A Matching Problem in Electricity Markets using Network Flows

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***Abstract*—This paper proposes a many-to-one matching algorithm between sellers and buyers in a deregulated electricity market model that can deal with the limited amount of supply of electricity. Our matching algorithm aspires safe supply of electricity and maximization of social welfare, which indicates overall satisfaction of participants in the markets. In order to satisfy these goals, the matching mechanism is developed based on the concept of a maximum flow problem in graph theory. Additionally, the piled knowledge in the field of economics about matching in markets is also applied into design of this algorithm. Simulation results shows that this algorithm can find whether there is the maximum many-to-one matching in a given model of electricity market or not.**

Keywords*—*bipartite graph; buyer-seller networks; many-to-one matching; maximum flow problem; social welfare.

I. Introduction

Many countries have constructed electricity grids with centralized structure, in which a few sellers generate electricity and supply it. This structure has arisen from need for providing electricity to buyers safely. Generators in that system must be responsible for safe supply, and they can get adequate remuneration for this responsibility.

The main concerns of electricity buyers are both safe supply and low prices. Recently, failures in the electricity grids have been decreasing more than ever [1]. Thus, the requests of buyers for reducing the electricity prices become more remarkable. However, some problems exist in electricity markets on the centralized electricity grids. First, the sellers do not tend to reduce the prices because they have few competitors. Second, buyers are doubtful about transparency of current prices because no buyers have sellers who can be compared with current sellers.

Recently, there have been some actions for restructuring electricity markets in many countries [1], and deregulation of the electricity market is one of them. This action aims to construct more competitive markets, which are expected to bring technological innovation and lower prices.

In deregulated electricity market, a Power Producer and Seller (PPS) conducts retail of electricity by using electricity grids owned by other generators. The PPS has small capacity to generate electricity; hence, they must get electricity from another generator when demand is beyond own capacity. PPS cannot get electricity as much as they want because they will face a severe business situation if they continue to do that.

Although buyers want to get the electricity from a seller offering a lower price than the others, some buyers cannot contract with such a seller if demand for the seller exceeds its capacity [2]. These buyers must pay higher costs to get electricity from one of the other sellers. In this situation, it is realized that some buyers who can purchase the electricity from the cheapest seller may be satisfied, but the satisfaction of the others will be diminished.

This research proposes an algorithm to find a *matching* between sellers and buyers in a static model of an electricity market. By changing prices offered by sellers in high demand, the algorithm tries to find the prices that satisfy all supply and demand, and give the highest social welfare to us.

In Section II, some related works are introduced. Section III explains definitions of methods used in this research. Section IV proposes an algorithm that finds a matching between the seller and the buyer. Section V discusses simulation results, and Section VI consists of conclusion and future work.

II. Related Work

In 1962, Gale and Shapley discussed a *stable marriage problem* and a *college admissions problem* [3]. The stable marriage problem is a classic problem of *one-to-one matching*, and the matching is called *perfect matching* when the number of elements is the same on both sides of the matching. The objective of the college admissions problem is to find a proper matching between colleges and students. Colleges have the capacity to accommodate some students, and, on the other hand, students must be linked to only one college; thus, this type of matching is called *many-to-one matching*.

After the research done by Gale et al. [3], various researches on matching problems have been conducted. A model of matching problems can be represented as a bipartite graph; hence some researchers consider the matching problems based on *graph theory*.

Matching market is a method that constructs a matching between sellers and buyers in the networked market with a pricing mechanism, and this method is explained by Easley and Kleinberg [4]. The network used in matching market is described as a *buyer-seller network* that is discussed in [5]. The utilization of this type of network provides two merits. First, a buyer can choose a seller independently based on its evaluation of the seller. This is practical for the use in real markets because the markets do not ever have the central coordination of the choice of buyers. Second, efficiency of the market can be examined by social welfare, which is the total of utilities of all market participants. Even though both matching market and general economics deal with markets, they differ in network structure because matching market does not use

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anonymous networks. Matching market deals with networks that has typical structure, in which there are some buyers, sellers, and links between them. Thus, matching market shows how market participants affect each other in the network.

In an ordinary method of matching market, sellers and buyers deal with a single item, and the matching algorithm proposed by [4] constructs only perfect one-to-one matching. This paper proposes an algorithm finding a many-to-one matching in which all buyers and sellers deal with different quantity of electricity. For that reason, this paper integrates methods of a *feasible flow problem* and a *single-source unsplittable flow problem* with the matching market algorithm. A feasible flow problem is an application of a maximum flow problem that finds whether there is network flow that satisfies supply and demand of every node or not [6]. This problem is solved by a *circulation problem* explained by Jungnickel [7]. Besides, a single-source unsplittable flow problem finds the paths from a source to sinks in a network [8][9][10].

III. Definitions

This section explains definitions on network structure and methods used in this research. In this paper, it is assumed that electricity does not decrease anywhere in market models.

A. Market Settings of Sellers and Buyers

This subsection discuses property of the networked electricity market model. A buyer-seller network is represented as a bipartite graph containing two node sets. One is a set N_s containing sellers s_i $(1 \leq i \leq n)$, and the other is a set N_b consisting of buyers b_j $(1 \leq j \leq m)$; thus, the sizes of N_s and N_b are $|N_s| = n$ and $|N_b| = m$.

Each seller s_i and buyer b_j has its *supply capacity* c_i and *consumption quantity* q_j respectively. Let $c_{\min}$ be the minimum supply capacity of all sellers, and let $q_{\max}$ be the maximum consumption quantity of all buyers; then, as in other researches that consider the single-source unsplittable flow problem, $c_{\min}$ and $q_{\max}$ must satisfy $q_{\max} \leq c_{\min}$. In addition, for safe supply, q_j and c_i must satisfy $\sum_{j=1}^{m} q_j \leq \sum_{i=1}^{n} c_i$.

In the rest of this subsection, *valuation*, *price*, and *payoff* will be defined based on the theory of [4]. Definitions of these variables in [4] consider only single item. However, electricity is not a single item. Then, this paper extends definition of these variables to deal with different unit of electricity.

b_j decides valuation v_j $(v_j \geq 0)$, which is the maximum acceptable price of one unit of electricity for b_j. The valuation is also called *reservation price* [11]. Additionally, every buyer has distinct valuation value, and all participants of the market do not act cooperatively; hence, valuation of one buyer is private information for the other buyers. Furthermore, s_i offers a price of one unit of electricity p_i $(p_i \geq 0)$ to all buyers.

s_i and b_j have a payoff $u(s_i)$ and $u(b_j)$ respectively. As in [4], even though payoff of a seller is generally calculated by subtracting supply cost from sales, for simplicity, all sellers produce electricity at zero cost in our model. Therefore, if s_i sells $y(s_i)$ units of electricity to buyers, $u(s_i)$ will be

$$u(s_i) = p_i y(s_i). \tag{1}$$

In terms of buyers, $u_{b_j s_i}$ represents payoff of b_j for one unit of electricity offered by s_i; hence, $u_{b_j s_i}$ is defined as

$$u_{b_j s_i} = v_j - p_i. \tag{2}$$

Because b_j needs to calculate payoffs towards each seller s_i with valuation v_j, let $\boldsymbol{u}_{b_j}$ be a payoff vector of b_j, such that

$$\boldsymbol{u}_{b_j} := (u_{b_j s_1} \ u_{b_j s_2} \ \cdots \ u_{b_j s_n}). \tag{3}$$

Consequently, if b_j purchases $y(b_j)$ units of electricity from s_i, the payoff of b_j will become

$$u(b_j) = u_{b_j s_i} y(b_j). \tag{4}$$

B. Structure of a Preferred-Seller Graph

A *preferred-seller* of a buyer b_j is sellers s_i whose price p_i brings $\max(\boldsymbol{u}_{b_j})$. However, if $\max(\boldsymbol{u}_{b_j}) \leq 0$, there is no preferred-seller for b_j. Each buyer purchases electricity from its preferred-sellers; in addition, for simplicity, no buyer purchases electricity from more than one seller in our model.

A *preferred-seller graph* is an undirected bipartite graph denoted by G $(N_s \cup N_b, E)$, in which an edge set E contains edges between every buyer and its preferred-sellers; therefore, a preferred-seller graph represents possible pairs between buyers and sellers in a market represented by G.

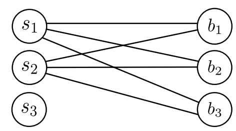
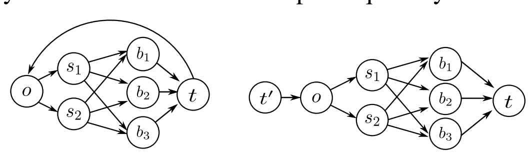


Figure 1. A preferred-seler graph ($|N_s| = 3, |N_b| = 3$).

In the graph G of Figure 1, the payoff of all buyers for both s_1 and s_2 is the maximum payoff, and the payoff of all buyers for s_3 is lower than the payoff for s_1 and s_2. As a result, Figure 1 has edges between every buyer and s_1 or s_2, and does not have edges between every buyer and s_3.

C. Flow Maximization on a Graph

Our algorithm realizes search for a many-to-one matching M on G. For this purpose, algorithm converts G into a directed graph H $((o \cup t \cup N_s \cup N_b), A)$, and considers a feasible circulation problem on H to find the flow that satisfies supply capacity of all sellers and consumption quantity of all buyers.



(a) A model of H. (b) A model of H'.

Figure 2. Graphs changed from the preferred-seller graph in Figure 1.

Any G can be converted to H by three steps. First, remove nodes whose degree is equal to zero, and add two nodes o and t. Second, make arcs (o, s_i) for all i, arcs (b_j, t) for all j, and an arc (t, o). Finally, change all edges in E to arcs $(s_i, b_j) \in A$ (let A be an arc set including all arcs in H). For instance, G in Figure 1 can be converted to H in Figure 2 (a).

Feasible circulation on H must satisfy following two constraints. One is a capacity constraint on every arc in H, and the other is a mass-balance constraint on every node in H.

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First, as the capacity constraint, every arc $(v, w) \in A$ must have capacity of x_{vw} that is a nonnegative value of *flow* on (v, w). Additionally, let $ub(v, w)$ and $lb(v, w)$ be upper and lower bound of x_{vw} respectively; therefore, x_{vw} must satisfy

$$lb(v, w) \le x_{vw} \le ub(v, w). \tag{5}$$

Figure 3 (a) shows a flow value x_{vw} on an arc (v, w).

(a) The flow on (v, w). (b) A model to show (10).

Figure 3. The flow notation and mass-balance constraint.

For any i, $lb(o, s_i) = 0$, and $ub(o, s_i) = c_i$. By (5),

$$0 \le x_{os_i} \le c_i. \tag{6}$$

In addition, for any j, $lb(b_j, t) = q_j$, and $ub(b_j, t) = q_j$; hence,

$$x_{b_j t} = q_j. \tag{7}$$

In terms of (t, o), $lb(t, o) = 0$, and $ub(t, o) = +\infty$. Therefore,

$$0 \le x_{to} \le +\infty. \tag{8}$$

As in (t, o), $lb(s_i, b_j) = 0$, and $ub(s_i, b_j) = +\infty$; Thus,

$$0 \le x_{s_i b_j} \le +\infty. \tag{9}$$

Second, to consider the mass-balance constraint, difference function $d(v)$ is utilized. $d(v)$ is denoted by

$$d(v) = \sum_{\{w:(v,w)\in A_k\}} x_{vw} - \sum_{\{w:(w,v)\in A_k\}} x_{wv}. \tag{10}$$

For any node $v \in (o \cup t \cup N_s \cup N_b)$, $d(v)$ represents difference between flow values current into v and current from v, and $d(v)$ must satisfy $d(v) = 0$. For instance, in Figure 3 (b), $d(o)$ must satisfy $d(o) = x_{os_1} + x_{os_2} - x_{t'o} = 0$.

The feasible circulation problem on H can be solved by using general methods of a maximum flow problem by changing H into $H'((t' \cup o \cup t \cup N_s \cup N_b), A')$. Any H can be converted to H' by following steps. First, add a node t' to H, and remove an reverse arc from H; then, add an arc (t', o) to H. For instance, H in Figure 2 (a) is converted to H' in Figure 2 (b). In terms of capacity bounds, $lb(t', o) = 0$, and $ub(t', o) = \sum_{j=1}^{m} q_j$. Hence, $x_{t'o}$ must satisfy

$$0 \le x_{t'o} \le \sum_{j=1}^{m} q_j. \tag{11}$$

To obtain the maximum flow on H', the algorithm solves following objective function with (6), (7), (8) and (11).

$$\text{Maximize} \quad x_{t'o}. \tag{12}$$

D. Construction of Unsplittable Flow

Even though the algorithm finds flow that satisfies (12), this flow does not necessarily construct a many-to-one matching. In Figure 4, both $t' - o - s_1 - b_2 - t$ and $t' - o - s_2 - b_2 - t$ are

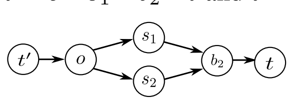


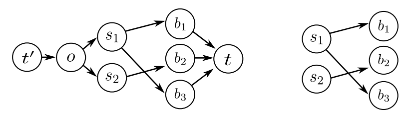
Figure 4. A model shows the constraint of unsplittability.

possible t'-t paths, and this flow is called *splittable flow* if both of these paths supply electricity to b_2. However, only one t'-t path must be selected to supply electricity to b_2 because no buyer can purchase electricity from more than one seller in our model; therefore, if the maximum flow is splittable flow, this flow must be changed into *unsplittable flow* by an algorithm proposed in [9]. This algorithm finds *alternating cycles* and adjusts flow on these cycles to get unsplittable flow.

In Figure 4, let $o - s_1 - b_2 - s_2 - o$ be an alternating cycle. In this alternating cycle, $o - s_1 - b_2$ is a forward path, and $o - s_2 - b_2$ is a backward path. Then, the algorithm decreases flow on the forward path, and augments flow on the forward path until flow on the forward path becomes zero.

E. Social Welfare with Flow Notation

Figure 5 (a) shows unsplittable flow on H' in Figure 2 (b). Because of the assumption that electricity does not



(a) Unsplittable flow. (b) A matching.

Figure 5. Unsplittable flow and a matching on H' in Figure 2 (b).

decrease anywhere in our model, in Figure 5 (a), t'-t path through (s_i, b_j) represents a transaction in which b_j purchases electricity from s_i. Furthermore, $x_{s_i b_j}$ represents quantity of electricity in the transaction between s_i and b_j. Thus, a many-to-one matching M between sellers and buyers can be acquired by extracting all arcs (s_i, b_j) from the unsplittable flow. The matching M is shown in Figure 5 (b).

In Section III-A, both $y(s_i)$ and $y(b_j)$ are defined as units of electricity in transactions of an electricity market. By using the flow notation, $y(s_i)$ can be described as

$$y(s_i) = \sum_{\{v:(s_i,v)\in A\}} x_{s_i v}. \tag{13}$$

By assigning (13) to $y(s_i)$ in (1), payoff of s_i is described by

$$u(s_i) = p_i \sum_{\{v:(s_i,v)\in A\}} x_{s_i v}. \tag{14}$$

Moreover, if b_j purchases electricity from s_i, $y(b_j)$ will be

$$y(b_j) = x_{s_i b_j}. \tag{15}$$

Therefore, (4) can be changed by

$$u(b_j) = u_{b_j s_i} x_{s_i b_j}. \tag{16}$$

Social welfare is defined as the total of all payoffs of buyers and sellers in a market; therefore, by summing up (14) and (16) for all buyers and sellers in an electricity market model, the social welfare of the model can be derived. Thus, social welfare of an electricity market model is represented by

$$W = \sum_{i=1}^{n} u(s_i) + \sum_{j=1}^{m} u(b_j). \tag{17}$$

For instance, social welfare W of the matching M in Figure 5 (b) is calculated in the following manner. s_1 supplies electricity to b_1 and b_3. Hence, $u(s_1) = p_1 x_{s_1 b_1} + p_1 x_{s_1 b_3}$. In terms of s_2, $u(s_2) = p_2 x_{s_2 b_2}$. b_1 and b_3 purchase $x_{s_1 b_1}$ units of electricity from s_1; thus, $u(b_1) = u_{b_1 s_1} x_{s_1 b_1}$ and $u(b_3) = u_{b_3 s_1} x_{s_1 b_3}$. Besides, $u(b_2) = u_{b_2 s_2} x_{s_2 b_2}$. Therefore, the social

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welfare will be $W = u(s_1) + u(s_2) + u(b_1) + u(b_2) + u(b_3)$.

IV. A Matching Algorithm

This section proposes an algorithm described below, which brings us to a matching between sellers and buyers.

Input: $|N_s|$, $|N_b|$, c_i, p_i, q_j, and v_j for all i, j.
Output: Updated prices and a many-to-one matching.
Seven steps of the algorithm:

1. Set the round number $k = 0$ when the algorithm starts. Construct G at round k, which is denoted by G_k, and let E_k is an edge set E of G_k. The algorithm terminates if there is one or more buyers that have no incident edges.
2. Convert G_k into H'_k that is H' at round k, and let A_k be an arc set A in H'_k.
3. Let M_k be the matching at round k. Discover M_k by solving the maximum splittable flow problem on H'_k. If there is feasible splittable flow, let this flow be called F. If there is not any feasible splittable flow, proceed to step 7.
4. Find alternating cycles in splittable flow F until no alternating cycles can be discovered in F; subsequently, augment or decrease flow along the alternating cycles.
5. Let $M_{\max}$ be a many-to-one matching that is feasible and maximizes social welfare. If M_k is $M_{\max}$, the algorithm terminates. The prices bring us to M are called *market-clearing prices*. If M_k is not $M_{\max}$, raise prices of sellers in H'_k by one unit.
6. Let W_k be social welfare derived from (17) on M_k.
7. Set $k = k + 1$, and back to the step 1 to start next round.

Before the algorithm starts, input must be initialized. The algorithm repeats its *round* that consists of seven steps and does not stop until discovering whether $M_{\max}$ exists or not in the preferred-seller graph with the given pattern of inputs.

The algorithm has two termination conditions; one is described in step 5, and the other is denoted in step 1. First one is trivial because it is the objective of this paper. Correctness of second one is described below. The prices offered by sellers do not decrease in the algorithm; for that reason, a buyer cannot purchase electricity from any seller if no seller offers a price that is lower than valuation of the buyer. Therefore, in that case, the algorithm terminates and finds that $M_{\max}$ does not exist in that graph and input pattern.

V. Simulation Results

To analyze accuracy of the proposed algorithm, a simulator of the algorithm has been developed with JAVA. This simulator was used to collect data about the transition of prices, preferred-seller graphs, and social welfare at every round.

Table I
The pattern of input for the simulation.

$\lvert N_s \rvert$	$\lvert N_b \rvert$	c_1	c_2	c_3	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8
3	8	160	160	160	10	20	30	40	50	60	70	80

p_1	p_2	p_3	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
4	2	1	5	6	7	8	9	10	11	12

Table I shows an example pattern of input for the simulation, and Table II displays one of the heuristic results on the given input. In Table II, prices at round 4 satisfy all supply capacity and consumption quantity, and these prices maximize social welfare. Therefore, the matching M_4 is $M_{\max}$, and the prices at round 4 are called market-clearing prices for the given input. Figure 6 shows the structure of M_4. Every arc in M_4 indicates

Table II
Transition of the variables in the simulation.

k	p_1	p_2	p_3	$y(s_i)$			$y(b_j)$							
				s_1	s_2	s_3	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
1	4	2	1	0	0	160	10	0	30	40	10	0	70	0
2	4	2	2	0	160	160	10	20	30	40	50	20	70	80
3	4	3	3	0	170	150	10	20	30	40	50	20	70	80
4	4	4	4	70	150	140	10	20	30	40	50	60	70	80

k	$u(s_i)$			$u(b_j)$								W
	s_1	s_2	s_3	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	
1	0	0	160	30	0	150	240	70	0	630	0	1120
2	0	320	320	20	60	120	200	300	140	560	720	2760
3	0	510	450	10	40	90	160	250	120	490	640	2760
4	280	600	560	0	20	60	120	200	300	420	560	3120

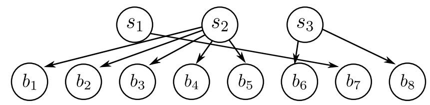

Figure 6. The matching M_4 in the simulation.

a transaction between a buyer and a seller, and flow on each arc is equal to the units of electricity dealt with in the transaction.

VI. Conclusion and Future Work

This paper proposed an algorithm discovering a many-to-one matching between buyers and sellers in a static electricity market model with the set of electricity prices. By the computer simulation, accuracy of the algorithm was examined.

In our definition of social welfare, the payoff of each seller is equal to the price offered by the seller. Nevertheless, in the real markets, sellers must pay the production costs that reduce a profit of the sellers; hence, more appropriate settings at that point is required to obtain more accurate social welfare in an electricity market. In addition, the concept of decrease of electricity on power lines will be integrated into the model of our algorithm. These assignments are our future work.

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