random	Uniform	2.50
Atlanta	15	44	210	Uniform	Random	Random	2.93
Newyork	16	98	240	Uniform	Random	Uniform	6.12
France	25	90	300	Random	Random	Uniform	3.60
Norway	27	102	702	Uniform	Random	Uniform	3.78
Nobel-us	14	42	91	Random	Random	Uniform	2.93
Nobel-ger	17	52	121	Random	Random	Uniform	3.06
Nobel-eu	28	82	378	Random	Random	Uniform	3.00

TABLE II. COMPARISON RESULTS.

Topology	InvCap	IGP-WO	NRPA	GNC	Exec time GNC	Exec time NRPA and IGP-WO	Exec time OPT
Abilene	48.12%	0%	0%	0%	0.3 min	10 min	1 min
Atlanta	54.58%	5.04%	5.04%	36.52% (0%)	2 min	10 min	4 min
Newyork	68.88%	37.77%	44.4%	28.8% (2.22%)	2 min	30 min	$>$ 4320 min
France	70.92%	19.50%	19.50%	60% (15.7%)	1 min	30 min	$>$ 4320 min
Norway	55.55%	7.40%	11.11%	59.31% (18.51%)	3 min	60 min	$>$ 4320 min
Nobel-us	53.51%	2.06%	2.06%	0%	6 min	10 min	58 min
Nobel-ger	43.15%	13.69%	13.69%	9% (6.75%)	1 min	10 min	122 min
Nobel-eu	24.74%	0.28%	0.28%	4.12% (1.31%)	8 min	30 min	1250 min

A. *Parameters*

The experiments were performed on networks with various sizes and characteristics taken from SNDlib [18]. The characteristics of the various network graphs tested are shown in Table I. A traffic pattern is considered uniform if it exists in K one commodity between each couple of nodes of the network (otherwise, it is a random one). A uniform traffic volume means that all the demand in the traffic matrix are equal (the same definition applies also for links capacities). Our proposal was implemented in Python using the NetworkX library for graph-related tasks, and Cplex (with the default settings) was used for the exact solving of USPR.

We compare our algorithm to the following common approaches from the literature:

- InvCap: a practical approach suggested by Cisco, this configuration sets the links weights inversely proportional to the capacities.
- IGP-WO: proposed in [19], this approach is based on a local search algorithm using dynamic graph algorithms to tackle the links' weights optimization.
- NRPA: in this approach [20], a Monte Carlo Search algorithm is used to solve the USPR problem through the application of Nested Rollout Policy Adaptation algorithm.

In order to evaluate the efficiency of our algorithm GNC and compare it to the other ones, we depict for each approach the execution time and the maximum load gap relative to the optimal value obtained by solving the MILP formulation of the problem using an exact approach OPT (or relative to a lower bound if no solution is found). The depicted results of IGP-WO and NRPA in Table II represent the average score of 5 executions as presented in [20].

B. *Performance evaluation*

The comparison results are shown in Table II. Regarding the execution time, we can see that GNC performs very well compared to NRPA, IGP-WO and especially OPT. The impact of topology aggregation on lightening the heavy computations to solve the USPR problem is very straightforward. In terms of maximum load gap, the performance depends on the tested topology network and its characteristics. For example, considering the Abilene results, all three approaches achieve a gap of 0%, which means that they achieve the optimal value. Looking at Atlanta topology, our algorithm does not perform well (36.52%) compared to IGP-WO and NRPA (5.04%). However, if a local search method is applied to the aggregations list generated by GNC (values reported between parentheses in GNC column), we can achieve excellent results (0%). Considering for example (N_0, N_1, N_2) as an initial aggregation, the local search method employed consists in replacing one node by another neighbor one (for instance to obtain (N_0, N_1, N_3)) or add a node to the aggregation list and check if it improves the obtained results. These results improvements show the potential of using our TA-based approach in general and the improvements that can be done by improving the aggregation level. We obtain similar results with France and Norway networks where a topology aggregation configuration can be found to achieve better results. For the remaining networks, our GNC algorithm outperforms IGP-WO and NRPA. For now, there is no clear correlation between

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the characteristics of the tested networks and the performance achieved by our algorithm (even in the case of a traffic matrix having a uniform pattern). Another point to be highlighted is that our approach (using the basic version of GNC) is deterministic compared to IGP-WO and NRPA, where results may differ with each experimental run.

V. Conclusion and future work

In this paper and in the context of traffic engineering, we tackle the Unsplittable Shortest Path Routing problem (USPR), which consists in finding the efficient weights of an IP network to handle traffics flows. The USPR problem is proven to be NP-hard and thus, many studies have used meta-heuristics to solve it. In this work, we propose a novel approach based on the Topology Aggregation (TA) paradigm in order to solve an USPR problem instance. The proposal consists of a TA-based methodology applicable to our use case in which an approach called GNC based on the Girvan-Newman community detection algorithm is implemented. The conducted experiments to evaluate GNC and compare it to other methods have shown the efficiency and the potential of using a TA-based method to solve the USPR problem. We intend in the future to improve our aggregation algorithm and conduct more extensive experiments to validate our work by testing other network configurations with challenging settings and analyze the correlation between the network's characteristics and the algorithm's performance. We also intend to study the delay-constrained version of USPR.

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