A Spectrum Sensing Technique Based on Variably Weighted Sensing Samples in the Presence of Random Traffic of Primary Users

Eunyoung Cho, Keunhong Chae, and Seokho Yoon
College of Information and Communication Engineering
Sungkyunkwan University
Suwon, South Korea
e-mail: syoon@skku.edu

***Abstract*—This paper investigates the spectrum sensing problem under the random traffic condition of the primary user (PU) in cognitive radio (CR) networks, where the PU may depart or arrive in a random way during the sensing period. Considering that the data transmission period of the secondary user (SU) starts right after the sensing period ends, we observe that, in the presence of the random traffic of the PU, the sensing samples in the latter part of the sensing period are more reliable in making a decision on whether the PU is present or not. Based on this observation, then, we propose spectrum sensing test statistics exploiting only sensing samples in the latter part the sensing period and assigning a larger weight to a sensing sample closer to the end of the sensing period. It is demonstrated in numerical results that the proposed methods offer a significant improvement in detection and receiver operating characteristic (ROC) performances over the conventional methods under the random traffic condition of the PU.**

Keywords- *Cognitive radio (CR); Random traffic; Spectrum sensing; Primary user (PU).*

I. Introduction

With the explosive demands for various high date rate services in wireless communications, recently, the radio frequency spectrum has rapidly become a scarce resource, and thus, the cognitive radio (CR) has gained much interest with its capability of offering a high degree of efficiency in using the radio frequency spectrum [1]-[3]. Spectrum sensing is an essential task in CR, which detects a spectral hole of the frequency spectrum allocated to the primary user (PU), thus allowing the secondary user (SU) to share the frequency spectrum with the PU [4].

Conventionally, the spectrum sensing techniques [5]-[7] have been designed under the assumption that the status of the PU does not change during the sensing period (i.e., the PU is present or absent during the whole sensing time). However, it is clear that the status of the PU may change in a real environment, i.e., the PU may depart or arrive in a random way during the sensing period. Although several spectrum sensing techniques [8]-[10] have been presented with considering this random traffic of the PU, the techniques require the channel knowledge, such as the distributions of the departure and arrival times of the PU signal [8] and noise variance [9], or they employ the sensing samples in the initial part of the sensing period causing a wrong spectral hole detection with a high likelihood [10].

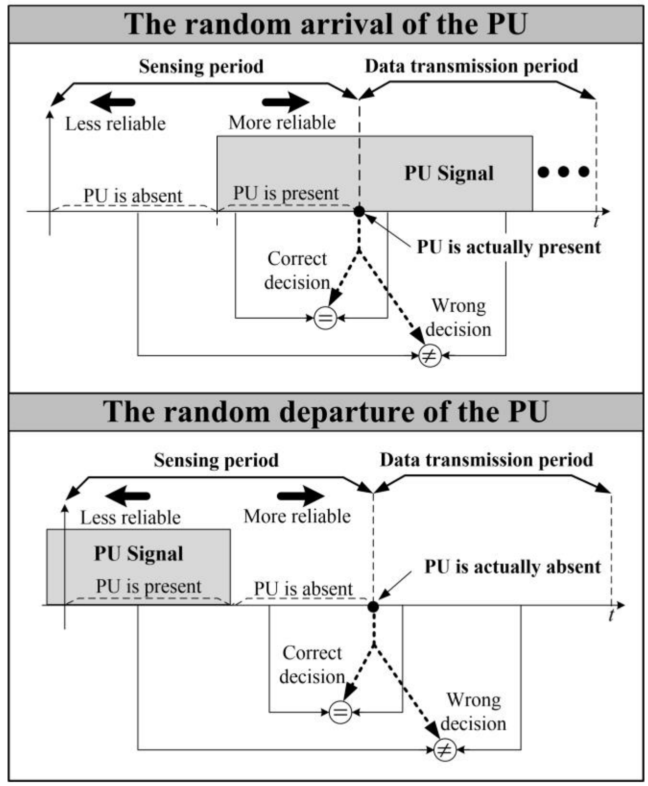

Figure 1. Spectrum sensing decision under the random traffic condition of the PU.

In this paper, we propose a spectrum sensing technique based on variably weighted sensing samples, where only sensing samples in the latter part of the sensing period are used in the spectral hole detection, and a larger weight is assigned to a sensing sample closer to the end of the sensing period (i.e., the sensing sample closest to the end of the sensing period has the largest weight). The proposed technique is expected to perform well in the presence of the random traffic of the PU signal, since a sensing sample closer to the end of the sensing period is more reliable in the decision on the presence and absence of the PU signal when the PU departs or arrives randomly during the sensing period, as shown in Figure 1 [11].

The remainder of this paper is organized as follows. Section 2 models the spectrum sensing problem under the random traffic condition of the PU as a binary hypothesis test. Section 3 describes the proposed technique. Section 4 compares the proposed and conventional techniques in terms of the detection probability and receiver operating characteristic (ROC) curve. Section 5 concludes this paper with a brief summary.

II. RANDOM TRAFFIC MODEL OF THE PU SIGNAL

The static and random traffic models of the PU signal are depicted in the left-hand and right-hand sides of Figure 2, respectively. In the static traffic model, the status of the PU signal remains unchanged during the whole sensing time, and thus, the spectrum sensing can be formulated as the following binary hypothesis testing problem [5]

$$H_0^s : z[i] = w[i] \qquad \text{for } i=1,2,\cdots,I, \tag{1}$$

and

$$H_1^s : z[i] = s[i] + w[i] \quad \text{for } i=1,2,\cdots,I, \tag{2}$$

where the hypotheses H_0^s and H_1^s represent the absence and presence of the PU signal during the whole sensing time, respectively, I is the total number of the sensing samples, and $z[i]$, $s[i]$, and $w[i]$ represent the ith samples of the received signal, the PU signal, and the additive noise, respectively.

In the random traffic model of the PU signal, on the other hand, the spectrum sensing is formulated as [8]

$$H_0^r : z[i] = \begin{cases} s[i] + w[i] & \text{for } i = 1, 2, \cdots, J_0, \\ w[i] & \text{for } i = J_0+1, J_0+2, \cdots, I, \end{cases} \tag{3}$$

and

$$H_1^r : z[i] = \begin{cases} w[i] & \text{for } n = 1, 2, \cdots, J_1, \\ s[i] + w[i] & \text{for } n = J_1+1, J_1+2, \cdots, I, \end{cases} \tag{4}$$

where the hypothesis H_0^r and H_1^r represent the absence and presence of the PU signal not during the whole sensing time but at the end of the sensing period, respectively, i.e., the PU signal is declared absent in the frequency band under consideration if it departs between the J_0th and (J_0+1) th samples, and thus, is eventually absent at the I th sample instant, whereas the PU signal is declared present if it arrives between the J_1th and (J_1+1) th samples and so is present at the I th sample instant. It is noteworthy that (3) and (4) reduce to (1) and (2), respectively, when $J_0 = J_1 = 0$.

III. PROPOSED SPECTRUM SENSING TECHNIQUE

A. *Test Statistics Based on Variably Weighted Sensing Samples*

To obviate the need for the knowledge on the distributions of the departure and arrival times of the PU signal, we consider a spectrum sensing technique based on the energy detection [6] with the sensing samples, and also, to exclude the sensing samples in the initial part of the sensing period causing a wrong decision on the presence and absence of the PU signal with a high likelihood, we exploit only the last L samples out of the I samples. In addition, we enable the spectrum sensing technique to assign a larger weight to a sensing sample closer to the end of the sensing period, since a sensing sample closer to the end of the sensing period is more reliable in the decision on the presence and absence of the PU signal under the random traffic condition of the PU signal, as mentioned in

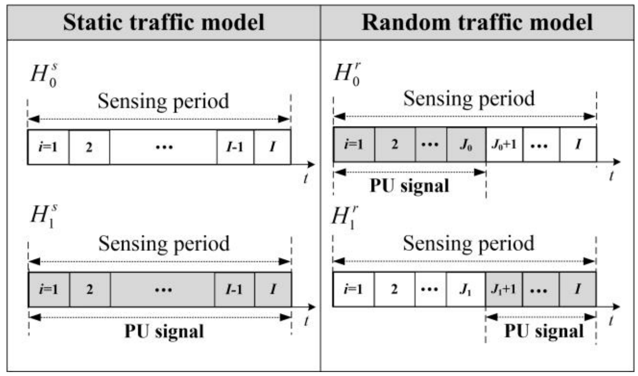

Figure 2. The static and random traffic models of the PU signal

Introduction. Bearing all of these desired features in mind, now, we propose the following two spectrum sensing test statistics

$$T_P = \sum_{i=I-L+1}^{I} \left(\frac{i-(I-L)}{L} \right)^a |z[i]|^2, \tag{5}$$

and

$$T_E = \sum_{i=I-L+1}^{I} b^{\frac{i-(I-L)}{L}} |z[i]|^2, \tag{6}$$

where $a > 0$ and $b > 1$. The two test statistics are similar in that both of them assign a larger weight to a sensing sample closer to the end of the sensing period; yet, they are different in their weights: The weights of (5) and (6) are the power and exponential functions, respectively, of the normalized indices of the last L samples.

B. *Distributions of Test Statistics*

Assuming that the L noise samples $\{w[i]\}_{i=I-L+1}^{I}$ are statistically independent and identically distributed Gaussian random variables with zero mean and variance σ^2, we can derive the characteristic functions of the test statistics as

$$\Phi_0(j\eta) = \prod_{l=1}^{L} \frac{1}{\sqrt{1 - j2\eta\sigma^2\lambda_l}} \tag{7}$$

under H_0^r and

$$\Phi_1(j\eta) = \prod_{l=1}^{L} \frac{1}{\sqrt{1 - j2\eta\sigma^2\lambda_l}} \exp\left(\frac{j\eta s^2[I-L+l]}{(1 - j2\eta\sigma^2\lambda_l)} \right) \tag{8}$$

under H_1^r, where $\lambda_l = (l/L)^a$ and $b^{(l/L)}$ for T_P and T_E, respectively. The inverse Fourier transforms of (7) and (8) would yield the probability density functions (PDFs) under H_0^r and H_1^r, respectively; however, it is highly complicated to express the PDFs in a closed form. Noting that the values of a and b do not change the general forms of the PDFs, thus, we verify the validity of the characteristic functions by deriving the PDFs for a simplified case (i.e., when $a = 0$ and $b = 1$, and so, when $T_P = T_E$), and then, by comparing the analytical detection probability based on the PDFs with the simulated detection probability. The characteristic functions $\Phi_0(j\eta)$ and $\Phi_1(j\eta)$ reduce to

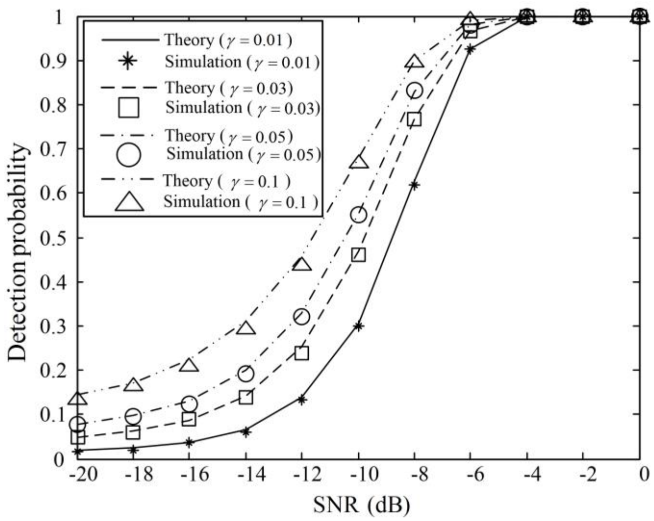

Figure 3. The analytical and simulated detection probabilities for T_P and T_E when γ =0.01, 0.03, 0.05, and 0.1 and L=50.

$$\frac{1}{(1-j2\eta\sigma^2)^{L/2}} \tag{9}$$

and

$$\frac{1}{(1-j2\eta\sigma^2)^{L/2}}\exp\left(\frac{j\eta\sum_{l=1}^{L}s^2[I-L+l]}{(1-j2\eta\sigma^2)}\right), \tag{10}$$

respectively, when $a=0$ and $b=1$. In fact, (9) and (10) are the characteristic functions of the central chi-square and non-central chi-square PDFs, respectively, with L degrees of freedom [12]. Thus, the detection probability is given by

$$\int_{\varepsilon}^{\infty}\left(\frac{1}{2\sigma^2}\right)\left(\frac{y}{\beta^2}\right)^{\frac{(L-2)}{4}} e^{\frac{-(\beta^2+y)}{2}} B_{L/2-1}(\frac{\beta\sqrt{y}}{\sigma^2})dy \tag{11}$$

for both T_P and T_E, where

$$\beta^2 = \sum_{l=1}^{L} s^2[I-L+l], \tag{12}$$

$$B_\alpha(x) = \sum_{k=0}^{\infty}\frac{(x/2)^{\alpha+2k}}{k!\Gamma(\alpha+k+1)}, \tag{13}$$

is the α th-order modified Bessel function of the first kind with $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$ and x>0, and ε is a threshold obtained from

$$\int_{\varepsilon}^{\infty}\frac{1}{\sigma^L 2^{L/2}\Gamma(L/2)} y^{L/2-1}e^{-y/2\sigma^2}dy = \gamma \tag{14}$$

with γ a pre-determined false alarm probability. Figure 3 shows the analytical and simulated detection probabilities for T_P and T_E when γ =0.01, 0.03, 0.05, and 0.1 and L=50, where we can clearly see that the analytical and simulated results agree with each other, thus verifying the validity of (7) and (8), and allowing us to use them in determining a threshold for the detection performance evaluation in the next section.

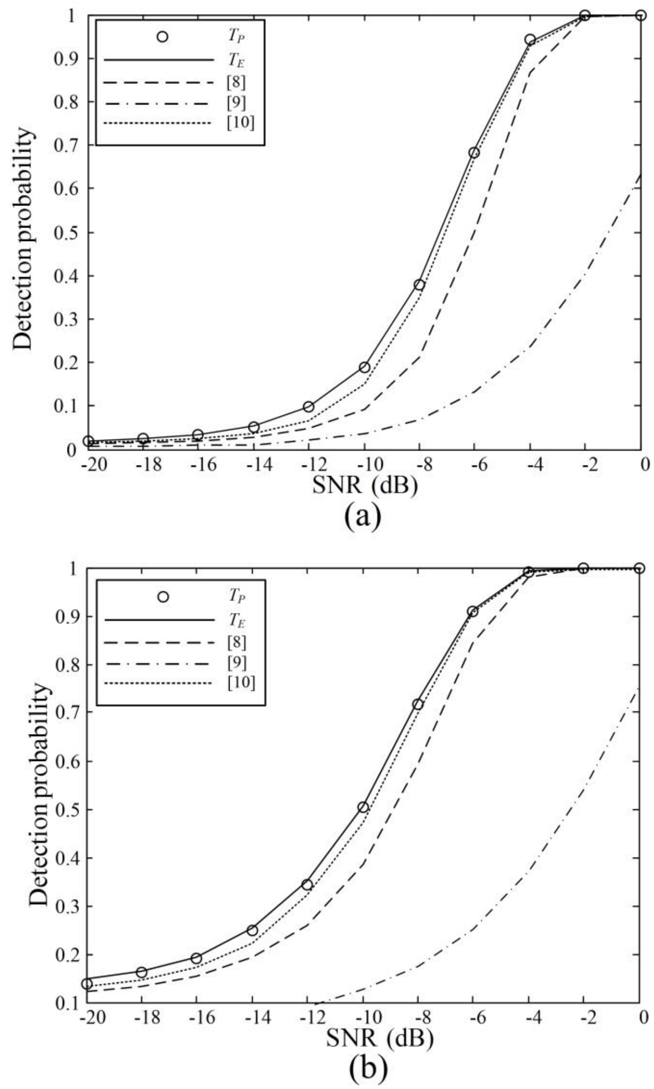

Figure 4. The detection probabilities as a function of the SNR of the proposed and conventional schemes when the false alarm probability is (a) 0.01 and (b) 0.1.

IV. NUMERICAL RESULTS

In this section, the proposed spectrum sensing technique is compared with the conventional spectrum sensing techniques in terms of the detection probability and ROC curve, where I is set to 200, J_0 and J_1 are assumed to be distributed uniformly over the sensing period, the signal-to-noise-ratio (SNR) is defined as $s^2[i]/E^2\{w[i]\}$ with $E\{\bullet\}$ denoting the statistical expectation, and L, a, and b are numerically optimized to maximize the detection probability for each of the given SNR values and false alarm probabilities.

TABLE I. THE OPTIMIZED VALUES OF $L, a,$ and b WHEN THE FALSE ALARM PROBABILITY IS 0.01 AND 0.1.

SNR (dB)			0	-2	-4	-6	-8	-10	-12	-14	-16	-18	-20
False Alarm Probability = 0.01	T_P	**Optimized *a***	0.4	0.3	0.7	1	1.6	2.2	2.7	2.4	2.6	2.2	2.3
		Optimized *L*	175	175	175	175	175	95	85	175	100	100	95
	T_E	**Optimized *b***	6.5	3	3.5	4.5	9.5	10.5	3	10.5	13	13	15
		Optimized *L*	175	175	175	175	150	125	65	95	105	125	110
False Alarm Probability = 0.1	T_P	**Optimized *a***	0.6	0.1	0.6	0.7	1.3	1.9	1.9	0.8	1.7	2.3	3
		Optimized *L*	150	150	175	175	175	175	150	95	110	150	90
	T_E	**Optimized b**	13.5	1.5	3	2.5	5	8.5	14.5	7.5	14.5	13.5	7
		Optimized *L*	175	150	175	150	175	150	175	150	175	80	150

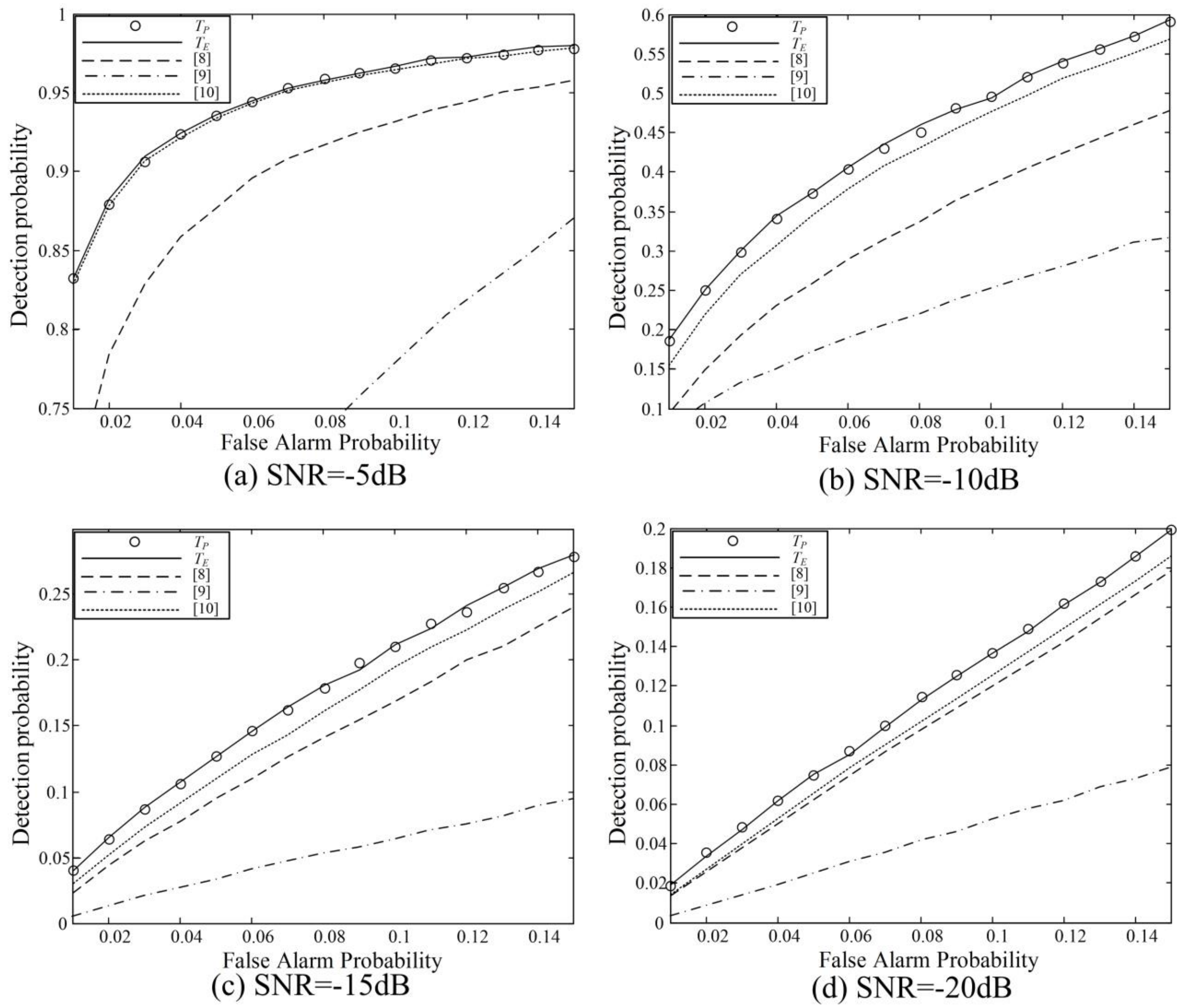

Figure 5. The ROC curves of the proposed and conventional techniques when the value of the SNR is (a) -5dB, (b) -10dB, (c) -15dB, and (d)-20dB.

Table I shows the optimized values of L, a, and b for various values of the SNR when the false alarm probability is 0.01 and 0.1, where it is observed that, as the value of the SNR decreases, the values of a and b generally increases to amplify the signal power, whereas the value of L generally decreases to exclude highly noise-contaminated sensing samples while preserving the reliable samples in the latter part of the sensing period.

Figure 4 shows the detection probabilities of the proposed and conventional spectrum sensing techniques as a function of the SNR when the false alarm probability is 0.01 and 0.1, where we can observe that the proposed techniques outperform the conventional techniques with a gain ranging approximately from 0.5 dB to 13 dB, which stems from the fact that the proposed techniques use only reliable sensing samples in the latter part of the sensing period unlike the conventional techniques. In addition, in the figure, we can see that the performance of T_E is slightly better than that of T_P, due to the effect of the weights on the sensing samples being slightly larger with T_E than with T_P.

Figure 5 shows the ROC curves of the proposed and conventional techniques when the value of the SNR is (a) -5dB, (b) -10dB, (c) -15dB, and (d) -20dB. It is seen in the figure that the proposed techniques offer an improvement in performance over the conventional techniques for all cases shown, and the improvement becomes more pronounced for a larger SNR value generally. This is because the reliable sensing samples are more efficiently utilized in the spectral hole detection through the proposed variable weighting methods, and the number L of the reliable sensing samples generally increases as the value of the SNR becomes larger, as shown in Table I.

Although the PU signal is assumed to arrive or depart only one time during the sensing period in this paper, the PU signal may arrive or depart several times [13] during the sensing period or even during the data transmission period [14]. So, we would like to address sensing techniques in such more realistic environments in the future work.

V. Conclusion

In this paper, we have proposed two novel detection test statistics based on variably weighted sensing samples for spectrum sensing under the random traffic condition of the PU signal. Using the power and exponential functions of the sensing samples in the latter part of the sensing period, we have designed weighting methods that enable the detection test statistics to assign a larger weight to a sensing sample closer to the end of the sensing period, and consequently, to improve their own decision reliability in the presence of the PU random traffic. Numerical results demonstrate that the proposed test statistics provide better detection and ROC performances than the conventional ones under the random traffic condition of the PU signal.

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