Schemes for Deterministic Joint Remotely Preparing an Arbitrary Three-qubit State

You-Bang Zhan*, Yuan-Shun Cui&, A-Long Yu$,
Xiao-Wei Li§, Gui-Bin Chen∮

School of Physics and Electronic Electrical Engineering,
Huaiyin Normal University, Huaian 223300, P. R. China
*e-mail: * ybzhan@hytc.edu.cn &cys571015@hytc.edu.cn $yal@hytc.edu.cn*
§lxw@hytc.edu.cn ∮gbchen@hytc.edu.cn

***Abstract*—Recently, a new scheme for joint remotely preparing an arbitrary three-qubit state based on two three-qubit projective measurements was proposed. In this paper, we put forward two novel schemes to complete the joint remote preparation for this class of three-qubit state with complex coefficients via three Greenberger-Horne-Zeilinger (GHZ) states as the quantum channel. In the present schemes, two senders share the original state which they wish to help the receiver to remotely prepare. To complete the schemes, some novel sets of mutually orthogonal basis vectors are introduced. It is shown that, only if two senders collaborate with each other, and perform projective measurements under suitable measuring basis and appropriate unitary operations on their own qubits respectively, the receiver can reconstruct the original state. Compared with the previous scheme, the advantage of the present schemes is that the total success probability can reach 1.**

Keywords-joint remote state preparation; arbitrary three-qubit state; three-qubit projective measurement; unit success probability

Ⅰ. INTRODUCTION

In the last decade, Lo [1], Pati [2], and Bennett *et al.* [3] presented a new quantum communication scheme that uses classical communication and a previously shared entangled resource to remotely prepare a quantum state. This communication scheme is called remote state preparation (RSP). Compared with teleportation [4], RSP requires less classical communication cost than teleportation. Since then, various theoretical protocols for generalization of RSP have been proposed and experimental implementations of RSP scheme have been presented [5-22]. One can note easily that the above schemes assume the case that only one sender knows the original state.

Recently, a novel aspect of PSP, called as the joint remote state preparation(JRSP), has been proposed [23-29]. In these schemes of the JRSP [23-29], two senders (or N senders) know partly of original state they want to remotely preparation, respectively. In a recent paper [30], joint remote preparation of an arbitrary three-qubit state with complex coefficients has been proposed. More recently, Chen *et al.* [31] pointed out that the scheme by Luo *et al.* [30] does not work for states with arbitrary complex coefficients, and then proposed a new scheme for JRSP of an arbitrary three-qubit state with complex coefficients via EPR-type pairs [31]. In the scheme [31], the coefficients of the original state are split into two symmetric subsets. For maximally quantum channel, Chen's scheme can be successfully realized only with the probability $\frac{1}{8}$ by two sets of three-qubit orthogonal basis projective measurement.

Now, we re-investigate the joint remote preparation of an arbitrary three-qubit state with complex coefficients. In this paper, two novel JRSP schemes are presented with unit successful probability. For clearly, we only consider maximally entangled channel. In Section 2, we propose the first scheme using three-qubit GHZ states as the quantum channel, and construct two sets of measuring basis which are same as [31]. Different from the Chens' scheme in [31], in our scheme, to acquire unit success probability, the coefficients of the original state are split into two non-symmetric subsets, and after first sender

 ISBN: 978-1-61208-214-1

(Alice) performs her projective measurement, the second sender (Bob) should make a suitable unitary operation on his qubits, and then perform another projective measurement on the qubits, the receiver can recover the original state by appropriate unitary operations, and the total successful probability of JPSP process being 1. In Section 3, we propose the other deterministic JRSP scheme via a novel three-qubit orthogonal basis projective measurement, and the total success probability is still 1. The required classical communication cost of each of these schemes is six bits. Some discussions and conclusions are given in the last Section.

Ⅱ. JRSP VIA TWO THREE-QUBIT PROJECTIVE MEASUREMENTS AND AN UNITARY OPERATION BY TWO SENDERS

Suppose that two senders Alice and Bob wish to help the receiver Charlie remotely prepare the state [31]

$$\begin{aligned}|p\rangle = {} & r_1 e^{i\varphi_1}|000\rangle + r_2 e^{i\varphi_2}|001\rangle + r_3 e^{i\varphi_3}|010\rangle \\ & + r_4 e^{i\varphi_4}|011\rangle + r_5 e^{i\varphi_5}|100\rangle + r_6 e^{i\varphi_6}|101\rangle \\ & + r_7 e^{i\varphi_7}|110\rangle + r_8 e^{i\varphi_8}|111\rangle, \end{aligned} \tag{1}$$

where r_j and φ_j $(j = 1, 2 \cdots, 8)$ are real, and $\sum_{j=1}^{8} r_j^2 = 1$. To acquire unit success probability, inspired by the schemes of [22, 32, 33], we assume that Alice and Bob share the state $|p\rangle$ and they know the state partly, *i.e.*, Alice knows r_j $(j = 1, 2 \cdots, 8)$, and Bob knows φ_j $(j = 1, 2 \cdots, 8)$, but Charlie does not know them at all. This means that the coefficients of the state (1) are split into two non-symmetric subsets, *i.e.*, modulus (r_j) and phase (φ_j) coefficients. We also suppose that the state shared by Alice, Bob, and Charlie as the quantum channel are three GHZ states

$$|\psi_{1(2,3)}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{a_1 b_1 c_1 (a_2 b_2 c_2, a_3 b_3 c_3)}, \tag{2}$$

where the qubits a_1, a_2, a_3 belong to Alice, qubits b_1, b_2, b_3 to Bob, and qubits c_1, c_2, c_3 to Charlie, respectively.

In order to complete the JRSP, Alice and Bob should construct their own measuring bases respectively. The first measuring basis chosen by Alice is a set of mutually orthogonal basis vectors (MOBVs) $\{|\mu_k\rangle\}(k = 1, 2, \cdots, 8)$, which is given by

$$\begin{pmatrix} |\mu_1\rangle \\ |\mu_2\rangle \\ |\mu_3\rangle \\ |\mu_4\rangle \\ |\mu_5\rangle \\ |\mu_6\rangle \\ |\mu_7\rangle \\ |\mu_8\rangle \end{pmatrix} = F \begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}, \tag{3}$$

where

$$F = \begin{pmatrix} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7 & r_8 \\ r_2 & -r_1 & r_4 & -r_3 & r_6 & -r_5 & r_8 & -r_7 \\ r_3 & -r_4 & -r_1 & r_2 & -r_7 & r_8 & r_5 & -r_6 \\ r_4 & r_3 & -r_2 & -r_1 & r_8 & r_7 & -r_6 & -r_5 \\ r_5 & -r_6 & r_7 & -r_8 & -r_1 & r_2 & -r_3 & r_4 \\ r_6 & r_5 & -r_8 & -r_7 & -r_2 & -r_1 & r_4 & r_3 \\ r_7 & -r_8 & -r_5 & r_6 & r_3 & -r_4 & -r_1 & r_2 \\ r_8 & r_7 & r_6 & r_5 & -r_4 & -r_3 & -r_2 & -r_1 \end{pmatrix}. \tag{4}$$

The second measuring basis chosen by Bob is a set of MOBVs $\{|\tau_m\rangle\}(m = 1, 2, \cdots, 8)$, which is given by

$$\begin{pmatrix} |\tau_1\rangle \\ |\tau_2\rangle \\ |\tau_3\rangle \\ |\tau_4\rangle \\ |\tau_5\rangle \\ |\tau_6\rangle \\ |\tau_7\rangle \\ |\tau_8\rangle \end{pmatrix} = \frac{1}{2\sqrt{2}} G \begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}, \qquad (5)$$

where

$$G = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ x_1 & -x_2 & x_3 & -x_4 & x_5 & -x_6 & x_7 & -x_8 \\ x_1 & -x_2 & -x_3 & x_4 & -x_5 & x_6 & x_7 & -x_8 \\ x_1 & x_2 & -x_3 & -x_4 & x_5 & x_6 & -x_7 & -x_8 \\ x_1 & -x_2 & x_3 & -x_4 & -x_5 & x_6 & -x_7 & x_8 \\ x_1 & x_2 & -x_3 & -x_4 & -x_5 & -x_6 & x_7 & x_8 \\ x_1 & -x_2 & -x_3 & x_4 & x_5 & -x_6 & -x_7 & x_8 \\ x_1 & x_2 & x_3 & x_4 & -x_5 & -x_6 & -x_7 & -x_8 \end{pmatrix}, \quad (6)$$

where $x_j = e^{-i\varphi_j}\,(j = 1, 2\cdots, 8)$.

Now, let Alice perform three-qubit projective measurement on the qubits a_1, a_2, a_3 by using the basis $\{|\mu_k\rangle\}(k = 1, 2, \cdots, 8)$ and publicly announces her measurement result. Next, according to Alice's result of measurement, Bob first makes a suitable unitary operation U_B on his qubits b_1, b_2, b_3, then he should perform three-qubit projective measurement on the qubits b_1, b_2, b_3 under the basis $\{|\tau_m\rangle\}(m = 1, 2, \cdots, 8)$. After these measurements, Alice and Bob inform Charlie of their outcomes of measurement by the classical channel. In accord with Alice's and Bob's results, Charlie can reconstruct the original state $|p\rangle$ by appropriate unitary operation. For instance, without loss of generality, assume Alice's result of measurement is $|\mu_3\rangle_{a_1a_2a_3}$, the qubits $b_1, c_1, b_2, c_2, b_3, c_3$ will collapse into the state

$$\begin{aligned} |\phi\rangle = \frac{1}{2\sqrt{2}} (r_3 |000000\rangle - r_4 |000011\rangle - r_1 |001100\rangle \\ + r_2 |001111\rangle - r_7 |110000\rangle + r_8 |110011\rangle \\ + r_5 |111100\rangle - r_6 |111111\rangle)_{b_1c_1b_2c_2b_3c_3}. \end{aligned} \qquad (7)$$

According to Alice's announcement, Bob should make an unitary operation $U_B = (\sigma_z)_{b_1} \otimes (-i\sigma_y)_{b_2} \otimes (\sigma_z)_{b_3}$ on his qubits b_1, b_2, b_3, the state (7) will become

TABLE 1: Corresponding relation between the measurement results (*MR*) of Alice and the local unitary operations U_B performed by Bob.

MR	U_B
$\|\mu_1\rangle_{a_1a_2a_3}$	$(I)_{b_1} \otimes (I)_{b_2} \otimes (I)_{b_3}$
$\|\mu_2\rangle_{a_1a_2a_3}$	$(I)_{b_1} \otimes (I)_{b_2} \otimes (-i\sigma_y)_{b_3}$
$\|\mu_3\rangle_{a_1a_2a_3}$	$(\sigma_z)_{b_1} \otimes (-i\sigma_y)_{b_2} \otimes (\sigma_z)_{b_3}$
$\|\mu_4\rangle_{a_1a_2a_3}$	$(I)_{b_1} \otimes (-i\sigma_y)_{b_2} \otimes (\sigma_x)_{b_3}$
$\|\mu_5\rangle_{a_1a_2a_3}$	$(-i\sigma_y)_{b_1} \otimes (I)_{b_2} \otimes (\sigma_z)_{b_3}$
$\|\mu_6\rangle_{a_1a_2a_3}$	$(-i\sigma_y)_{b_1} \otimes (\sigma_z)_{b_2} \otimes (\sigma_x)_{b_3}$
$\|\mu_7\rangle_{a_1a_2a_3}$	$(\sigma_x)_{b_1} \otimes (-i\sigma_y)_{b_2} \otimes (\sigma_z)_{b_3}$
$\|\mu_8\rangle_{a_1a_2a_3}$	$(-i\sigma_y)_{b_1} \otimes (\sigma_x)_{b_2} \otimes (\sigma_x)_{b_3}$

$$\begin{aligned} |\phi'\rangle = \frac{1}{2\sqrt{2}} (r_1 |000100\rangle + r_2 |000111\rangle + r_3 |001000\rangle \\ + r_4 |001011\rangle + r_5 |110100\rangle + r_6 |110111\rangle \\ + r_7 |111000\rangle + r_8 |111011\rangle)_{b_1c_1b_2c_2b_3c_3}. \end{aligned} \qquad (8)$$

Then, Bob measures his qubits b_1, b_2, b_3 in the basis $\{|\tau_m\rangle\}(m = 1, 2, \cdots, 8)$, and informs Charlie of his result by the classical channel. Assume Bob's results of measurement is $|\tau_2\rangle_{b_1b_2b_3}$, the state of qubits c_1, c_2, c_3

will collapse into the state

$$
\begin{aligned}
|\varphi\rangle = \frac{1}{8}(& r_1 e^{i\varphi_1}|010\rangle - r_2 e^{i\varphi_2}|011\rangle + r_3 e^{i\varphi_3}|000\rangle \\
& - r_4 e^{i\varphi_4}|001\rangle + r_5 e^{i\varphi_5}|110\rangle - r_6 e^{i\varphi_6}|111\rangle \\
& + r_7 e^{i\varphi_7}|100\rangle - r_8 e^{i\varphi_8}|101\rangle). \qquad (9)
\end{aligned}
$$

In accord with Alice's and Bob's outcomes, Charlie can perform an unitary operation $U_C = (I)_{c_1} \otimes (\sigma_x)_{c_2} \otimes (\sigma_z)_{c_3}$

on qubits c_1, c_2 and c_3, and the original state $|p\rangle$ can be recovered. If Alice's measurement results are the other 7 cases, Bob should perform the appropriate unitary operation on the qubits b_1, b_2, b_3 and then measure these qubits in the basis $\{|\tau_m\rangle\}$. The relation between the results obtained by Alice and the appropriate unitary transformation performed by Bob is shown in the Table 1. It is easily found that, for all the 64 measurement outcomes of Alice and Bob, the receiver Charlie can reconstruct the original state $|p\rangle$ and the total successful probability of the present JRSP process being 1. So, our scheme is deterministic. It requires classical communication cost is six bits in this scheme.

III. JRSP VIA TWO THREE-QUBIT PROJECTIVE MEASUREMENTS BY TWO SENDERS

Now, let us further propose the scheme for remote preparation of an arbitrary three-qubit state by only two three-qubit projective measurements. Assume the state that Alice and Bob wish to help the receiver Charlie remotely prepare is still in state $|p\rangle$ (see (1)), and the quantum channel shared by Alice, Bob and Charlie is still in states (2).

In order to realize the JRSP, two senders need to construct their own measuring bases respectively. The first measuring basis chosen by Alice is still in a set (3). The second measuring bases by Bob are eight sets of MOBVs $\{|\eta_m^{(k)}\rangle\}$, which are given by

$$
\begin{pmatrix} |\eta_1^{(k)}\rangle \\ |\eta_2^{(k)}\rangle \\ |\eta_3^{(k)}\rangle \\ |\eta_4^{(k)}\rangle \\ |\eta_5^{(k)}\rangle \\ |\eta_6^{(k)}\rangle \\ |\eta_7^{(k)}\rangle \\ |\eta_8^{(k)}\rangle \end{pmatrix} = \frac{1}{2\sqrt{2}} H^{(k)} \begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}, \qquad (10)
$$

where $k = 1, 2, \cdots, 8$, and $H^{(k)}$ are 8×8 matrices given in appendix.

Alice first performs the three-qubit projective measurements on her qubits a_1, a_2, a_3 under the basis $\{|\mu_k\rangle\}$ (see Eqs. (3)) and publicly announces her outcomes of measurement. In accord with Alice's result, Bob should choose suitable measuring basis in the MOBVs $\{|\eta_m^{(k)}\rangle\}$ to measure his qubits b_1, b_2, b_3 and then inform Charlie of his result of measurement by the classical channel. According to Alice's and Bob's outcomes, Charlie can reconstruct the original state $|p\rangle$ by appropriate unitary operation. For example, without loss of generality, assume Alice's measurement result is $|\mu_2\rangle_{a_1a_2a_3}$, then Bob should choose measuring basis $\{|\eta_m^{(2)}\rangle\}$ (see Eq.(10) and the Appendix), which is given by

$$
\begin{pmatrix} |\eta_1^{(2)}\rangle \\ |\eta_2^{(2)}\rangle \\ |\eta_3^{(2)}\rangle \\ |\eta_4^{(2)}\rangle \\ |\eta_5^{(2)}\rangle \\ |\eta_6^{(2)}\rangle \\ |\eta_7^{(2)}\rangle \\ |\eta_8^{(2)}\rangle \end{pmatrix} = \frac{1}{2\sqrt{2}} H^{(2)} \begin{pmatrix} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \\ |110\rangle \\ |111\rangle \end{pmatrix}, \qquad (11)
$$

where

$$H^{(2)} = \begin{pmatrix} x_2 & x_1 & x_4 & x_3 & x_6 & x_5 & x_8 & x_7 \\ x_2 & -x_1 & x_4 & -x_3 & x_6 & -x_5 & x_8 & -x_7 \\ x_2 & -x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_2 & x_1 & -x_4 & -x_3 & x_6 & x_5 & -x_8 & -x_7 \\ x_2 & -x_1 & x_4 & -x_3 & -x_6 & x_5 & -x_8 & x_7 \\ x_2 & x_1 & -x_4 & -x_3 & -x_6 & -x_5 & x_8 & x_7 \\ x_2 & -x_1 & -x_4 & x_3 & x_6 & -x_5 & -x_8 & x_7 \\ x_2 & x_1 & x_4 & x_3 & -x_6 & -x_5 & -x_8 & -x_7 \end{pmatrix}, \quad (12)$$

(here $x_j = e^{-i\varphi_j}$, $j = 1, 2, \cdots, 8$), to measure the qubits b_1, b_2, b_3. After these measurements, Alice and Bob inform Charlie of their outcomes by the classical channel. If Bob's measurement result is $\left|\eta_4^{(2)}\right\rangle_{b_1b_2b_3}$, the qubits c_1, c_2, c_3 will collapse into the state

$$\begin{aligned} |\psi\rangle = \frac{1}{8}(& r_2 e^{i\varphi_2}|000\rangle + r_1 e^{i\varphi_1}|001\rangle - r_4 e^{i\varphi_4}|010\rangle \\ & - r_3 e^{i\varphi_3}|011\rangle - r_6 e^{i\varphi_6}|100\rangle - r_5 e^{i\varphi_5}|101\rangle \\ & + r_8 e^{i\varphi_8}|110\rangle + r_7 e^{i\varphi_7}|111\rangle)_{c_1c_2c_3}. \end{aligned} \quad (13)$$

According to Alice's and Bob's public announcements, Charlie can perform the local unitary operation $(\sigma_z)_{c_1} \otimes (\sigma_z)_{c_2} \otimes (\sigma_x)_{c_3}$ on his qubits c_1, c_2 and c_3, and the original state $|p\rangle$ can be recovered. If Alice's measurement results are the other 7 cases in the basis $\{|\mu_k\rangle\}(k = 1, 2, \cdots, 8)$, Bob should choose appropriate measuring bases $\{\left|\eta_m^{(k)}\right\rangle\}(k = 1, 2, \cdots, 8)$ to measure his qubits b_1, b_2 and b_3, then Charlie can recover the original state $|p\rangle$ by suitable unitary operations. Here we no longer depict them one by one. The corresponding relation of Alice's measurement result $|\mu_k\rangle_{a_1a_2a_3}$ and the measuring basis $\{\left|\eta_m^{(k)}\right\rangle\}$ performed by Bob can be described explicitly as

$$\begin{aligned} &|\mu_1\rangle_{a_1a_2a_3} \to \{\left|\eta_m^{(1)}\right\rangle\}, \quad |\mu_2\rangle_{a_1a_2a_3} \to \{\left|\eta_m^{(2)}\right\rangle\}, \\ &|\mu_3\rangle_{a_1a_2a_3} \to \{\left|\eta_m^{(3)}\right\rangle\}, \quad |\mu_4\rangle_{a_1a_2a_3} \to \{\left|\eta_m^{(4)}\right\rangle\}, \\ &|\mu_5\rangle_{a_1a_2a_3} \to \{\left|\eta_m^{(5)}\right\rangle\}, \quad |\mu_6\rangle_{a_1a_2a_3} \to \{\left|\eta_m^{(6)}\right\rangle\}, \\ &|\mu_7\rangle_{a_1a_2a_3} \to \{\left|\eta_m^{(7)}\right\rangle\}, \quad |\mu_8\rangle_{a_1a_2a_3} \to \{\left|\eta_m^{(8)}\right\rangle\}, \end{aligned} \quad (14)$$

where $m = 1, 2, \cdots, 8$. It is easily found that for all the 512 measurement outcomes of Alice and Bob, the receiver Charlie can reconstruct the original state $|p\rangle$, and the total successful probability *P* is

$$P = 512 \times \frac{1}{8} \times \frac{1}{64} = 1. \quad (15)$$

So, the JRSP scheme is also deterministic. The required classical communication cost is six bits.

Ⅳ.CONCLUSION

In conclusion, we have presented two novel schemes for joint remote preparation of an arbitrary three-qubit states with complex coefficients. In these schemes, two senders share the arbitrary three-qubit states, but each sender only partly knows the state, and two three-qubit GHZ states are exploited as the quantum channel. To complete the JRSP schemes, some novel sets of three-qubit mutually orthogonal basis vectors have been introduced. In the first scheme, the first sender performs a three-qubit projective measurement on her qubits. According to the measurement result of the first sender, the second sender should perform a suitable unitary operation on his qubits, then makes another three-qubit projective measurement on those qubits. In accord with the measurement outcomes of two senders, the receiver can reconstruct the original state by appropriate unitary operation. Next, we have proposed second scheme for JRSP of arbitrary three-qubit state with two senders. In this scheme, the first sender performs a three-qubit projective measurement on her qubits and the measuring basis is still in Section 2. Different from the scheme in Section 2, according to measurement result of first sender, the second sender should choose one of the novel eight sets of the measuring basis to measure his qubits. After these projective measurements by two senders, the original state can be recovered by the receiver. Compared with the previous scheme of JRSP in [31], the advantage of the present schemes is that the total success probability can reach 1. Thus, our present schemes are useful in expanding RSP field in quantum information science.

REFERENCES

[1] K. H. Lo, “Classical-communication cost in distributed quantum-information processing: A generalization of quantum-communication complexity" Phys. Rev. A, vol. 62, 2000, 012313, doi: 10.1013/PhysRevA. 62. 012313.

[2] A. K. Pati, “Minimun classical bit for remote preparation and measurement of a qubit" Phys. Rev. A, vol. 63, 2001, 014302, doi: 10.1013/PhysRevA. 63. 014302.

[3] C. H. Bennett, D. P. Divincenzo, P. W. Shor, J. A. Smolin, B. M. Terhal, and W. K. Wootters, “Remote state preparation" Phys. Rev. Lett., vol. 87, 2001, 077902, doi: 10.1013/PhysRevLett. 87. 077902.

[4] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, “Teleporating an unknown quantum state via dual classical and Einstein-Podolsky -Rosen channels" Phys. Rev. Lett., vol. 70, 1993, pp. 1895-1899, doi: 10.1013/PhysRev Lett. 70. 1895.

[5] I. Devetak and T. Berger, “Low-entanglement remote state preparation" Phys. Rev. Lett., vol. 87, 2001, 197901, doi: 10.1013/PhysRevLett. 87. 197901.

[6] B. Zeng and P. Zhang, “Remote-state preparation in higher dimension and the parallelizeable manifold S^{n-1}" Phys. Rev. A, vol. 65, 2002, 022316, doi: 10.1013 /PhysRevA. 65. 022316.

[7] D. W. Berry and B. C. Sanders, “Optimal remote state preparation" Phys. Rev. Lett., vol. 90, 2003, 057901, doi: 10.1013/PhysRevLett. 90. 057901.

[8] A. Abeyesinghe and P. Hayden, “Generalized remote state preparation: trading cbits, qubits, and ebits in quantum communication" Phys.Rev. A, vol. 68, 2003, 062319, doi: 10.1013/PhysRevA. 68. 062319.

[9] D. W.Leung and P. W. Shor, “Oblivious remote state preparation" Phys. Rev. Lett., vol. 90, 2003, 127905, doi: 10.1013/PhysRev Lett. 90. 127905.

[10] M. G. A. Paris, M. Cola, and R. Bonifacio, “Remote state preparation and teleportation in phase space " J.Opt.B: Quantum Semiclass. Opt., vol. 5, 2003, pp. S360-S364, doi: 10.1088/ 1464-4266/5/3/370.

[11] M. Y. Ye , Y. S. Zhang, and G. C. Guo, “Faithful remote state preparation using finite classical bits and a nonmaximally entangled state” Phys. Rev. A, vol. 69, 2004, pp. 022310, doi: 10.1013/PhysRevA. 69. 022310.

[12] A. Hayashi, T. Hashimoto, and M. Horibe, “Remote state preparation without oblivious conditions” Phys. Rev. A, vol. 67, 2003, 052302, doi: 10.1013/ PhysRevA. 67. 052302.

[13] Z. Kurucz, P. Adam, Z. Kis, and J. Janszky, “Continous variable remote state preparation” Phys.Rev. A, vol. 72, 2005, 052315, doi: 10.1013 /PhysRevA.72. 052315.

[14] J. M. Liu, X. L. Feng, and C. H. Oh, “Remote preparation of arbitrary two- and three-qubit states” EPL, vol. 87, 2009, 30006, doi: 10.1209/0295-5075/87/30006.

[15] P. C. Ma and Y. B. Zhan, “Scheme for remotely preparing a four-particle entangeld cluster-type state” Opt.Commun., vol. 283, 2010, pp. 2640-2643, doi: 10.1016/j.optcom 2010. 02. 034.

[16] M. X. Luo, X. B. Chen, S. Y. Ma, Y. X. Yang, and Z. M. Hu, “Deterministic remote preparation of an arbitrary W-class state with multiparty" J. Phys. B: At.Mol.Opt.Phys., vol. 43, 2010, 065501, doi: 10.1088/0953-0475/43/6/065501.

[17] X. H. Peng, X. W. Zhu, X. M. Fang, M. Feng, M. L. Liu, and K. L. Gao,“Experimental implementation of remote state preparation by nuclear magnetic resonance” Phys. Lett. A, vol. 306, 2003, pp. 271-276, doi: 10.1016/S0375-9601(02) 01604-3.

[18] S. A. Babichev, B. Brezger, and A. I. Lvovsky,

"Remote preparation of a single-mode photonic qubit by measuring field quadrature noise" Phys. Rev. Lett., vol. 92, 2004, 047903, doi: 10.1013 /PhysRev Lett. 92.047903.

[19] G. Y. Xiang, J. Li, B. Yu, and G. C. Guo, "Remote preparation of mixed states via noisy entanglement" Phys. Rev. A, vol. 72, 2005, 012315, doi: 10.1013/PhysRevA. 72. 012315.

[20] N. A. Peters, J. T. Barreiro, M. E. Goggin, T. C. Wei, and P. G. Kwiat, "Remote state preparation: arbitrary remote control of photon polarization" Phys. Rev. Lett., vol. 94, 2005, 150502, doi: 10.1013/PhysRevLett. 94. 150502.

[21] W. Rosenfeld, S. Berner, J. Volz, M. Weber, and H. Weinfurter, "Remote preparation of an atomic quantum memory" Phys. Rev.Lett., vol. 98, 2007, 050504, doi: 10.1013/PhysRevLett. 98. 050504.

[22] M. Y. Wang and F. L. Yan, "Two-step deterministic remote preparation of an arbitrary quantum state" Commun.Theor.Phys., vol. 54, 2010, pp. 792-796, doi: 10.1088/0253-6102/54/5/03.

[23] Y. Xia, J. Song, and H. S. Song, "multiparty remote state preparation" J. Phys. B: At.Mol.Opt.Phys., vol. 40, 2007, pp. 3719-3724, doi: 10.1088/0953-4075/40/18/011.

[24] N. B. An and J. Kim, "Joint remote state preparation" J. Phys. B: At.Mol.Opt.Phys., vol. 41, 2008, 095501, doi: 10.1088/0953 -4075/41/9 /095501.

[25] N. B. An and J. Kim, "Collective remote statepreparation" Int. J. Quantum. Inf., vol. 6, 2008, pp. 1051-1066, doi: 10.1142 /s0219749908004304.

[26] K. Hou, J. Wang, Y. L. Lu, and S. H. Shi, "Joint remote preparation of a multipartite GHZ-class state" Inter. J. Theor. Phys., vol. 48, 2009, pp. 2005-2015, doi: 10.1007/s10073-009 -9975-3.

[27] N. B. An, "Joint remote preparation of a general two-qubit state" J. Phys. B: At.Mol.Opt.Phys., vol. 42, 2009, 125501, doi: 10.1088 /0953-4075/42/12 /125501.

[28] Q. Q. Chen, Y. Xia, J. Song, and N. B. An, "Joint remote state preparation of a W-type state via W-type states" Phys. Lett. A, vol.374, 2010, pp. 4483-4487，doi: 10.1016 /j.physleta. 2010.09.013.

[29] N. B. An, "Joint remote state preparation via W and W-type states" Opt. Commun., vol. 283, 2010, pp. 4113-4117, doi: 10.1016/j.optcom.2010.06.016.

[30] M. X. Luo, X. B. Chen, S. Y. Ma, X. X. Niu, and Y. X. Yang, "Joint remote preparation of an arbitrary three-qubit state" Opt. Commun., vol. 283, 2010, pp. 4796-4801, doi: 10.1016/j.optcom .2010.07.043.

[31] Q. Q. Chen, Y. Xia, and N. B. An, "Joint remote preparation of an arbitrary three-qubit state via EPR-type pairs" Opt. Commun., vol. 284, 2011, pp. 2617-2601, doi: 10.1016/j.optcom. 2011.01.033.

[32] X. Q. Xiao, J. M. Liu, and G. H. Zeng, "Joint remote state preparation of arbitrary two- and three-qubit states" J. Phys. B: At.Mol.Opt.Phys., vol. 44, 2011, 075501, doi: 10.1088/0953 -4075/44/7/075501.

[33] N. B. An, C. T. Bich, and N. V. Don, "Deterministic joint remote state preparation" Phys. Lett. A, vol. 375, 2011, pp. 3570-3573, doi: 10.1016/j.physleta. 2011.08.045.

Appendix

The matrices $H^{(k)}(k=1,2,\cdots,8)$ in Eq.(10) are of the form

$$H^{(1)}=\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ x_1 & -x_2 & x_3 & -x_4 & x_5 & -x_6 & x_7 & -x_8 \\ x_1 & -x_2 & -x_3 & x_4 & -x_5 & x_6 & x_7 & -x_8 \\ x_1 & x_2 & -x_3 & -x_4 & x_5 & x_6 & -x_7 & -x_8 \\ x_1 & -x_2 & x_3 & -x_4 & -x_5 & x_6 & -x_7 & x_8 \\ x_1 & x_2 & -x_3 & -x_4 & -x_5 & -x_6 & x_7 & x_8 \\ x_1 & -x_2 & -x_3 & x_4 & x_5 & -x_6 & -x_7 & x_8 \\ x_1 & x_2 & x_3 & x_4 & -x_5 & -x_6 & -x_7 & -x_8 \end{pmatrix}, \quad (A1)$$

$$H^{(2)}=\begin{pmatrix} x_2 & x_1 & x_4 & x_3 & x_6 & x_5 & x_8 & x_7 \\ x_2 & -x_1 & x_4 & -x_3 & x_6 & -x_5 & x_8 & -x_7 \\ x_2 & -x_1 & -x_4 & x_3 & -x_6 & x_5 & x_8 & -x_7 \\ x_2 & x_1 & -x_4 & -x_3 & x_6 & x_5 & -x_8 & -x_7 \\ x_2 & -x_1 & x_4 & -x_3 & -x_6 & x_5 & -x_8 & x_7 \\ x_2 & x_1 & -x_4 & -x_3 & -x_6 & -x_5 & x_8 & x_7 \\ x_2 & -x_1 & -x_4 & x_3 & x_6 & -x_5 & -x_8 & x_7 \\ x_2 & x_1 & x_4 & x_3 & -x_6 & -x_5 & -x_8 & -x_7 \end{pmatrix}, \qquad (A.2)$$

$$H^{(3)}=\begin{pmatrix} x_3 & x_4 & x_1 & x_2 & x_7 & x_8 & x_5 & x_6 \\ x_3 & -x_4 & x_1 & -x_2 & x_7 & -x_8 & x_5 & -x_6 \\ x_3 & -x_4 & -x_1 & x_2 & -x_7 & x_8 & -x_5 & -x_6 \\ x_3 & x_4 & -x_1 & -x_2 & x_7 & x_8 & -x_5 & -x_6 \\ x_3 & -x_4 & x_1 & -x_2 & -x_7 & x_8 & x_5 & x_6 \\ x_3 & x_4 & -x_1 & -x_2 & -x_7 & -x_8 & x_5 & x_6 \\ x_3 & -x_4 & -x_1 & x_2 & x_7 & -x_8 & -x_5 & x_6 \\ x_3 & x_4 & x_1 & x_2 & -x_7 & -x_8 & -x_5 & -x_6 \end{pmatrix}, \qquad (A.3)$$

$$H^{(4)}=\begin{pmatrix} x_4 & x_3 & x_2 & x_1 & x_8 & x_7 & x_6 & x_5 \\ x_4 & -x_3 & x_2 & -x_1 & x_8 & -x_7 & x_6 & -x_5 \\ x_4 & -x_3 & -x_2 & x_1 & -x_8 & x_7 & -x_6 & -x_5 \\ x_4 & x_3 & -x_2 & -x_1 & x_8 & x_7 & -x_6 & -x_5 \\ x_4 & -x_3 & x_2 & -x_1 & -x_8 & x_7 & x_6 & x_5 \\ x_4 & x_3 & -x_2 & -x_1 & -x_8 & -x_7 & x_6 & x_5 \\ x_4 & -x_3 & -x_2 & x_1 & x_8 & -x_7 & -x_6 & x_5 \\ x_4 & x_3 & x_2 & x_1 & -x_8 & -x_7 & -x_6 & -x_5 \end{pmatrix}, \qquad (A.4)$$

$$H^{(5)}=\begin{pmatrix} x_5 & x_6 & x_7 & x_8 & x_1 & x_2 & x_3 & x_4 \\ x_5 & -x_6 & x_7 & -x_8 & x_1 & -x_2 & x_3 & -x_4 \\ x_5 & -x_6 & -x_7 & x_8 & -x_1 & x_2 & -x_3 & -x_4 \\ x_5 & x_6 & -x_7 & -x_8 & x_1 & x_2 & -x_3 & -x_4 \\ x_5 & -x_6 & x_7 & -x_8 & -x_1 & x_2 & x_3 & x_4 \\ x_5 & x_6 & -x_7 & -x_8 & -x_1 & -x_2 & x_3 & x_4 \\ x_5 & -x_6 & -x_7 & x_8 & x_1 & -x_2 & -x_3 & x_4 \\ x_5 & x_6 & x_7 & x_8 & -x_1 & -x_2 & -x_3 & -x_4 \end{pmatrix}, \qquad (A.5)$$

$$H^{(6)}=\begin{pmatrix} x_6 & x_5 & x_8 & x_7 & x_2 & x_1 & x_4 & x_3 \\ x_6 & -x_5 & x_8 & -x_7 & x_2 & -x_1 & x_4 & -x_3 \\ x_6 & -x_5 & -x_8 & x_7 & -x_2 & x_1 & -x_4 & -x_3 \\ x_6 & x_5 & -x_8 & -x_7 & x_2 & x_1 & -x_4 & -x_3 \\ x_6 & -x_5 & x_8 & -x_7 & -x_2 & x_1 & x_4 & x_3 \\ x_6 & x_5 & -x_8 & -x_7 & -x_2 & -x_1 & x_4 & x_3 \\ x_6 & -x_5 & -x_8 & x_7 & x_2 & -x_1 & -x_4 & x_3 \\ x_6 & x_5 & x_8 & x_7 & -x_2 & -x_1 & -x_4 & -x_3 \end{pmatrix}, \qquad (A.6)$$

$$H^{(7)}=\begin{pmatrix} x_7 & x_8 & x_5 & x_6 & x_3 & x_4 & x_1 & x_2 \\ x_7 & -x_8 & x_5 & -x_6 & x_3 & -x_4 & x_1 & -x_2 \\ x_7 & -x_8 & -x_5 & x_6 & -x_3 & x_4 & -x_1 & -x_2 \\ x_7 & x_8 & -x_5 & -x_6 & x_3 & x_4 & -x_1 & -x_2 \\ x_7 & -x_8 & x_5 & -x_6 & -x_3 & x_4 & x_1 & x_2 \\ x_7 & x_8 & -x_5 & -x_6 & -x_3 & -x_4 & x_1 & x_2 \\ x_7 & -x_8 & -x_5 & x_6 & x_3 & -x_4 & -x_1 & x_2 \\ x_7 & x_8 & x_5 & x_6 & -x_3 & -x_4 & -x_1 & -x_2 \end{pmatrix}, \qquad (A.7)$$

$$H^{(8)}=\begin{pmatrix} x_8 & x_7 & x_6 & x_5 & x_4 & x_3 & x_2 & x_1 \\ x_8 & -x_7 & x_6 & -x_5 & x_4 & -x_3 & x_2 & -x_1 \\ x_8 & -x_7 & -x_6 & x_5 & -x_4 & x_3 & -x_2 & -x_1 \\ x_8 & x_7 & -x_6 & -x_5 & x_4 & x_3 & -x_2 & -x_1 \\ x_8 & -x_7 & x_6 & -x_5 & -x_4 & x_3 & x_2 & x_1 \\ x_8 & x_7 & -x_6 & -x_5 & -x_4 & -x_3 & x_2 & x_1 \\ x_8 & -x_7 & -x_6 & x_5 & x_4 & -x_3 & -x_2 & x_1 \\ x_8 & x_7 & x_6 & x_5 & -x_4 & -x_3 & -x_2 & -x_1 \end{pmatrix}, \qquad (A.8)$$

where $x_j = e^{-i\varphi_j}$ $(j = 1, 2, \cdots, 8)$.