An Efficient Estimation Scheme of Timing Offset for OFDM Transmission Over Multipath Fading Channels

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Abstract— **This paper proposes a novel estimation scheme of the timing offset for orthogonal frequency division multiplexing transmission over multipath fading channels. We first design an impulse-like correlation function and obtain the sample standard deviation of the correlation values, and then, develop a decision metric based on the reciprocals of the sample standard deviations for the timing offset estimation, which has the maximum value when the timing offset is estimated correctly. Unlike the conventional schemes, the proposed scheme exploits the change in statistics by the first-arriving path rather than the magnitude itself of the first-arriving path. Numerical results show that the proposed timing offset estimation scheme offers a performance improvement over the conventional schemes in various multipath fading channels.**

Keywords- OFDM; timing offset estimation; multipath fading channels; decision metric; standard deviation

I. INTRODUCTION AND RELATED WORKS

Compared with the single carrier communication systems, Orthogonal Frequency Division Multiplexing (OFDM) has many advantages such as simple equalization and high spectral efficiency [1]. However, the OFDM is very sensitive to the timing synchronization error [2]: Specifically, the timing offset causes an error in determining the starting point of the Fast Fourier Transform (FFT) window at the receiver, eventually resulting in a serious Signal-to-Noise Ratio (SNR) degradation [1]. Although several timing offset estimation techniques [3]-[7] were proposed, all of them do not perform well under the influence of the multipath fading common in wireless channels, since they often mistake one of the timings of the delayed paths for the (correct) timing of the first-arriving path, i.e., they suffer from a problem of ambiguity in timing. Thus, the timing offset estimation techniques alleviating the influence of the multipath fading were developed: [8] takes the earliest among the paths with magnitudes exceeding a pre-determined threshold as the first-arriving path, and on the other hand, [9] and [10] perform a preprocessing based on power normalization to increase the relative magnitude of the first-arriving path to those of the delayed paths. Although these techniques offer an improvement in timing offset estimation to some degree, all of them would perform poorly when the first-arriving path is attenuated severely or when the first-arriving path is not the strongest one.

In this paper, thus, we propose a novel timing offset estimation scheme with a higher degree of robustness to the multipath fading, where we exploit the change in statistics by the first-arriving path rather than the magnitude itself of the first-arriving path, unlike in the conventional schemes. Specifically, we estimate the timing offset based on the change in the reciprocal of the standard deviation of the correlation samples of the OFDM symbol. From numerical results, the proposed scheme is confirmed to provide a better estimation performance over the conventional schemes in multipath fading environments.

In the following sections, we describe the OFDM system model in multipath channel environments (Section II), propose a novel timing offset estimation scheme (Section III), compare the performance of the proposed and conventional schemes (Section IV), and finally, conclude this paper with a brief summary (Section V).

II. SYSTEM MODEL

The baseband equivalent of the n-th received OFDM sample $y(n)$ can be expressed as

$$y(n) = \sum_{l=0}^{L-1} m(l)s(n-\varepsilon-l)e^{j2\pi\Delta f n/N} + w(n), \qquad (1)$$

where $\{s(n)\}_{n=-N_P}^{N-1}$ is a transmitted OFDM signal comprising the data part $\{s(n)\}_{n=0}^{N-1}$ with size N and the Cyclic Prefix (CP) part $\{s(n)\}_{n=-N_P}^{-1}$ with size N_P, ε is the timing offset normalized to the sample interval, Δf is the carrier frequency offset normalized to the subcarrier spacing $1/N$, $m(l)$ is the l-th channel coefficient of a multipath fading channel with length L, and $w(n)$ is the complex Gaussian noise sample with mean zero and variance $\sigma_w^2 = E[|w(n)|^2]$, where $E[\cdot]$ denotes the statistical expectation. In this paper, we consider the timing offset estimation based on training symbols and it is assumed that a training symbol $\{s(n)\}_{n=0}^{N-1}$ with two identical halves (i.e., $s(n) = s(n+N/2)$ for $n = 0,1,\cdots,N/2-1$) is used, as in other studies.

III. PROPOSED SCHEME

A. Generation of Impulse-like Correlation Function

Considering that the ideal form of the correlation function for the timing offset estimation is the impulse-like

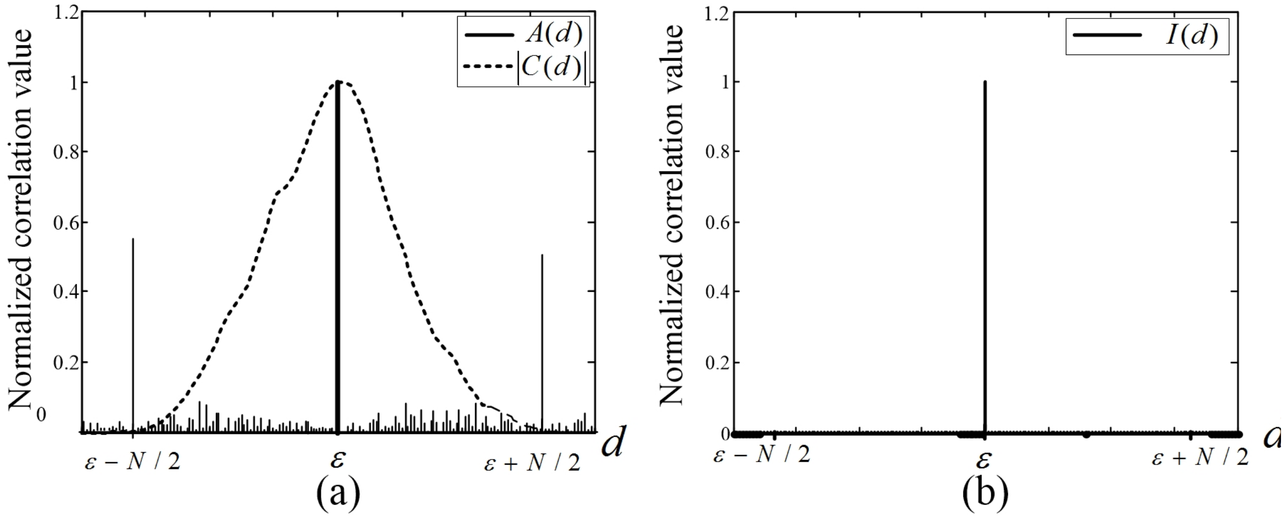

Figure 1. (a) $A(d)$ and $|C(d)|$ in the absence of channel distortion. (b) $I(d)$ in the absence of channel distortion.

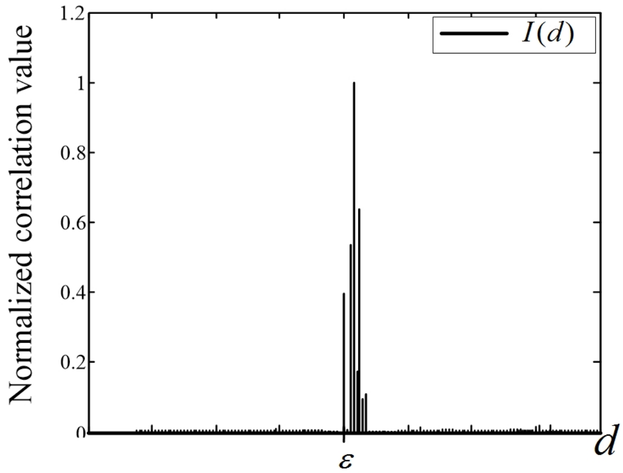

Figure 2. $I(d)$ in a multipath Rayleigh fading channel with an 8-path exponential power delay profile.

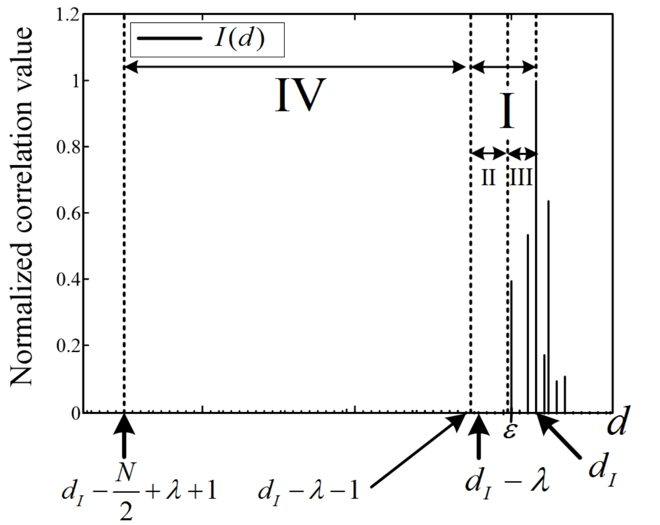

Figure 3. A magnified version of Figure 2.

one, where the correlation is very high at the correct timing, whereas the correlation remains very low at the incorrect timings, we first generate

$$A(d) = \sum_{k=0}^{N-1} y(d+k)s^*(k) \quad (2)$$

by correlating the received OFDM samples and a locally generated training symbol samples, where d is the candidate for the timing offset ε. As shown in Figure 1(a), $A(d)$ has three correlation peaks at $d=\varepsilon$ and $\varepsilon \pm (N/2)$, and the two correlation peaks at $\varepsilon \pm (N/2)$ can be significantly reduced by the product of $|A(d)|^2$ and

$$C(d) = \frac{1}{N_P+1}\sum_{i=0}^{N_P}\left|\sum_{k=0}^{N/2-1} y^*(d-i+k)y(d-i+k+N/2)\right|^2 \quad (3)$$

with $(\bullet)^*$ the complex conjugate operation, as shown in Figure 1(b), where

$$I(d) = |A(d)|^2 \cdot C(d) \quad (4)$$

and it should be noted that $C(d)$ is the moving average of the absolute-squared correlation between the received OFDM samples with length $N/2$. Although $I(d)$ has an impulse-like form, it is severely distorted by multipath fading as shown in Figure 2, where the multipath channel is assumed to be Rayleigh distributed with an 8-path exponential Power Delay Profile (PDP). Yet, we will propose a novel timing offset estimation scheme exploiting the distorted $I(d)$, since it still keeps its impulse-like feature to some degree.

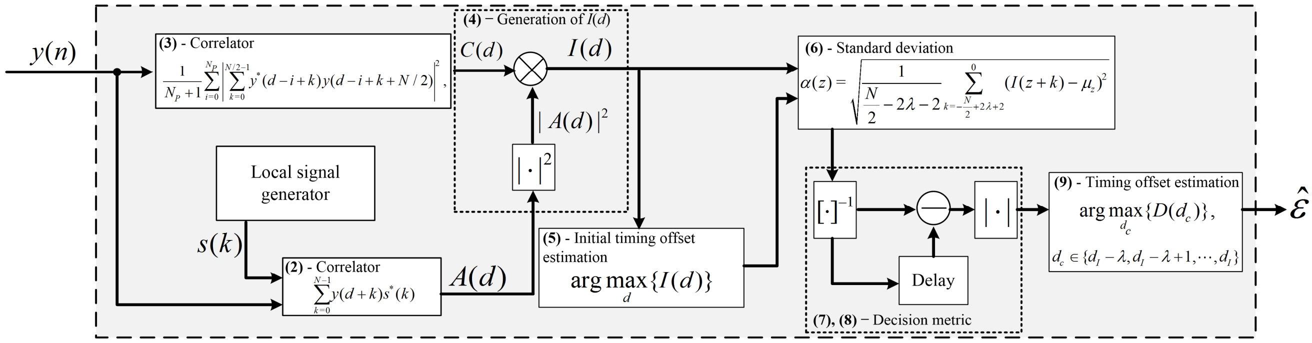

Figure 4. The overall structure of the proposed timing estimator.

B. *Proposed Timing Offset Estimation Scheme*

Using $I(d)$, first, we obtain an initial timing offset estimate d_I as

$$d_I = \arg\max_{d}\{I(d)\}. \quad (5)$$

From Figure 3, then, we can expect that $I(\varepsilon)$ would be within the interval I ($d_I - \lambda \le d \le d_I$), where λ is such that all channel paths are expected to be received within $\lambda+1$ samples (i.e., $\lambda \in \{L-1, L, \cdots, N_P\}$). Denoting d in the interval I by d_c (i.e., $d_c \in \{d_I - \lambda, d_I - \lambda + 1, \cdots, d_I\}$), thus, we can make the following three observations: (i) The values of $I(d_c)$ in the sub-interval II ($d_I - \lambda \le d_c \le \varepsilon - 1$) are much smaller than those in the sub-interval III ($\varepsilon \le d_c \le d_I$), (ii) the values of $I(d)$ in the sub-interval IV ($d_I - N/2 + \lambda + 1 \le d \le d_I - \lambda - 1$) have statistics similar to those of the values of $I(d_c)$ in the sub-interval II, but different from those of the values of $I(d_c)$ in the sub-interval III, and (iii) the timing offset ε is the smallest value of d_c in the sub-interval III. Based on the observations, now, we can model the estimating problem of the timing offset as the detecting problem of the variation between the values of $I(d)$ for $d_I - N/2 + \lambda + 1 \le d \le \varepsilon - 1$ and those of $I(d)$ for $\varepsilon \le d \le d_I$, and we exploit the sample standard deviation

$$\alpha(z) = \sqrt{\frac{1}{\frac{N}{2} - 2\lambda - 2} \sum_{k=-\frac{N}{2}+2\lambda+2}^{0} (I(z+k) - \mu_z)^2}, \quad (6)$$

of $I(d)$ for $z \in \{d_I - \lambda - 1, d_I - \lambda, \cdots, d_I\}$ as a measure for detecting the variation, where μ_z is the sample mean of $I(d)$ for $z - \frac{N}{2} + 2\lambda + 2 \le d \le z$. It should be noted that the standard deviation is more sensitive to the variation of $I(d)$ compared with the variance, due to it being expressed in the same unit as that of $I(d)$, and thus, the standard deviation would offer a better performance in detecting the variation over the variance. Since ε is the smallest value of d_c in the sub-interval III and $I(d_c)$ in the sub-interval III often varies significantly, $|\alpha(\varepsilon) - \alpha(\varepsilon - 1)|$ is expected to be larger than $|\alpha(d_c) - \alpha(d_c - 1)|$ in the sub-interval II, whereas to be smaller than $|\alpha(d_c) - \alpha(d_c - 1)|$ in the sub-interval III. In this paper, we thus propose to use

$$D(d_c) = \left| \frac{1}{\alpha(d_c)} - \frac{1}{\alpha(d_c - 1)} \right| = \left| \frac{\alpha(d_c - 1) - \alpha(d_c)}{\alpha(d_c)\alpha(d_c - 1)} \right| \quad (7)$$

as the decision metric for detecting the variation for $d_c \in \{d_I - \lambda, d_I - \lambda + 1, \cdots, d_I\}$. Unlike $|\alpha(d_c) - \alpha(d_c - 1)|$, $D(d_c)$ has its maximum value at $d_c = \varepsilon$ as shown in the following: (i) $\alpha(d_c) \cong \alpha(d_c - 1)$ when $d_c < \varepsilon$, and thus, $D(d_c) \cong \left| \frac{0}{\alpha(d_c)\alpha(d_c - 1)} \right| = 0$ (ii) $\alpha(\varepsilon) \gg a(\varepsilon - 1)$ when $d_c = \varepsilon$, and thus, $D(\varepsilon) \cong \left| -\frac{\alpha(\varepsilon)}{\alpha(\varepsilon)\alpha(\varepsilon - 1)} \right| = \frac{1}{\alpha(\varepsilon - 1)}$, and (iii) $\alpha(d_c) \gg \alpha(\varepsilon - 1)$ and $\alpha(d_c - 1) \gg \alpha(\varepsilon - 1)$ when $d_c > \varepsilon$, and thus, $D(d_c) = \left| \frac{1}{\alpha(d_c)} - \frac{1}{\alpha(d_c - 1)} \right| < \frac{1}{\alpha(\varepsilon - 1)} = D(\varepsilon)$. The discussions above can be summarized as

$$D(d_c) = \begin{cases} \left| \frac{\alpha(d_c - 1) - \alpha(d_c)}{\alpha(d_c)\alpha(d_c - 1)} \right| \cong 0, & \text{when } d_c < \varepsilon, \\ \left| \frac{\alpha(d_c - 1) - \alpha(d_c)}{\alpha(d_c)\alpha(d_c - 1)} \right| \cong \frac{1}{\alpha(\varepsilon - 1)}, & \text{when } d_c = \varepsilon, \\ \left| \frac{1}{\alpha(d_c)} - \frac{1}{\alpha(d_c - 1)} \right| < \frac{1}{\alpha(\varepsilon - 1)}, & \text{when } d_c > \varepsilon, \end{cases} \quad (8)$$

for $d_c \in \{d_I - \lambda, d_I - \lambda + 1, \cdots, d_I\}$, where we can see that $D(d_c)$ has the maximum value when $d_c = \varepsilon$ regardless of the multipath components, and thus, the proposed scheme is anticipated to achieve the robustness to the channel attenuation and randomness. Finally, we can obtain the timing offset estimate

$$\hat{\varepsilon} = \arg\max_{d_c}\{D(d_c)\} \quad (9)$$

for $d_c \in \{d_I - \lambda, d_I - \lambda + 1, \cdots, d_I\}$. Figure 4 depicts the structure of the proposed timing estimator comprising the operations in (2)-(9).

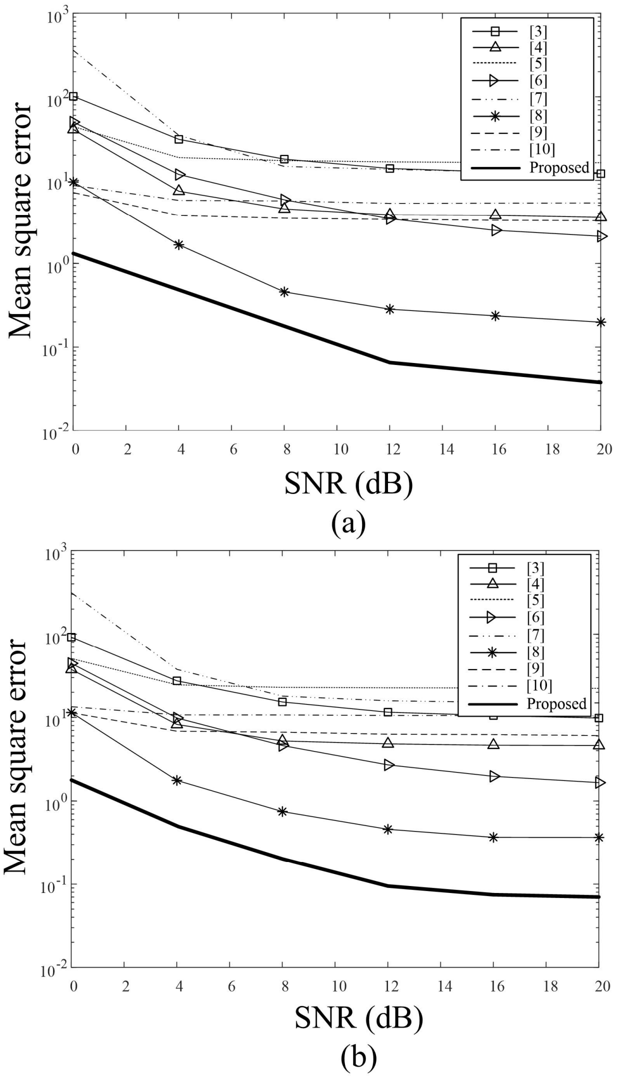

Figure 5. The MSE performances of the timing offset estimation schemes in an 8-path multipath fading channel with (a) exponential power delay profile (E-PDP) and (b) uniform power delay profile (U-PDP).

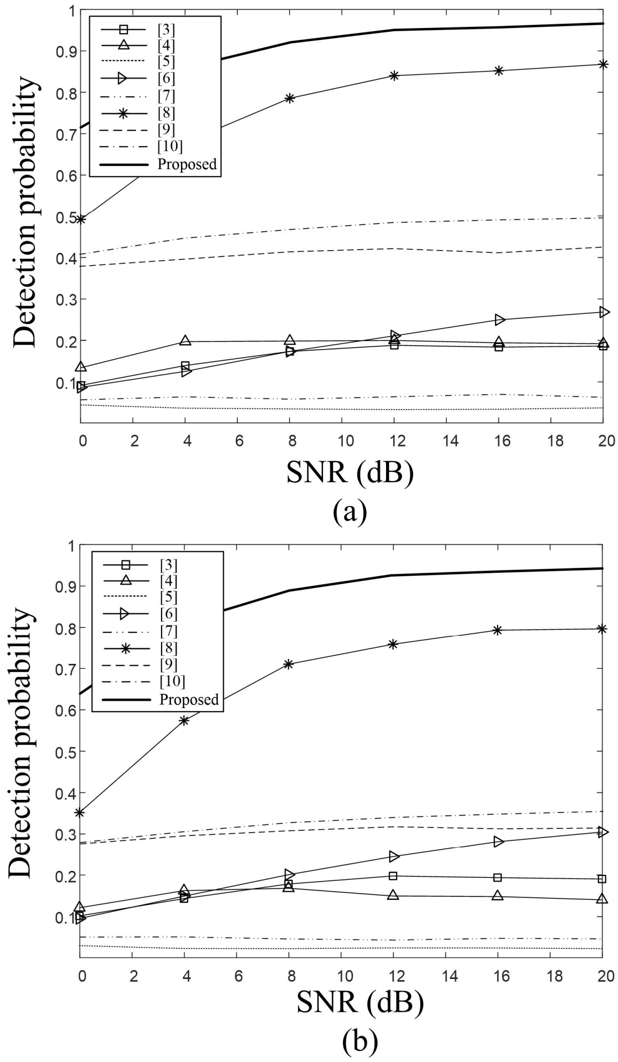

Figure 6. The detection probabilities of the timing offset estimation schemes in an 8-path multipath fading channel with (a) exponential power delay profile (E-PDP) and (b) uniform power delay profile (U-PDP).

IV. NUMERICAL RESULTS

In this section, the timing offset estimation performance of the proposed scheme is compared with those of the conventional schemes in [3]-[10] in terms of the Mean Square Error (MSE) and detection probability in multipath fading environments. We assume the following real transmission parameters: The IFFT size $N = 128$, the CP length $N_P = 8$, the carrier frequency offset $\Delta f = 0.1$, and 8-path multipath Rayleigh fading channels with exponential (E-) and uniform (U-) PDPs having the average power of $e^{-l/8} / \{\sum_{l=0}^{7} e^{-l/8}\}$ and $1/8$, respectively. In addition, the probability of false alarm and λ are set to 10^{-5} and N_P, respectively.

From Figure 5 showing the MSE and detection performances of the timing offset estimation schemes as a function of the SNR defined as $E[|s(k)|^2]/\sigma_w^2$ over the multipath fading channels, we can clearly see that the proposed scheme provides a better performance in timing estimation over the conventional schemes as expected. Especially, from the figures, it is observed that the difference in performance is more significant with the E-PDP than with the U-PDP of the multipaths, which stems from the fact that the variation in statistics by the first-arriving path is more pronounced compared with those of the delayed paths with the E-PDP than with the U-PDP of the multipaths, and thus, the advantage of the proposed scheme based on the change in statistics by the first-arriving path is more prominent with E-PDP over the conventional schemes based on the magnitude itself of the first-arriving path. In addition, the difference in performance increases as the value of SNR becomes larger. This is because the influence of the multipaths becomes more dominant over the noise in high SNR region, and thus, the advantage of the proposed scheme (designed to alleviate the influence of the multipaths) becomes more obvious than it is in low SNR region.

In short, we can conclude that the proposed scheme overcomes the multipath fading more efficiently than the conventional schemes.

V. Conclusion

In this paper, we have proposed a novel timing offset estimation scheme for OFDM transmission in multipath fading channels. We have first designed a correlation function having an impluse-like form, and then, have developed a decision metric based on the standard deviation of the correlation values to estimate the timing offset. From numerical results, it is confirmed that the proposed scheme provides a better MSE and detection performance compared with the conventional schemes in various multipath fading environments. Our future work is to verify the applicability of the proposed scheme to more realistic channel environments.

Acknowledgment

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References

[1] M. Morelli, C.-C. J. Kuo, and M.-O. Pun, “Synchronization techniques for orthogonal frequency division multiple access (OFDMA): a tutorial review," *Proc. IEEE*, vol. 95, no. 7, pp. 1394-1427, July 2007.

[2] R. V. Nee and R. Prasad, OFDM for Wireless Multimedia Communications, Boston, MA: Artech House, 2000.

[3] T. M. Schmidl and D. C. Cox, “Robust frequency and timing synchronization for OFDM," *IEEE Transactions on Communications*, vol. 45, no. 12, pp. 1613-1621, Dec. 1997.

[4] H. Minn, M. Zeng, and V. K. Bhargava, “On timing offset estimation for OFDM systems," *IEEE Communications Letters*, vol. 4, no. 7, pp. 242-244, July 2000.

[5] K. Shi and E. Serpedin, “Coarse frame and carrier synchronization of OFDM systems: a new metric and comparison," *IEEE Transactions on Wireless Communications*, vol. 3, no. 4, pp. 1271-1284, July 2004.

[6] J. W. Choi, Q. Zhao, and H. L. Lou, “Joint ML estimation of frame timing and carrier frequency offset for OFDM systems employing time-domain repeated preamble,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 1, pp. 311-317, 2010.

[7] J. Liu, C. Wei, X. Zeng, J. Lu, and M. Wang, “Research on timing synchronization algorithms for CO-OFDM systems,’ in Proc. International Conference on Optical Communications and Networks (ICOCN), 2016.

[8] A. Awoseyila, C. Kasparis, and B. G. Evans, “Improved preamble-aided timing estimation for OFDM systems," *IEEE Communications Letters*, vol. 12, no. 11, pp. 825-827, Nov. 2008.

[9] S. Suyoto, A. Kurniawan, and S. Sugihartono “Timing estimation based on normalized 3rd-order central moment for symmetric correlator in OFDM systems,” in Proc. International Conference on Information Technology Systems and Innovation (ICITSI), pp. 135-158, 2014.

[10] S. Suyoto, I. Iskandar, S. Sugihartono, and A. Kurniawan, “Improved Timing Estimation Using Iterative Normalization Technique for OFDM Systems,” *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 7, no. 2, pp 905-911, 2017.