

# An Electricity Flow Optimization Problem for Effective Operation of Storage Devices in Microgrid

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**Abstract**—An electrical energy storage device has been expected to alleviate negative effects of the integration of renewable energy resources into electricity supply systems; however, the appropriate storage technology along with the control characteristics as a component in the electricity supply system has not been clarified yet. As one of approaches, an effective control of electricity flow and an operation of storage devices have been developing. This paper formulates an electricity flow optimization problem in reducing transmission loss to analyze the fluctuation of it followed by the adjustment of the amount of electricity stored in storage devices.

**Keywords**—*electrical energy storage device; microgrid; renewable energy resource; transmission loss; transportation problem.*

## I. INTRODUCTION

Smart Grid, a concept of incorporating ICT (Information and Communication Technology) into electricity supply system, is being developed from various perspectives, such as enhancing the reliability of electricity supply system and realizing low carbon society with renewable energy resources. On the other hand, to reduce the amount of transmission loss, physical loss of electrical energy while it is transmitted though transmission line is also main objective of Smart Grid technology [1].

In a traditional power grid, the large amount of electricity can be lost since electricity is supplied from a few large-scale power plants located in rural areas to consumer sides. In contrast, by exploiting renewable energy resources as distributed energy resources and controlling them effectively with the concept of Smart Grid, the reduction of transmission loss has been expected [2].

However, the generation patterns of those renewable energy resources fluctuate irregularly due to weather conditions and it may result in the fluctuation of frequency and voltage in a power grid. As a result, the degradation of the stability in power grid might be caused when a large amount of them are introduced in electricity supply system [1][2][3].

To deal with the drawback of the renewable energy resource, management strategies of them are studied in many research works [3]; Microgrid (MG), forming a small-scale power grid composed of local electricity consumers and distributed energy resources has been proposed to realize a self-sufficient supply system in local areas [2]. In [4], it is stated that MG with its locality can make energy management of renewable energy

resources improved as well as reducing transmission loss. In addition, as a way to mitigate negative effects of the integration of renewable energy resources, utilizing grid-integrated electrical energy storage devices has been considered as well [5][6]. This is because it can be used to stabilize the short-time frequency and voltage fluctuation, manage peak loads, and improve the power quality [7][8].

### A. Related Work of Storage Device

A lot of research related to the use of the storage device in electricity supply systems have been developing in order to improve the efficiency and reliability of the storage technology [9]. Moreover, although there are various types and sizes of storage devices for different purposes, a large-scale storage device installed beside renewable energy resources has been developing. This is because if renewable energy resources and storage devices are installed in geographically separated places, transmission loss may arise to send surplus electricity from renewable energy resources to storage devices to store it. Furthermore, storage devices have been also expected as a backup source of electricity for an emergency, such as when natural disaster occurs from the perspective of the stability and durability of the electricity supply system [10].

Many of the current research related to storage technology mainly focus on the power balance for stable and reliable electricity supply [6][11]; however, a full potential of the storage device and an appropriate storage technology along with the control characteristics as a component of the electricity supply system have not been clarified yet and remained as a research task [11] [12]. As one of approaches to analyze the control characteristics, an effective operations of the storage device by optimizing electricity flow has been studied [9][13].

### B. Research Objective

This paper assumes the situation that storage devices are installed beside renewable energy resources which generate surplus electricity in MG. Then, this paper tries to smooth the amount of electricity stored in storage devices by adjusting the amount of electricity supplied from renewable energy resources. This is to distribute the amount of surplus electricity as backup sources for the realization of a durable electricity supply system for an emergency.

In this situation, the amount of electricity generated in a renewable energy resource can be divided into two types of use, the amount of electricity supplied from a renewable energy resource and the amount of electricity stored in a storage device; that is, there is a dependency between the amount of electricity supplied from the renewable energy resource and the amount of electricity stored in the storage device. Moreover, transmission loss depends on how much and how far electricity is transmitted. As a result, it might be indicated that smoothing the amount of electricity stored in storage devices may affect the total transmission loss. Therefore, this paper analyzes how transmission loss is influenced by various patterns of the scatter of the amount of electricity stored in each storage device.

The rest of the paper is organized as follows: In Section I I, the definition of MG graph, such as types of nodes and the amount of electricity, and problem formulation are mentioned. A simulation procedure and result are presented in section II I. Section IV concludes the paper with future work.

## II. MODEL DEFINITION AND PROBLEM FORMULATION

In this section, firstly, the model of MG graph is defined. Then, an electricity flow optimization problem is formulated.

### A. Definition of Microgrid Graph

This paper assumes MG graph composed of different types of node and edge. First of all, let a node  $G$  be Central Generation Facility (CGF) node utilizing the chemical energy stored in fossil fuels such as coal, fuel oil, natural gas which is converted into electrical energy. In reality, CGF will be assigned to each MG as a supplemental power resource for renewable energy resources; accordingly, this paper considers as though a single  $G$  node is included as a component of MG.

Secondly, for renewable energy resources, although there are different kinds of them, such as solar or wind power generation in reality, this paper does not consider those differences but assume them just as a renewable energy resource. Furthermore, this paper supposes an installation of the storage device beside renewable energy resources mentioned in Section I-B. Thus, let a node representing renewable energy resources contain the function of the storage device and be defined as renewable energy resource with storage device node,  $RS_i$ .

Finally, a node for electricity demand, such as houses, factories, hospitals and so on is defined as  $D_j$ . This paper assumes that the number of electricity demand nodes in MG graph,  $m$ , is greater than that of RS nodes,  $n$ .

Those three kinds of nodes are connected in the following manners: Every electricity demand node is connected to  $G$  so that they can receive electricity from the stable power resource. Furthermore, each electricity demand node is connected to all RS nodes in MG graph to simplify the problem discussed in the following subsections.

### B. Problem Formulation

MG graph describing node types and electricity flows is shown in Figure 1. Let  $d_j$  be the amount of electricity demand

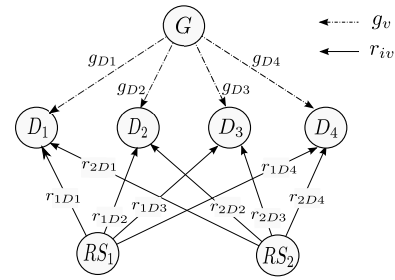


Fig. 1. A structure of MG graph.

in node  $D_j$ , and the amount of electricity derived from  $G$  node to  $D_j$  is  $g_j$ . The amount of electricity generated in  $RS_i$  is  $r_i$ , and the amount of electricity supplied from  $RS_i$  to  $D_j$  is  $r_{ij}$ . The surplus electricity generated in renewable energy resources can be stored in a storage device. Hence, let the amount of surplus electricity stored in  $RS_i$  be  $s_i$  which can be described as the subtraction of the total amount of  $r_{ij}$  for  $j \in D$  from the amount of  $r_i$  as in (1). The sum of  $r_{ij}$  for  $j \in D$  is also defined as  $r'_i$  as in (2). Finally, let a rate of transmission loss along with electricity transmission between  $RS$  nodes and  $D$  nodes be defined as  $loss_{ij}$ .

$$s_i = r_i - \sum_{j=1}^m r_{ij} \quad (1)$$

$$r'_i = \sum_{j=1}^m r_{ij} \quad (2)$$

There are several constraints to match demand and supply. Firstly, the amount of electricity demand in each  $D$  node has to be fully met with the total amount of electricity supplied from  $G$  node and  $RS$  nodes stated in (3).

$$d_j = g_j + \sum_{i=1}^n r_{ij} \quad (3)$$

This paper focuses on optimizing electricity flows from RS nodes to D nodes. Thus, let  $d'_j$  be the amount of the electricity subtracted  $g_j$  from  $d_j$ , denoting the amount of electricity that RS nodes need to supply to  $D_j$ . In order to match demand and supply in MG, the condition in (4) must be satisfied.

$$\sum_{j=1}^m d'_j = \sum_{i=1}^n r'_i \quad (4)$$

In this paper, for randomly given  $d_j$  and  $r_i$ , the value of  $s_i$  is set at first, and  $r'_i$  is determined accordingly. Afterward,  $r_{ij}$  is optimized.

1) *Transportation Problem*: To decide  $r_{ij}$ , this paper applies Transportation Problem (TP). TP is based on supply and demand of commodities transported from several sources to different destinations [14]. In general, TP tries to minimize total transportation cost for the commodities transported from source to destinations. Applying TP for an electricity flow

optimization, this paper considers to minimize total transmission loss from  $RS$  to  $D$  nodes. The objective function of the electricity flow optimization can be formulated as in (5).

$$\left\{ \begin{array}{l} \text{Minimize } \sum_{i=1}^n \sum_{j=1}^m r_{ij} \text{loss}_{ij} \\ \text{Subject to } \sum_{j=1}^m r_{ij} = r'_i, \quad \text{for } i = 1, 2, \dots, n \\ \sum_{i=1}^n r_{ij} = d'_j, \quad \text{for } j = 1, 2, \dots, m \\ r_{ij} \geq 0 \quad \text{for } i = 1, 2, \dots, n \text{ and } j = 1, 2, \dots, m \end{array} \right. \quad (5)$$

2) *Method to Solve Transportation Problem*: There are mainly two methods to obtain the solution for TP, Simplex method and Transportation method. This paper applies Transportation method since it is more efficient one that yields results faster and with less computational effort [14]. Transportation method consists of the following three steps.

- 1) Obtaining an initial feasible solution.
- 2) Testing Optimality.
- 3) Revising the solution until an optimal solution is obtained.

For conducting those three steps, in general, TP can be portrayed in a tabular form by means of a transportation table shown in (2). In a transportation table, the number of row represents that of RS node,  $n$ , and the number of column denotes that of D node,  $m$ .

RS node( $i$ )	D node( $j$ )				Supply( $r'_i$ )
	1	2	...	$m$	
1	$r_{11}$ $\text{loss}_{11}$	$r_{12}$ $\text{loss}_{12}$	...	$r_{1m}$ $\text{loss}_{1m}$	$r'_1$
2	$r_{21}$ $\text{loss}_{21}$	$r_{22}$ $\text{loss}_{22}$	...	$r_{2m}$ $\text{loss}_{2m}$	$r'_2$
...	...	...	...	...	...
$n$	$r_{n1}$ $\text{loss}_{n1}$	$r_{n2}$ $\text{loss}_{n2}$	...	$r_{nm}$ $\text{loss}_{nm}$	$r'_n$
Demand( $d'_j$ )	$d'_1$	$d'_2$	...	$d'_m$	$\sum d'_j = \sum r'_i$

Fig. 2. A transportation table.

3) *North-West Corner Method*: The first step is to obtain an initial feasible solution which satisfies the requirement of demand and supply. Although an initial feasible solution can be obtained by several methods, this paper applies North-West Corner method (NWC), a method to compute an initial feasible solution, which begins selecting a basic variable in the upper left-hand corner of the transportation table. The algorithm of NWC is described in Algorithm 1.

4) *Modified Distribution Method*: Once an initial solution is obtained by NWC, the next step is to check its optimality. Although there are several methods to find an optimal solution of TP, such as Vogel's method or Stepping Stone method,

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#### Algorithm 1 North-West Corner Method (NWC)

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**Require:**  $\sum r'_i = \sum d'_j$   
 Start with the cell at the upper left-hand corner  
 $i, j \leftarrow 0$   
**while**  $i < n$  **do**  
   **if**  $r_{ij} > d'_j$  **then**  
      $r_{ij} \leftarrow r_{ij} - d'_j$   
   **else if**  $r_{ij} < d'_j$  **then**  
      $d'_j \leftarrow d'_j - r_{ij}$   
      $i \leftarrow i + 1$   
   **else**  
      $i \leftarrow i + 1$   
      $j \leftarrow j + 1$   
   **end if**  
**end while**  
 Calculate initial transmission loss

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Modified Distribution method (MODI) has been used as a standard technique for obtaining an optimal solution [14]. This paper applies MODI for obtaining optimal solution, and the algorithm of MODI is described in Algorithm 2.

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#### Algorithm 2 Modified Distribution Method (MODI)

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**Require:** An initial feasible solution by NWC  
**while** all  $p_{ij}$  are not a positive value **do**  
   Determine the values of dual variables,  $u_i$  and  $v_j$   
   **for** each unassigned cell **do**  
      $p_{ij} \leftarrow c_{ij} - (u_i + v_j)$   
   **end for**  
   **if** all  $p_{ij} \geq 0$  **then**  
     Optimization is completed  
     Calculate optimized transmission loss  
   **else**  
     **if**  $p_{ij} < 0$  and  $\min(p_{ij})$  **then**  
       Draw a closed path starting from  $r_{ij}$   
       Assign + or - sign on the cells in it alternately  
       Find  $\min(r_{ij})$  in cells with - sign  
     **end if**  
     **for** each cells in closed path **do**  
       **if** sign is + **then**  
          $r_{ij} \leftarrow r_{ij} + \min(r_{ij})$   
       **else**  
          $r_{ij} \leftarrow r_{ij} - \min(r_{ij})$   
       **end if**  
     **end for**  
   **end while**

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### III. SIMULATION AND RESULT

In this section, the simulation procedure and result are presented.

#### A. Simulation Procedure

In this simulation, MG graph is created with 4 RS nodes, 8 D nodes, and a single G node. The sum of  $r_i$  and  $d_j$  are set

as 1200 and 1000. Then, assigning a random value in each  $r_i$  for  $i = 1$  to 4 and  $d_j$  for  $j = 1$  to 8 such that the sum of  $r_i$  is equal to 1200, that of  $d_j$ , 1000, respectively. Note that it is assumed that about 80% of each  $d_j$  is supplied by  $G$  node in this simulation. As a result,  $d'_j$  is set as the subtraction of  $g_j$  from  $d_j$ . The total amount of surplus electricity is determined by the difference of the sum of  $r_i$  and that of  $d_j$ , which is 200. Based on this value, the value of  $s_i$  is given. This simulation creates 2000 patterns of the set of  $s_i$  for  $i = 1$  to 4 such that the sum of  $s_i$  is 200. A pattern of  $s_i$  is, for instance,  $s_1=37.54045$ ,  $s_2=62.13593$ ,  $s_3=45.95469$ ,  $s_4=54.36893$ . Once the value of  $s_i$  is given, set the value of  $r'_i$ , subtracting  $s_i$  from  $r_i$ . Then, decide  $r_{ij}$  such that transmission loss is minimized by means of NWC and MODI method discussed in Section II-C. Note that  $loss_{ij}$  is randomly assigned to each edge between  $RS$  and  $D$  nodes with the value of the second decimal place in the range shown in (6). This is because about 5% of electricity would be lost during transmission to consumer sides in general although it depends on the distance it is transmitted.

$$0 < loss_{ij} \leq 0.05 \quad (6)$$

### B. Simulation Result

A correlation between scattering of  $s_i$  and transmission loss in Figure 3. In Figure 3, the horizontal axis describes the standard deviation of the pattern of  $s_i$ , and the vertical axis represents the transmission loss optimized by MODI.

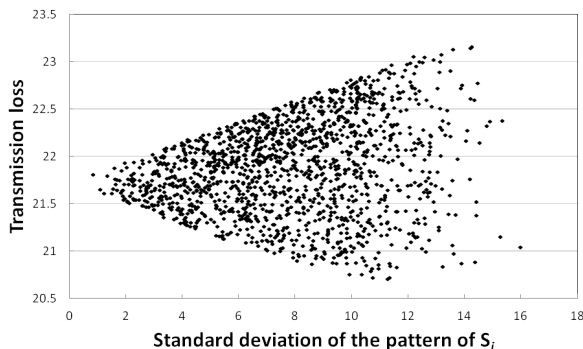


Fig. 3. Correlation between scattering of  $s_i$  and transmission loss.

As seen in Figure 3, when the value of standard deviation of the pattern of  $s_i$  is close to 0, the transmission loss gets the value close to the average and does not get the worst value. In addition, the difference of the best and worst value of the transmission loss is smaller as a standard deviation of the pattern of  $s_i$  is close to 0.

This result may indicate that even if the amount of electricity stored in several storage devices is smoothed by adjusting electricity flows, the transmission loss would almost take the average value although it is not the best improved value.

### IV. CONCLUSION AND FUTURE WORK

In summary, this paper formulates an electricity flow optimization problem, presents its solution, and analyzes the

fluctuation of transmission loss followed by the adjustment of the amount of electricity stored in storage devices.

The simulation result implies that when the amount of electricity stored in storage device is smoothed by adjusting the electricity flows, the value of the transmission loss almost gets the mean value compared to other patterns. However, this result comes out under a certain situation given in this paper, and other result might be obtained in different situations. To examine it, therefore, further simulation in various situations, such as different node numbers and their values need to be conducted. Moreover, since this paper does not cover all aspects of prospective electricity supply system, there are remained issues to make this research more practical. For example, this paper assumes that each  $D$  node is connected to all  $RS$  nodes in MG for the simplification of the electricity flow optimization problem. Hence, the paper needs to reorganize the structure of the MG which has less transmission lines by considering the effective connections between  $D$  nodes and  $RS$  nodes.

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