Resource Allocation Algorithm for MISO-OFDMA Systems with QoS Provisioning

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***Abstract*—The problem of user selection and resource allocation for the downlink of wireless systems operating over a frequency-selective channel is investigated. It is assumed that the Base Station (BS) uses many antennas, whereas a single antenna is available to each user (Multiple Input Single Output - MISO case). To relieve heavy computational burden, a suboptimal, but efficient algorithm is devised that is based on Zero Forcing (ZF) beamforming. The algorithm maximizes the sum of the users' data rates subject to constraints on total available power and individual data rate requirements for each user. Simulation results are provided to indicate that the algorithm can satisfy the fairness criterion. Thus, the algorithm can be applied to latest-generation wireless systems that provide Quality-of-Service (QoS) guarantees.**

***Index Terms*—MISO, OFDMA, resource allocation, Zero-Forcing, minimum data rate constraints.**

I. Introduction

Orthogonal Frequency Division Multiple Access (OFDMA) [1] is a multi-user version of the popular Orthogonal Frequency Division Multiplexing (OFDM) digital modulation scheme. In OFDMA, multiple access is achieved by first dividing the spectrum of interest into a number of subcarriers and then assigning subsets of the subcarriers to individual users. OFDMA helps exploit multiuser diversity in frequency-selective channels, since it is very likely that some subcarriers that are "bad" for a user are "good" for at least one of the other users. Because of its superior performance in frequency-selective fading wireless channels, OFDMA is the modulation and multiple access scheme used in latest wireless systems such as IEEE 802.16e (Mobile WiMAX) [1].

In recent years, many dynamic resource allocation algorithms have been developed for the Single Input Single Output (SISO)-OFDMA systems. In [2] [3], the system throughput is maximized with a total power constraint and in [4]-[6], the total power consumption is minimized with constraints on the users' data rates. In [7], minimum data rate is maximized while in [8]-[10], proportional data rate constraints are introduced. In [11] [12], the fulfillment of every user's data rate constraints are guaranteed in order to maximize the sum of the users' data rates and in [13], the sum throughput is maximized with long term access proportional fairness. In addition, in [14] weighted sum data rate is maximized with "self-noise" and phase noise. Finally, in [15], system throughput is maximized but the resource allocation unit is not the subcarrier, as in previous algorithms [2]-[14], but a time/frequency unit (slot), in accordance with WiMAX systems [1].

An additional major advance in recent wireless systems is the use of Multiple Input Multiple Output (MIMO) transmission to improve communication performance [1]. In fading environments MIMO technology offers significant increase in the data throughput and the link range without additional bandwidth or transmit power requirements by opening up multiple data pipes in the same frequency band of operation [16]. Because of these properties, MIMO systems have received increasing attention in the past decade. MIMO related algorithms can be implemented in each subcarrier and by combining OFDMA with MIMO transmission, wireless systems can offer larger system capacities and improved reliability.

In general, in order to transmit on the boundary of the capacity region, the BS needs to transmit to multiple users simultaneously in each subchannel employing Dirty Paper Coding (DPC) [16]. However, DPC has large implementation complexity. In [17] [18], user selection and beamforming algorithms, that are based on ZF [19], are proposed in order to maximize the system capacity without guaranteeing any kind of fairness among the users' data rates. In [20], proportional data rate constraints are applied and in [21] a kind of fairness is supported.

In this paper, a user selection and resource allocation algorithm for multiuser downlink MISO-OFDMA is developed that is less complex than exhaustive search algorithm and incorporates fairness by imposing minimum data rate constraints among users. As in [20], the beamforming scheme of [17] is applied in each subcarrier, where each user experiences flat fading [1], but the user selection procedure takes minimum data rate constraints into account. A complexity analysis is also presented in order to further support our statements and simulation results indicate that the algorithm can satisfy the fairness criterion.

The remainder of the paper is organized as follows. A description of the MISO-OFDMA system model is introduced in Section II, whereas the problem of sum data rate maximization using minimum data rate constraints is formulated in Section III. The proposed algorithm is introduced in Section IV and Section V contains the complexity analysis of the proposed algorithm and a complexity comparison with exhaustive search algorithm. Simulation results, analysis and a comparison between the proposed algorithm and previous resource allocation schemes are provided in Section VI. Finally, Section VII contains concluding remarks.

In the following, $(\cdot)^T$ denotes transpose, whereas $(\cdot)^*$ denotes conjugate transpose. $\mathbf{x}$ denotes a column vector, $\mathbf{A}$ denotes a matrix, $\|\cdot\|$ represents the Euclidean norm, and $\mathbb{E}$ is the mean value. Finally, $[x]_+ = \max\{0, x\}$.

II. System Model

Consider an OFDMA downlink transmission with N subcarriers, T transmit antennas at the BS and K active users, each equipped with a single receive antenna. Also, let B be the overall available bandwidth, and $\mathbf{h}_{k,n} = [h^1_{k,n} \ \dots \ h^T_{k,n}]^T$ be the $T \times 1$ baseband equivalent gain vector of the channel between the BS and user k in subcarrier n. Thus, for each subcarrier n, the baseband equivalent model for the system can be written as

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{x}_n + \mathbf{z}_n, \tag{1}$$

where $\mathbf{H}_n = [\mathbf{h}_{1,n} \ \mathbf{h}_{2,n} \ \dots \ \mathbf{h}_{K,n}]^T$ is a $K \times T$ matrix with complex entries, $\mathbf{x}_n = [x_{1,n} \dots x_{T,n}]^T$ is the $T \times 1$ transmitted signal vector in subcarrier n, $\mathbf{y}_n = [y_{1,n} \dots y_{K,n}]^T$ is a $K \times 1$ vector containing the received signal of each user, and $\mathbf{z}_n = [z_{1,n} \dots z_{K,n}]^T$ is a $K \times 1$ vector denoting the noise that is assumed to be independent identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian with covariance matrix $\sigma^2 \mathbf{I}_K$.

It is also assumed that the channel vectors are statistically independent and that their distribution is continuous. Hence, $\text{rank}(\mathbf{H}_n) = \min(T, K)$. Moreover, the practically important case where $K \geq T$ is considered. Hence, $\text{rank}(\mathbf{H}_n) = T$. The total transmitted power, in the entire OFDM symbol, is P_{tot} and equal power is allocated to each subcarrier. Hence, $\text{trace}[\mathbf{C}_n] \leq \frac{P_{tot}}{N}$, where $\mathbf{C}_n = \mathbb{E}[\mathbf{x}_n (\mathbf{x}_n)^*]$ is the covariance matrix of the transmitted signal $\mathbf{x}_n$.

Using only transmit beamforming, which is a suboptimal strategy, the following model is obtained. Let $\mathbf{w}_{k,n} = [w^1_{k,n} \ w^2_{k,n} \ \dots \ w^T_{k,n}]^T$ be the $T \times 1$ beamforming weight vector for user k in subcarrier n. Then, the baseband model (1) can be written as

$$\mathbf{y}_n = \mathbf{H}_n \mathbf{W}_n \mathbf{D}_n \mathbf{s}_n + \mathbf{z}_n, \tag{2}$$

where $\mathbf{W}_n = [\mathbf{w}_{1,n} \ \mathbf{w}_{2,n} \ \dots \ \mathbf{w}_{K,n}]$ is the $T \times K$ beamforming weight matrix, $\mathbf{s}_n = [s_{1,n} \dots s_{K,n}]^T$ is a $K \times 1$ vector containing the signals destined to each user, and $\mathbf{D}_n = \text{diag}(\sqrt{p_{1,n}}, \sqrt{p_{2,n}}, \dots, \sqrt{p_{K,n}})$ accounts for the distribution of the power allocated to subcarrier n among the K users. According to (2), the resulting received signal vector for user k in subcarrier n, is given by

$$\begin{aligned} y_{k,n} &= \sum_{i=1}^{K} \mathbf{h}_{k,n} \mathbf{w}_{i,n} \sqrt{p_{i,n}} s_{i,n} + z_{k,n} = \\ &= \mathbf{h}_{k,n} \mathbf{w}_{k,n} \sqrt{p_{k,n}} s_{k,n} + \\ &+ \sum_{i=1, i \neq k}^{K} \mathbf{h}_{k,n} \mathbf{w}_{i,n} \sqrt{p_{i,n}} s_{i,n} + z_{k,n}, \end{aligned} \tag{3}$$

where the term in third line in (3) represents the multi-user interference caused by the simultaneous transmission of data to other users in subcarrier n. Concerning (3), a graphic representation of the MISO downlink beamforming block diagram is shown in Fig. 1.

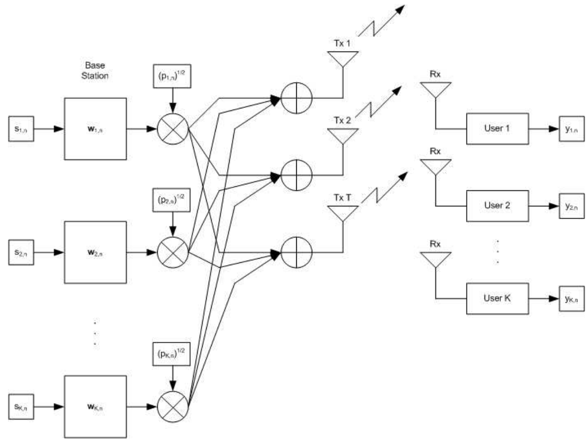

Fig. 1. MISO-OFDMA block diagram.

III. Problem Formulation

ZF beamforming is a spatial signal processing by which the multiple antenna transmitter can null multiuser interference signals in wireless communications. It inverts the channel matrix at the transmitter in order to create orthogonal channels between the transmitter and the receiver. The beamforming vectors are selected such that $\mathbf{h}_{i,n} \cdot \mathbf{w}_{j,n} = 0$, for $i \neq j$, and (3) becomes $y_{k,n} = \mathbf{h}_{k,n} \mathbf{w}_{k,n} \sqrt{p_{k,n}} s_{k,n} + z_{k,n}$. It is then possible to encode users individually, and with smaller complexity compared to DPC. ZF at the transmitter incurs an excess transmission power penalty relative to ZF-DPC and the (optimal) MMSE-DPC transmission scheme. If $K \leq T$ and $\text{rank}(\mathbf{H}_n) = K$, the ZF beamforming matrix is the pseudo-inverse of $\mathbf{H}_n$, namely $\mathbf{W}_n = \mathbf{H}^*_n (\mathbf{H}_n \mathbf{H}^*_n)^{-1}$.

However, if $K > T$, it is not possible to use it because $\mathbf{H}_n \mathbf{H}^*_n$ is singular and low complexity SDMA approaches are required. In that case, it is necessary to select $t \leq T$ out of K users in each subcarrier. Hence, there are I possible combinations of users transmitting in the same subcarrier, denoted as A_i, where $A_i \subset \{1, 2, \dots, K\}$, $0 < |A_i| \leq T$, where $|A_i|$ denotes the cardinality of set A_i, and $I = \sum_{l=1}^{T} \binom{K}{l}$.

Let a set of users $A_i = \{s_1, \dots, s_t\}$, that produce the row-reduced channel matrix $\mathbf{H}_n(A_i) = [\mathbf{h}_{s_1,n} \ \mathbf{h}_{s_2,n} \ \dots \ \mathbf{h}_{s_t,n}]^T$ in each subcarrier. When ZF is used, the data rate of user $k \in A_i$, in subcarrier n, is given by [16] [20]

$$r_{k,i,n} = \log_2(\mu_n c_{k,n}(A_i)), \tag{4}$$

where $c_{k,n}(A_i) = \{[(\mathbf{H}_n(A_i)\mathbf{H}_n(A_i)^*)^{-1}]_{k,k}\}^{-1}$ and μ_n is obtained by solving the water-filling equation [20] $\sum_{k \in A_i} \left[\mu_n - \frac{1}{c_{k,n}(A_i)}\right]_+ = \frac{P_{tot}}{N}$. The power loading then yields $p_{k,i,n} = c_{k,n}(A_i) \left[\mu_n - \frac{1}{c_{k,n}(A_i)}\right]_+, \forall k \in A_i$.

By applying the conclusions above, the linear beamforming optimization problem, that performs user selection in each

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subcarrier and resource allocation in the entire OFDM symbol, can be formulated as

$$\max_{\rho_{k,i,n},p_{k,i,n}} \frac{B}{N}\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{i=1}^{I}\rho_{k,i,n}r_{k,i,n} \tag{5}$$

subject to

$$\rho_{k,i,n} \in \{0,1\},\ \forall k,i,n,$$
$$p_{k,i,n} \geq 0,\ \forall k,i,n,$$
$$\sum_{k=1}^{K} p_{k,i,n} \leq \frac{P_{tot}}{N},\ \forall n,i,$$
$$\sum_{k=1}^{K} \rho_{k,i,n} \leq T,\ \forall n,i,$$
$$R_k \geq mr_k,\ \forall k,$$

where mr_k is the minimum data rate required by the kth user. $\rho_{k,i,n}$ is the subcarrier allocation indicator such that $\rho_{k,i,n} = 1$ if user $k \in A_{i,n}$, and $A_{i,n}$ is selected in subcarrier n; otherwise $\rho_{k,i,n} = 0$, $\forall k,i,n$. The total data rate for user k, denoted as R_k, is defined as

$$R_k = \frac{B}{N}\sum_{n=1}^{N}\sum_{i=1}^{I}\rho_{k,i,n}r_{k,i,n}. \tag{6}$$

The problem above (5) is an NP-hard combinatorial optimization problem with non-linear constraints. The optimal solution can be obtained by exhaustive search of all possible user assignment sets in all subcarriers but the complexity is given by I^N, which is extremely complicated even for moderate K, N.

IV. The Proposed Resource Allocation Algorithm

In the following, a suboptimal, two-step, low-complexity user selection and resource allocation algorithm is proposed, that selects users independently in each subcarrier, it is based on ZF beamforming and guarantees individual minimum data rates.

1) Step 1: Algorithm without minimum data rate constraints: It is a modification of [17] implemented in each subcarrier.

- Set $R_k = 0$, $\forall k$, $\rho_{k,i,n} = 0$, $\forall k,i$, and $\forall n \in \mathcal{S}$.
- For $n = 1,2,\ldots,N$:
  - Set $\mathcal{U} = \{1,2,\ldots,K\}$, $|A_{i,n}| = \emptyset$.
  - Find user $k = \text{argmax}_{j\in\mathcal{U}} \parallel \mathbf{h}_{j,n} \parallel$.
  - Set $t = 1$, $\rho_{k,i,n} = 1$, $A_{i,n}(t) = \{k\}$, $\mathcal{U} = \mathcal{U} - \{k\}$, and compute R_k, according to (4), (6). $A_{i,n}(t)$ means the allocation result of the t step in subcarrier n.
  - For $t = 2,3,\ldots,T$:
    - ∗ Find a user, $s_t \in \mathcal{U}$, such that
    $$\sum_{k\in A_i(t-1)\cup\{s_t\}} r_{k,i,n} > \sum_{k\in A_i(t-1)} r_{k,i,n}$$
    - ∗ If user s_t is found, set $\rho_{s_t,i,n} = 1$, $A_{i,n}(t) = A_{i,n}(t-1)\cup s_t$, and $\mathcal{U} = \mathcal{U} - \{s_t\}$.
    - ∗ Compute R_k, $\forall k \in A_{i,n}(t)$, according to (4), (6).

After the initialization, the algorithm finds user k with the best channel condition in subcarrier n. Subcarrier n is then assigned to additional users if the sum data rate in subcarrier n increases [17]. If there are more than one candidate users s_t in each step, pick the one with the maximum sum data rate. Procedure continues for all subcarriers.

2) Step 2: Subcarrier reallocation: In Step 1, a subcarrier allocation solution is obtained which does not guarantee the fulfillment of every user's data rate constraints. So, some subcarriers need to be allocated to the users whose minimum data rate constraints have not been satisfied yet, a procedure that causes an inevitable decrease in the overall data rate since these subcarriers were first allocated to users with the best channel condition on them. During the reallocation procedure each subcarrier reallocation should cause the least reduction in the sum of the users' data rates and the number of reallocation operations should be kept as low as possible. Thus,

$$e_{k,t,n} = \max\left(\frac{r_{t,i,n}-r_{k,i,n}}{r_{k,i,n}}, \frac{r_{k^*,i,n}-r_{k^*,i',n}}{r_{k^*,i',n}}\right), \tag{7}$$
$$\forall k^* \in A_{i,n}\cap A_{i',n},\ \forall n \in \mathcal{S},$$

is defined as the cost function of reallocating subcarrier n to user k, forming $A_{i',n}$, instead of the originally assigned user t. If user k occupies subcarrier n, instead of user t, then the data rates of the other users (each $k^* \in A_{i,n}\cap A_{i',n}$) occupying subcarrier n will be affected too. Hence, the cost function (7) takes into account the change of the data rate of all users in each subcarrier. In addition, max function, in (7), consists of at most T elements, $r_{k^*,i,n}$ means the data rate of user $k^* \in A_{i,n}$ and $r_{k^*,i',n}$ means the data rate of user $k^* \in A_{i',n}$, both in subcarrier n. The subcarrier reallocation algorithm is as follows.

- For $k = 1,2,\ldots,K$:
  - Set $\mathcal{S} = \{1,2,\ldots,N\}$.
  - While $R_k < mr_k$:
    - ∗ Calculate the cost function according to (7).
    - ∗ Find $[t^*,n^*] = \underset{t\in A_{i,n},n\in\mathcal{S}}{\text{argmin}}\ e_{k,t,n}$.
    - ∗ If $R_{t^*} - r_{t^*,i,n} \geq mr_{t^*}$ and $R_m \geq mr_m$, $\forall m \in A_{i,n}\cap A_{i',n}$: Set $\rho_{t^*,i,n} = 0$, $\rho_{k,i,n} = 1$, $\mathcal{S} = \mathcal{S} - \{n\}$, $A_{i,n} = A_{i',n}$ and compute R_m, $\forall m \in A_{i,n}$, according to (4), (6).
    - ∗ Else: $\mathcal{S} = \mathcal{S} - \{n\}$.
    - ∗ If $\mathcal{S} = \emptyset$: break.

In Step 2, R_k, $A_{i,n}$ and $\rho_{k,i,n}$, $\forall k,i,n$ are known from Step 1. Subcarrier reallocation is carried out on a user-by-user basis for all users whose minimum data rate requirements have not been satisfied in Step 1. Consider user k for example. In each stage, user t^* and subcarrier n^* with the lowest cost function are selected which cause the least reduction in the sum of the users' data rates. Subcarrier n^* will be allocated to user k instead of the originally assigned user t^*, if this reallocation does not cause $R_{t^*} - r_{t^*,i,n} < mr_{t^*}$ and $R_m < mr_m$, $\forall m \in A_{i,n}\cap A_{i',n}$. Otherwise, the reallocation will not be done and new t^*, n^* will be identified from the rest of the subcarriers. This subcarrier reallocation process repeats for user k until its data rate requirement is satisfied, otherwise an outage is occured.

V. Complexity Analysis

In order to analyze the computational complexity of the proposed algorithm, recall that K refers to the total number of users in the system and T refers to the number of transmit antennas at the BS. N on the other hand refers to the number of subcarriers, which is much larger than both K and T.

In Step 1 of the proposed algorithm, the best user k among K users is found for subcarrier $n = 1, 2, \ldots, N$, which requires $O(KN)$ operations. Then, at most $T-1$ other users are found for subcarrier n which requires the evaluation of at most T data rates $r_{k,i,n}$. In order to evaluate $r_{k,i,n}$, inversion of $\mathbf{H}_n(A_i(t-1)\cup s_t)\mathbf{H}_n(A_i(t-1)\cup s_t)^*$ is required which can be done in time $O(T^2)$, for the worst case, when T users occupy each subcarrier, using the matrix inversion lemma as described in [17]. Repeating this over at most $K-1$ users ($s_t \notin A_i(t-1)$) in each one of the $t=2$ to T steps, and over all subcarriers of set $\mathcal{S}$, the overall complexity of Step 1 is obtained to be $O(KNT^3)$.

In Step 2, while loop runs for at most N times for each user. In the while loop, cost function is calculated which requires T comparisons for the max function, $O(NT^2)$ time for the data rates ($r_{k,i,n}$, $r_{k^*,i,n}$, $r_{k^*,i',n}$) for all subcarriers and TN multiplications. Therefore, cost function requires $O(T^4N^2)$. Finding t^*, n^*, requires $O(TN)$. Thus, complexity of Step 2 is $O(KN^2T^4)$ which is also the complexity of the whole proposed algorithm.

As was mentioned the complexity of exhaustive search for the optimal solution of the original problem is given by I^N, where $I = \sum_{l=1}^{T}\begin{pmatrix} K \\ l \end{pmatrix}$. Alternatively, the complexity is $O(K^{NT})$ and is prohibitive even for moderate values of K, N, and T. Thus, it is easily observed that the proposed algorithm has a very dramatic reduction in complexity compared to $O(K^{NT})$ required by the exhaustive search.

VI. Simulation Results

The proposed algorithm is compared with the algorithms proposed in [17]-[21], Round Robin (RR) algorithm, and Maximal Ratio Combining (MRC) transmission, only to the user with the strongest channel. $mr_k = 1.5$ bits/s/Hz, $\forall k$, and in [20], proportional data rate constraints are $\gamma_k = \frac{mr_k}{\sum_{k=1}^{K} mr_k}$, $\forall k$ and system parameters are $D = 0.1$, $L = T$. In RR algorithm, each user is given a fair share of the channel resource regardless of the channel state and T users are selected in each subcarrier. Both equal power (EQ) allocation and water-filling (WF) power allocation over the parallel subchannels are considered.

In all simulations presented in this section, the frequency-selective channel consists of six independent Rayleigh multipath components. As in [20], an exponentially decaying power delay profile is assumed, the ratio of the energy of the lth tap to the first tap being equal to e^{-2l}. A maximum delay spread of 5μs and maximum doppler of 30Hz is assumed. The channel information is sampled every 0.5ms to update the proposed algorithm, $T = 4$, $N = 128$ and the number of channel realizations is equal to 10^5.

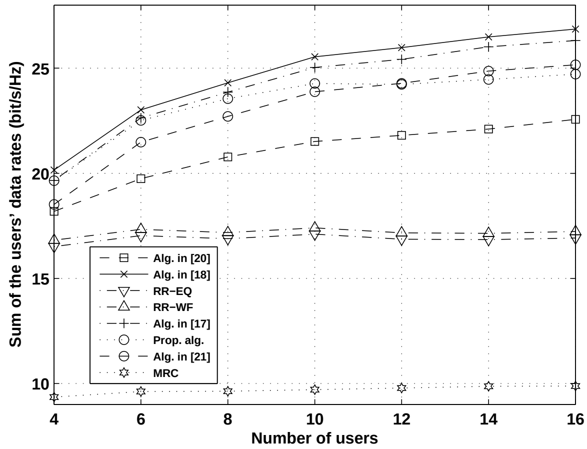

Fig. 2. Sum of the users' data rates vs K.

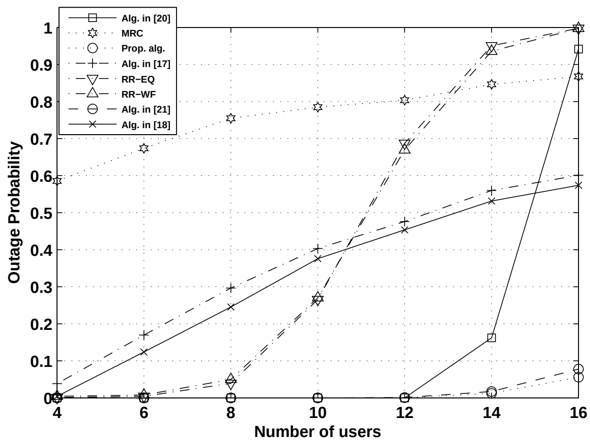

Fig. 3. Outage probability vs K.

In Figs. 2, 3, K varies from $4-16$ in increment of 2 and $SNR = 20$, while in Figs. 4, 5, $K = 16$. Also, Figs. 2, 4 depict the sum of the users' data rates vs number of users and average channel SNR, respectively, and Figs. 3, 5 depict the outage probabily for different values of number of users and average channel SNR, respectively. Outage probability is defined as the ratio of the users that have not reached their target data rate over K.

In Figs. 2, 4, it can be seen the reasonable price being paid in order to guarantee minimum data rate requirements by using the proposed algorithm. In Fig. 2, as the number of users increases, the difference in sum data rates increases because additional multiuser diversity is available to [17] [18] that only target sum data rate maximization. On the other hand, more users put more constraints to the proposed algorithm, because new users need to share the same resources. Algorithm

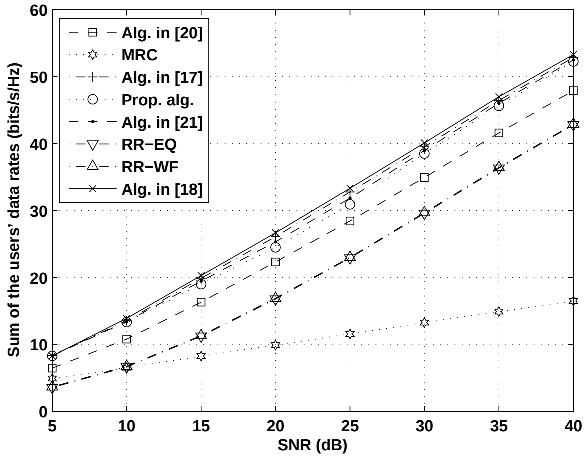

Fig. 4. Sum of the users' data rates vs SNR.

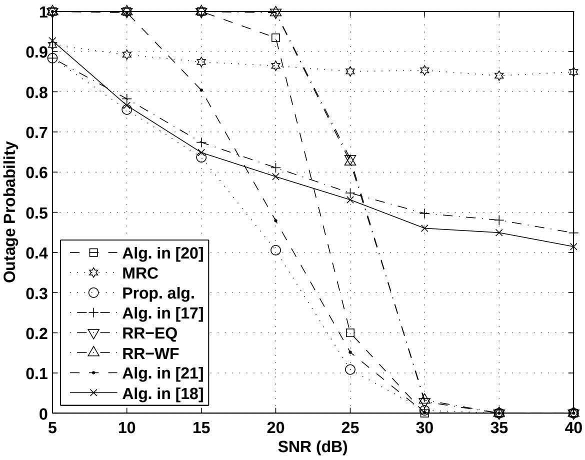

Fig. 5. Outage probability vs SNR.

in [17] is only the first Step of the proposed algorithm where minimum data rate requirements are not yet considered while MRC algorithm is the lower bound of the proposed algorithm as in MRC each subcarrier is allocated to only one user. In [20], although $\gamma_k = \frac{mr_k}{\sum_{k=1}^{K} mr_k}$, $\forall k$, it does not mean that minimum data rate requirements are satisfied, as seen in Figs. 3, 5. Moreover, sum data rate of the proposed algorithm is significantly enhanced over both RR-WF and RR-EQ algorithm, wherein the channel gain information is not exploited. Furthermore, algorithm in [21] imposes a kind of fairness between users' data rates. In addition, in Fig. 3, the outage probability achieved by the proposed algorithm is smaller than that achieved from the reference algorithms. Finally, in Fig. 5, it is also shown that the outage probability achieved by the proposed algorithm decreases quickly over the average channel SNR and is lower than that of the reference algorithms.

VII. Conclusion

A fairness-aware user selection and resource allocation algorithm, which is based on ZF beamforming, for the MISO downlink over frequency-selective channels was introduced. The main goal was to satisfy the minimum data rate requirements of users despite the loss with respect to the unconstrained case where the only target is the maximization of the sum data rate. Simulation results provide proofs about these statements and complexity analysis shows the dramatic reduction in complexity compared with exhaustive search.

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