# Subspace Identification Methods and Multivariable Control for a Doubly-fed

# **Induction Generator**

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Abstract—Doubly-Fed Induction Generators (DFIG) are being widely used on Wind Turbine Generator System (WTGS), although synchronous generators are being extensively utilized too. Thus, there are different types of identification techniques to obtain an estimated model of system. Orthogonal Decomposition (ORT) and Multivariable Output Error State Space (MOESP) algorithms are two well-known subspace identification techniques, discussed in this paper. These identification techniques are often implemented for multivariable systems. Subspace identification algorithms are attractive since the state space form is highly suitable to estimate, predict, filters, as well as for control design. The Doubly-Fed Induction Generator is widely used in the variable-speed constant-frequency Wind Power Generation System, using the vector control scheme which provides good performance in maximum wind energy capturing. In the traditional vector control scheme, the reduced doubly-fed induction generator model, which neglects the stator flux transients, is employed to simplify the rotor current inner-loop controller using the standard Proportional-Integral (PI) regulation. The approach proposed in this paper must be approached in order to obtain an approximate model influenced by the behaviour of the system with certain functional characteristics and environmental changes.

Keywords-subspace identification; parameters estimated; wind turbine; multivariable system; integral controller.

### I. INTRODUCTION

Subspace identification methods have proven to be such a valuable tool in identification area since past few years. This interest is due to the ability of the subspace approach in providing accurate state-space models for multivariable linear systems directly from input-output data [1][2]. In the identification procedure, the main step is to compute the Singular Value Decomposition (SVD) of a block Hankel matrix H [3] constructed with input-output data. The computation of SVD can be done offline or online. The offline can be easily converted to an adaptive online version for a slow time Vaihingen system, such as process control system [3]-[4].

In control systems design with advanced model-based control methods, lots of time and cost are required for system identification to construct accurate models like a variable-speed induction machine. To save the cost for modeling of dynamical systems, it is important to choose proper test signals so that the leading dynamics are stimulated efficiently [5]. A doubly-fed induction generator is a variable-speed induction machine widely utilized in the modern wind power industry. The reasons for using variable speed wind turbines are fourfold: 1) a higher energy yield, 2) a reduction of mechanical loads and a simpler pitch control, 3) an extensive controllability of both active and reactive powers, and 4) less fluctuation in output power [5][6].

There are several vector control strategies for a wind turbine driven DFIG's; one common method is to control the rotor current with Stator Flux Orientation (SFO) [7]. In all the existing control schemes, the reduced DFIG model, which neglects the transient terms in the stator flux and most of them use the traditional PI regulator in their rotor current controllers [8].

In this paper, two subspace approaches are observed in order to perform open-loop system identification. These two approaches are the Orthogonal Decomposition (ORT) method and Multivariable Output Error State Space (MOESP) method. The objective of this paper is to analyse the performance of the models in identifying the state space model of a doubly-fed induction generator (DFIG) and the performance of the Integral multivariable controller applied to the estimated system, as shown in Section 3 and Section 4, respectively.

### II. MODELING OF DFIG

### A. General Descriptions

Figure 1 illustrates the DFIG wind power generation system consisting of a wound rotor induction generator and a back-to-back converter between the rotor slip rings and the grid.



Figure 1. Variable-speed wind turbine system using DFIG.

#### B. Mathematical Model of DFIG

Figure 2 shows the equivalent circuit of a DFIG in the reference frame rotating at the synchronous angular speed  $\omega_1$  [7].



Figure 2. T-equivalent circuit of DFIG in reference frame rotating at  $\omega_1$ .

According to Figure 2, the stator and rotor space-vector fluxes  $\Psi_s$  and  $\Psi_r$  are given respectively by

$$\begin{cases} \boldsymbol{\Psi}_{s} = \mathbf{L}_{s}\mathbf{I}_{s} + \mathbf{L}_{m}\mathbf{I}_{r} \\ \boldsymbol{\Psi}_{r} = \mathbf{L}_{m}\mathbf{I}_{s} + \mathbf{L}_{r}\mathbf{I}_{r} \end{cases}$$
(1)

where  $L_{\sigma s}$ ,  $L_{\sigma r}$  and  $L_m$  are the stator and rotor leakage and mutual inductances, and  $L_s = L_{\sigma s} + L_m$  and  $L_r = L_{\sigma r} + L_m$  the self inductances of the stator and rotor windings, respectively.

The stator and rotor space-vector voltages  $V_s$  and  $V_r$  in the same reference frame can be expressed as

$$\begin{cases} \mathbf{V}_{s} = R_{s}\mathbf{I}_{s} + \frac{d\boldsymbol{\Psi}_{s}}{dt} + j\boldsymbol{\omega}_{1}\boldsymbol{\Psi}_{s} \\ \mathbf{V}_{r} = R_{r}\mathbf{I}_{r} + \frac{d\boldsymbol{\Psi}_{r}}{dt} + j(\boldsymbol{\omega}_{1} - \boldsymbol{\omega}_{r})\boldsymbol{\Psi}_{r} \end{cases}$$
(2)

where  $\mathbf{I}_s$  and  $\mathbf{I}_r$  are the stator and rotor current vectors,  $\mathbf{R}_s$  and  $\mathbf{R}_r$  the stator and rotor resistances,  $\omega_r$  is the rotor angular speed, and  $\omega_s = \omega_1 - \omega_r$  the slip angular speed.

The model of DFIG uses the SVO scheme described in [7], in which we can rewrite the rotor voltage equation in the synchronous d - q reference frame as

$$\begin{pmatrix}
\mathbf{V}_{rd} = V'_{rd} + \frac{L_m}{L_s} \left( \mathbf{V}_s - \frac{R_s}{L_s} \boldsymbol{\Psi}_{sd} + \boldsymbol{\omega}_r \boldsymbol{\Psi}_{sq} \right) \\
\mathbf{V}_{rq} = V'_{rq} - \frac{L_m}{L_s} \left( \frac{R_s}{L_s} \boldsymbol{\Psi}_{sq} + \boldsymbol{\omega}_r \boldsymbol{\Psi}_{sd} \right)$$
(3)

where

$$\begin{cases} \mathbf{V}_{rd}^{'} = R_{r}^{'} I_{rd} + \sigma L_{r} \frac{dI_{rd}}{dt} - \omega_{s} \sigma L_{r} I_{rq} \\ \mathbf{V}_{rq}^{'} = R_{r}^{'} I_{rq} + \sigma L_{r} \frac{dI_{rq}}{dt} + \omega_{s} \sigma L_{r} I_{rd} \end{cases}$$
(4)

The system in state space representation is presented as

follows:

$$\frac{dI_{rd}}{dt} = \frac{1}{\sigma L_r} \left( V_{rd}^{'} - R_r^{'} I_{rd} + \omega_s \sigma L_r I_{rq} \right) 
\frac{dI_{rq}}{dt} = \frac{1}{\sigma L_r} \left( V_{rq}^{'} - R_r^{'} I_{rq} - \omega_s \sigma L_r I_{rd} \right) 
y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_{rd} \\ I_{rq} \end{bmatrix}$$
(5)

### III. SUBSPACE IDENTIFICATION METHOD

The model presented in Section 2 is nonlinear. The linear model of the DFIG can be obtained by applying an subspace approach as the ORT (Orthogonal Decomposition) or MOESP (Multivariable Output Error State Space) methods to the system around an operating point [3], as follows:

$$\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k]$$
  
$$\mathbf{y}[k] = \mathbf{C}\mathbf{x}[k] + \mathbf{D}\mathbf{u}[k]$$
 (6)

being  $\mathbf{x} \in \mathbb{R}^{n \times 1}$ , where n = 6 is the number of state variables,  $\mathbf{A} \in \mathbb{R}^{n \times n}$ ,  $\mathbf{B} \in \mathbb{R}^{n \times p}$ , where p = 2 is the number of inputs,  $\mathbf{u} \in \mathbb{R}^{p \times 1}$ , and  $\mathbf{C} \in \mathbb{R}^{m \times n}$ , where m = 2 is the number of outputs.

Suppose that the input-output data  $\{u(t), y(t), t = 0, 1, \dots, N + 2k - 2\}$  are given with N sufficiently large and k > n. Based on the input-output data [1], we define as usual block Hankel matrices of past input is defined as

$$U_{p} = \begin{bmatrix} u(0) & u(1) & \cdots & u(N-1) \\ u(1) & u(2) & \cdots & u(N) \\ \vdots & \vdots & \ddots & \vdots \\ u(k-1) & u(k) & \cdots & u(N+k-2) \end{bmatrix}$$
(7)

and future input as

$$U_{f} = \begin{bmatrix} u(k) & u(k+1) & \cdots & u(k+N-1) \\ u(k+1) & u(k+2) & \cdots & u(k+N) \\ \vdots & \vdots & \ddots & \vdots \\ u(2k-1) & u(2k) & \cdots & u(N+2k-2) \end{bmatrix}$$
(8)

where  $U_p, U_f \in \mathbb{R}^{km \times N}$ . Similarly, we define past and future output  $Y_p, Y_f \in \mathbb{R}^{kp \times N}$  respectively. In [9], the constructed input and output are given as

$$U = \begin{bmatrix} U_p \\ U_f \end{bmatrix}, Y = \begin{bmatrix} Y_p \\ Y_f \end{bmatrix}$$
(9)

The extended observability matrix of order i is defined as

$$\Gamma_i = \begin{bmatrix} C^T & (CA)^T & \cdots & (CA^{i-1})^T \end{bmatrix}^T$$
(10)

and the block lower triangular Toeplitz matrix as

$$H_{i} = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D \end{bmatrix}$$
(11)

we consider a related LQ decomposition

$$\begin{bmatrix} U_f \\ U_p \\ Y_p \\ Y_f \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \\ Q_4^T \end{bmatrix}$$
(12)

- Step 1: Construct data matrices of  $U_p, U_f, Y_p$  and  $Y_f$
- Step 2: Perform LQ factorization by (12)
- Step 3: Perform SVD to the working matrix

ORT: 
$$G = [L_{42}]$$
  
MOESP:  $G = [L_{42}L_{43}]$   
where  
 $G = \begin{bmatrix} \hat{U} & \bar{U} \end{bmatrix} \begin{bmatrix} \hat{S} & 0\\ 0 & \bar{S} \end{bmatrix} \begin{bmatrix} \hat{V}\\ \bar{V} \end{bmatrix} \simeq \hat{U}\hat{S}\hat{V}^T$ 

Step 4: Compute the estimates of *A* and *C* by (10)  $\Gamma_i = \hat{U}\hat{S}^{1/2}$ 

Step 5: Compute the estimates of *B* and *D* by (11)  $\bar{U}^T L_{41} L_{11}^{-1} = \bar{U}^T H_i$ 

## IV. MULTIVARIABLE CONTROL

## A. Integral multivariable controller

In Figure 3, the controller structure is defined by considering accumulated for the error in the signal control [10] is shown.



Figure 3. Block diagram of the integral action controller.

In [11], the error in the signal control is defined as follows

$$\mathbf{u}[k] = -\mathbf{K}\mathbf{x}[k] + \mathbf{K}_i \mathbf{v}[k]$$
(13)

with

$$\mathbf{v}[k+1] = \mathbf{v}[k] + \mathbf{e}[k] \tag{14}$$

where the error  $\mathbf{e}[k]$  is defined as follows

$$\mathbf{e}[k] = \mathbf{r}[k] - \mathbf{y}[k] \tag{15}$$

and  $\mathbf{v}[0] = 0$ . Therefore, an extended system can be defined as follows

$$\begin{bmatrix} \mathbf{x}[k+1] \\ \mathbf{v}[k+1] \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{x}[k] \\ \mathbf{v}[k] \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{B} \\ -\mathbf{D} \end{bmatrix} \mathbf{u}[k] + \begin{bmatrix} \mathbf{0} \\ -\mathbf{I} \end{bmatrix} \mathbf{r}[k]$$

$$\mathbf{y}[k] = \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}[k] \\ \mathbf{v}[k] \end{bmatrix}$$
(16)
(17)

where the control signal is computed as follows

$$\mathbf{u}[k] = -\begin{bmatrix} \mathbf{K} & -\mathbf{K}_i \end{bmatrix} \begin{bmatrix} \mathbf{x}[k] \\ \mathbf{v}[k] \end{bmatrix}$$
(18)

### V. RESULTS

The parameters of the doubly-fed induction generator used in the simulations are  $R_s = 0.01(p.u.)$ ,  $R_r = 0.009(p.u.)$ ,  $L_{\sigma r} = 0.18(p.u.)$ ,  $L_{\sigma s} = 0.07(p.u.)$ ,  $L_m = 3.015(p.u.)$  and  $\omega_s = 1.2(p.u.)$ .

The wind energy conversion system is identified from behavior shown in Figure 4 and Figure 5. The estimated model by the ORT and MOESP method are shown in (19) and (20), respectively.



Figure 4. Output behavior.



Figure 5. Input behavior.

$$\mathbf{A} = \begin{bmatrix} 0.9992 \ 0.0021 \\ -0.0684 \ 0.9992 \end{bmatrix} \mathbf{B} = \begin{bmatrix} -0.0179 \ 0.0007 \\ -0.0026 \ -0.1041 \end{bmatrix}$$
(19)  
$$\mathbf{C} = \begin{bmatrix} -2.3294 \ -0.0199 \\ 0.1133 \ -0.4085 \end{bmatrix} \mathbf{D} = \begin{bmatrix} 0.0594 \ -0.0790 \\ 0.0054 \ 0.0165 \end{bmatrix}$$
(19)  
$$\mathbf{A} = \begin{bmatrix} 0.9992 \ 0.0112 \\ -0.0129 \ 0.9992 \end{bmatrix} \mathbf{B} = \begin{bmatrix} -0.0566 \ 0.0620 \\ -0.1034 \ -0.0494 \end{bmatrix}$$
(20)  
$$\mathbf{C} = \begin{bmatrix} -0.2080 \ -0.2871 \\ 0.3077 \ -0.1940 \end{bmatrix} \mathbf{D} = \begin{bmatrix} 0.0467 \ -0.2088 \\ 0.0014 \ -0.0063 \end{bmatrix}$$
(20)

The graph of superimpose between simulated true system data and simulated outputs obtained from ORT and MOESP models are shown in Figure 6 and Figure 7. The input data system is shown in Figure 8. In Figure 6 and Figure 7, the



Figure 6. Superimpose between simulated true data system of output 1 and simulated output 1 obtained from estimated system matrices of ORT and MOESP methods.



Figure 7. Superimpose between simulated true data system of output 2 and simulated output 2 obtained from estimated system matrices of ORT and MOESP methods.

MOESP method gives the best performance overall in open loop compared to the ORT model over the real response of system.

In Figure 9, Figure 10 and Figure 11, the outputs tracking performance and the control inputs are shown using the ORT model.

In Figure 12, Figure 13 and Figure 14, the outputs tracking performance and the control inputs are shown using the MOESP model.

As shown in Figure 14, the results for the control inputs by using the ORT method is better than the obtained results shown in Figure 11, the MOESP method, in terms of control signal amplitude.



Figure 9. Output 1 tracking performance of ORT model by using integral multivariable controller.



Figure 8. Input data of true system and estimated system matrices of ORT and MOESP methods.



Figure 10. Output 2 tracking performance of ORT model by using integral multivariable controller.



Figure 11. Control signal of ORT model.



Figure 12. Output 1 tracking performance of MOESP model by using integral multivariable controller.



Figure 13. Output 2 tracking performance of MOESP model by using integral multivariable controller.



Figure 14. Control signal of MOESP model.

### VI. CONCLUSION AND FUTURE WORK

In this paper, two subspace identification models of a doubly-fed induction generator are obtained based on two subspace algorithms which are ORT and MOESP; they show an advantages when the parameters of system are unknown and are known only the input-output values, because they make it possible to obtain an estimated model of system. The MOESP method gives the best performance overall in open loop compared to the ORT method with the same inputs of real system. However, the output tracking performance by using the ORT model is better than the obtained results with the MOESP model in terms of overshoot and control signal amplitude when it is applied the integral multivariable controller. The settling time in tracking performance in both models is the same.

Since practically many real-world systems are time-varying, the approach proposed in this paper must be considered in order to obtain an approximate model influenced by the behaviour of the system with certain functional characteristics and environmental changes. These simulations can be obtained with a programming tool, like MATLAB/Simulink [12] for testing the results of identification and control algorithms designed before being applied on real-world systems with time-varying.

The proposed methodologies for identification, the MOESP and ORT methods, shown a state space representation of the system without rearranging the estimated parameters in each vector and obtaining the parameters matrices (**A**, **B**, **C** and **D**) with a minor dimensions compared to matrices shown in [13], improving the computational cost with the algorithms.

A future work includes the implementation of algorithms for identification coupled multivariable systems, where couplings are considered between inputs and outputs as possible disturbances present in each subsystem, allow it to obtain an approximate model of system and propose a control strategy of the system.

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