Investigation of the Correlation Effect on the Performance of V-BLAST and OSTBC MIMO Systems

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***Abstract*—In this paper, we study the effect of the spatial correlation between multiple antennas on the performance of the MIMO systems in flat fading channels. As performances metrics, the Bit Error Rate (BER) and Symbol Error Rate (SER) are investigated for Orthogonal Space Time Block Coding (OSTBC) and Vertical Bell Laboratories Layered Space-Time code (V-BLAST) architecture respectively. It is assumed that all antennas have the same radiation pattern and the spatial correlation is independent of the position of an antenna in the array. In this investigation, it is assumed that the correlation matrix of the MIMO channel is the Kronecker product of the receive and transmit correlation matrices. The desired correlated channel is obtained by the Cholesky decomposition of the correlation matrix.**

Keywords-Multi-antenna MIMO system, OSTBC, V-BLAST, Bit Error Rate

I. INTRODUCTION

Multiple-input multiple-output (MIMO) systems are those that have multiple antenna at both transmitter and receiver. They were first investigated by computer simulation in the 1980s, and later papers explored them analytically. Since that time, interest in MIMO systems has exploded. Multiple antennas can be used for increasing capacity or for increasing diversity. To improve the diversity in a MIMO system, the Orthogonal Space Time Block Coding are used [10]. On the other hand, to improve the capacity, independent data streams are transmitted simultaneously from multiple antennas [2]. The Vertical Bell Laboratories Layered Space-Time code (V-BLAST) is an architecture for realizing very high data rates over the rich scattering wireless channel [3]. The detection algorithms for V-BLAST architecture are traditionally based on the Order Successive Interference Cancellation (OSIC) [4]. The V-BLAST OSIC algorithm is an iterative algorithm where the the strongest signal is decoded first, then the effect of this strongest signal is cancelled from each of the received signals [8]. The correlation at the receiver and the transmitter side can considerably reduce the performance of the MIMO systems. It is important to quantify the effect of the correlation to properly design MIMO systems [12]. In [5], the BER expressions of optimum combining and maximal ratio combining in the presence of one co-channel interferer was derived. In [6], the effect of transmission design and spatial correlation on the symbol error rate (SER) was analyzed for MIMO communication systems. While most previous work considered the analysis for OSTBC and V-BLAST with separately correlation model, this work considers the effect of the channel correlation using the jointly-correlated model. With this approach an analytical solution can be derived for the BER and SER performances. When all the antennas in the MIMO systems have the same radiation pattern, which is the case in almost the practical cases, and when the spatial correlation is independent of the position of an antenna in the array, it is shown in [1] that the (4x4) MIMO correlation matrix can be approximated by the Kronecker product of the transmitter correlation matrix and the receiver correlation matrix.

In this paper, the BER and the SER of the OSTBC and OSIC V-BLAST, respectively, are investigated with respect to the spatial correlation effect. The kronecker model of the correlated channel and the Cholesky decomposition of the cross-correlated matrix are considered for the study.

The remainder of this paper is organized as follows. In Section II, we present the system model and give a brief review of the detection algorithm of OSIC. A brief description of the OSTBC is also reviewed in this section. In Section III, we present the correlation model. The simulation results and discussion of results will be presented in Section IV. Finally, we conclude our paper in Section V.

II. SYSTEM AND CHANNEL MODEL

We consider a narrow band MIMO wireless communication system with 2 transmit antennas and 2 receive antennas. It is assumed that the channel experiences quasi-static flat Rayleigh fading. We also assume that CSI is known at the receiver. For the OSTBC, at each time the two transmit antennas send

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Alamouti coded blocks [10] and two receive antennas are used to receive the encoded transmitted block. Let $H = [h_{i,j}]$ denote the 2x2 MIMO channel matrix where $h_{i,j}$ is the fading coefficient between transmit *j* and receive antenna *i*, $j = 1, 2$, $i = 1, 2$. $h_{i,j}$ is a sample of independent complex Gaussian random variables with zero mean and variance 1/2 per dimension. For decoding purposes, a 2x2 matrix H is used to denote the channel between the two transmit and two receive antennas. The complex envelope of the received signal at the antenna array after matching filter is given by [2]:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where **y** represent a two dimensional received signal, **n** is a 2 dimensional complex additive white Gaussian noise (AWGN) vector, of which each component is statistically independent and has zero mean and a variance $\sigma^2/2$ per dimension, **x** a two dimensional of transmitted signal and **H** is a 2x2 MIMO fading channel [11]:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \tag{2}$$

To introduce the correlation effect in the system model, a correlated matrix should be generated according to [7]

$$\mathbf{vec}(\mathbf{H}_c) = R_{MIMO}^{1/2}\ \mathbf{vec}(\mathbf{H}) \tag{3}$$

where $\mathbf{H}_c$ represent the correlated matrix of the MIMO channel and is given by [7]:

$$\mathbf{H}_c = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \tag{4}$$

The correlation matrix of the MIMO channel, R_{MIMO}, is obtained by the Kronecker product of the transmit and receive correlation matrices as follow [11]:

$$R_{MIMO} = R_{MS} \otimes R_{BS} \tag{5}$$

Using the properties of the Kronecker product the model of (2) is simplified to the form [7]:

$$\mathbf{H}_c = R_{MS}^{1/2}\ \mathbf{H}\ (R_{BS}^{1/2})^T \tag{6}$$

The correlation matrices of the transmitter and the receiver are given by [11]:

$$R_{BS} = \begin{bmatrix} 1 & \rho_{12}^{BS} \\ \rho_{21}^{BS} & 1 \end{bmatrix} \tag{7}$$

$$R_{MS} = \begin{bmatrix} 1 & \rho_{12}^{MS} \\ \rho_{21}^{MS} & 1 \end{bmatrix} \tag{8}$$

To generate the desired correlated elements, the elements of (4) are arranged in vector form as:

$$\mathbf{vec}(\mathbf{H}_c) = \begin{bmatrix} \alpha_{11} \\ \alpha_{12} \\ \alpha_{21} \\ \alpha_{22} \end{bmatrix} \tag{9}$$

The correlated and no-correlated elements are then related by [11]:

$$\begin{bmatrix} \alpha_{11} \\ \alpha_{12} \\ \alpha_{21} \\ \alpha_{22} \end{bmatrix} = C \begin{bmatrix} h_{11} \\ h_{12} \\ h_{21} \\ h_{22} \end{bmatrix} \tag{10}$$

where C is the result of the Cholesky decomposition of the R_{MIMO} [11]:

$$R_{MIMO} = CC^T \tag{11}$$

III. V-BLAST OSIC DETECTION ALGORITHM

The simple transmission and detection mechanism of the V-BLAST and its ability to achieve a high data rate performances have made the technic a popular solution for signal processing. In this section, we briefly describe the V-BLAST OSIC algorithm. Letting $\mathbf{x} = [x_1, x_2]^T$ represent the vector of the transmitted symbols, then the corresponding received vector is given by [9]:

$$\mathbf{y}_1 = \mathbf{H}_c\mathbf{x} + \mathbf{n} \tag{12}$$

In the above equation, **x** and $\mathbf{y}_1$ are two vectors of transmitted and received signals respectively. To be able to detect the different stream send by the transmitter, an iterative process in which stream by stream detection is necessary. In other words, the detection process should be performed on one stream at time. This process is based on the linear combinatorial nulling on the received vector and an operation of cancelling the obtained symbol operation is followed [9]. The process should have the following steps:

In a first step, the received vector, $\mathbf{y}_1$, should be multiplied by a nulling vector, $\mathbf{w}_{k1}$ to obtain the quantity y_{k1} [9]:

$$y_{k1} = \mathbf{w}_{k1}^T \mathbf{y}_1 \tag{13}$$

In the next step, y_{k1} is sliced using the quantization, Q(.), operation appropriate to the the constellation in use and the a_{k1} is obtained [9]:

$$a_{k1} = Q(y_{k1}) \tag{14}$$

Finally, a modified received vector $\mathbf{y}_2$ is obtained by cancelling a_{k1} from the received vector $\mathbf{y}_1$, resulting [9]:

$$\mathbf{y}_2 = \mathbf{y}_1 - a_{k1}(\mathbf{H}_c)_{k1} \tag{15}$$

where $(\mathbf{H}_c)_{k1}$ represents the k_1th column of $\mathbf{H}_c$. The previous steps are then performed on the modified received vector $\mathbf{y}_2$ to detect the following symbol. The process should be stopped until all symbols are detected.

IV. SIMULATION RESULTS

In this section, we illustrate the results of the effect of spatial correlation of MIMO channel on the performances of V-BLAST and OSTBC MIMO architecture. The Monte-Carlo simulations are used to obtain the results. In V-BLAST architecture, a frame of 100 symbols is transmitted over the MIMO channel. The channel is represented by a 2x2 matrix with correlated elements. The performances of the non linear detection are obtained for 5000 iterations. The detected symbols at the output of the receiver are compared with the ones transmitted from the transmitter side so that the SER is calculated. The results of the simulations are represented in the Fig. 1. In this figure, the curve which correspond to zero correlation is identical to the litterature results [2]. From theses results it is shown that the correlation between sub channels degrade considerably the performances of V-BLAST architecture. In fact, for a fixed SNR of 15 dB, the SER of $\rho = 0.8$ is much important than the one of $\rho = 0.7$. For a fixed value of correlation, say $\rho = 0.4$, the SER decrease when the value of the signal to noise ratio is increased. We also notice that for $\rho = 0.999$, the SER is almost constant and doesn't depend on the SNR. This can be explained by the fact that with strong correlation, the algorithm become less efficient and hence the importance of the spatial correlation on the hole performances. For the performances of the OSTBC, the results are presented in the Fig. 2. The one rate Alamouti code and BPSK modulation with real constellation were used for the simulations [10]. In the simulation, a random 2x2 MIMO channel Matrix is generated. The results show also that the strong correlation decrease considerably the BER performances of the OSTBC.

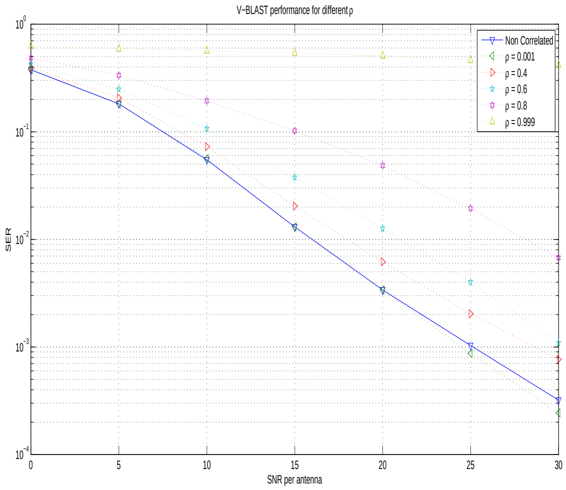

Fig. 1. SER Performances of V-BLAST OSIC algorithm with respect to different spatial correlation ρ

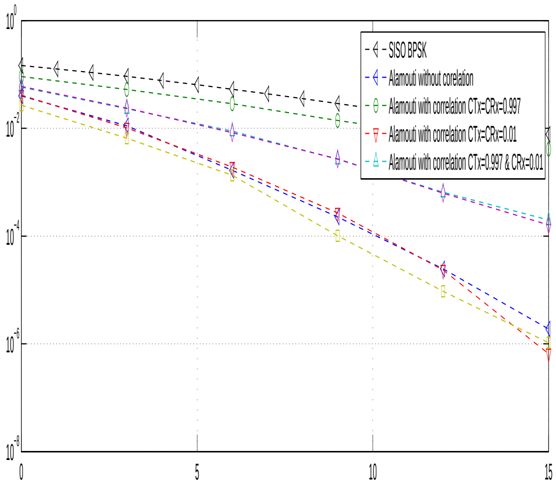

Fig. 2. BER Performances of OSTBC using Alamouti code with respect to different spatial correlation ρ

V. CONCLUSION

In this paper, we investigated the effect of the spatial correlation on the performances of two MIMO architectures known as the V-BLAST and OSTBC. The Kronecker model for the MIMO chanel combining with Cholesky decompossition were used to the simulation. The BER and the SER are used as performances metrics for the study. The results shows clearly that the correlation decrease considerably the performances of the MIMO systems.

REFERENCES

[1] A. Hotinen, O. Tirkkonen, and R. Wichman “ Multi-antenna transceiver techniques for 3 G and beyond, West Sussex, England; Hoboken, NJ: J. Wiley, 2003”

[2] G. J. Foschini and M. G. Gans “ On limits of wireless communications in a fading environment when using multiple antennas” *IEEE Wireless Personal Communications*, pp. 311–335, Mars 1998.

[3] G. J. Foschini “ Layered space time architecture for wireless communication in a fading environment using multi-element antennas” *IEEE Bell Labs. Tech. J. vol. 1, no. 2. Communications*, pp. 41–59, 1996.

[4] Z. Hufei, W. Chen, B. Li, and F. Gao “ An Improved Square Root Algorithm for V-BLAST Based on Efficient Inverse Cholesky Factorization” *IEEE Trans. on Wireless Communications*, vol. 10, no. 1, pp. 43–48, January 2011.

[5] J. Cui, A.U.H. Sheikh, and D.D. Falconer “ BER analysis of optimum combining and maximal ratio combining with channel correlation for dual antenna systems” *IEEE Vehicular Technology Conference, IEEE 47th*, vol. 1, no. 1, pp. 43–48, May 1997.

[6] E. Bjornson, E. Jorswieck, and B. Ottersten ”Impact of Spatial Correlation and Precoding Design in OSTBC MIMO Systems,” *Wireless Communications, IEEE Transactions on*, vol. 9, no. 11, pp.3578-3589, November 2010

[7] H. Ozcelik, M. Herden, W. Weichselberger, J. Wallace, and E. Bonek “ Deficiencies of Kronecker MIMO radio channel model” *IEEE Electronics Letters.*, vol. 39, no. 16, pp. 1209–1210, August 2003.

[8] P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuella “ V-BLAST an architecture for realizing very high data rate over the rich-scattering wirless channel” *URSI International Symposium on Signals, Systems and Electronics.*, 1998.

[9] G.D. Golden, C.J. Foschini, R.A. Valenzuela, and P.W. Wolniansky “ Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture” *IEEE Electronics Letters*, vol. 35, no. 1, pp. 14–16, Jan. 1999.

[10] S. M. Alamouti “A Simple Transmit Diversity Technique for Wireless Communications”, *IEEE Journal on Select Areas in Communications*, vol. 16, no. 8, pp. 1451-1458, October 1998.

[11] Kermoal, J. P., Schumacher, L., Pedersen, K. I., Mogensen, P. E., and Frederiksen, F “A stochastic MIMO radio channel model with experimental validation”, *Selected Areas in Communications, IEEE Journal on*, vol 20, No 6, August 2002, pages 1211-1226.

[12] Zhemi Xu, Sana Sfar, and Rick S. Blum “Receive Antenna Selection for Closely Spaced Antennas with Mutual Coupling”, *IEEE Transaction on WirelessCommunications*, vol. 9, no. 2, pp. 652-661, February 2010.