Amplifying Matrix Design for Cooperative Relay Networks under Channel Uncertainty and Power Constraint

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***Abstract*—This paper presents an optimum amplifying relay matrix based on minimum mean square error (MMSE) criteria for the cooperative distributed wireless relay networks under channel uncertainty and total power constraint at the relay nodes. A one-source-one-destination pair node and N relay nodes are considered, which can be extended for a general case. All relays are assumed to exchange their received signals with no errors for full cooperation. With the derived optimum amplifying relay matrix, bit error rate (BER) of the relay network under channel uncertainty is simulated and compared with that of the network under a certain channel condition to observe the degradation.**

***Index Terms*—AF, MMSE, channel-uncertainty, power-constraint.**

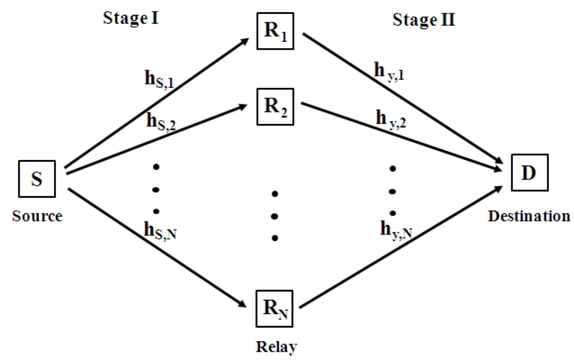

Fig. 1. Wireless relay network with one-source-one-destination pair and N relay nodes under no channel uncertainty ($\mathbf{h}_s$, $\mathbf{h}_y$).

I. Introduction

Due to the restricted power, relay nodes in wireless networks can only retransmit their received signal from a source node within a limited distance [1]. To overcome this limitation, various protocols were proposed. Relay protocols can be classified as amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF) [2]–[8]. The AF relay protocol only retransmits a scaled version of their received signals from the source node according to their power constraint. Therefore, AF relay protocol employed in this current paper is a reasonable strategy when relay nodes have limited power. Compared to the DF and CF relay protocols, AF relay protocol results in a lower complexity. This is due to the fact it does not require signal processing at the relay for decoding and compressing. In addition, AF relay protocol shows a shorter delay at the relay node. This is also due to the same fact unlike the DF and CF protocols.

In practice, due to wireless communication characteristics during information transmission, all relay nodes can have an inaccurate knowledge of their local channels, either from the source node to relay nodes or from relay nodes to the destination node. In [5], the DF relay protocol with single-input single-output was considered for a multibranch cooperative distributed wireless relay network with direct link between the source and the destination under no channel uncertainty. While, in [6], a multibranch noncooperative AF relay network without direct link between the source and the destination was considered for single-input single-output with N relays under channel uncertainty. AF relay protocol in [6] did not employ cooperation among distributed relay nodes even though authors considered channel uncertainty. Furthermore, total power constraint at relay nodes was not included in [6]. In [7], [8], relay nodes exchanged their received signals for cooperation, and channels were assumed to be perfectly known at each relay node. While, in [9], [10], total power constraint at the destination node instead of relay nodes was considered for noncooperative AF wireless relay networks under certain channel condition to minimize mean square error. In addition, channel uncertainty for noncooperative AF wireless relay networks under the received power constraint at the destination node was studied in [11]. Namely, recently, cooperative AF wireless relay networks under channel uncertainty with power constraint at relay nodes was not considered. Therefore, the objective of this current paper is to find an optimum relay amplifying matrix for the cooperative distributed wireless

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relay networks under channel uncertainty and total power constraint at relay nodes. The minimum mean square error (MMSE) criterion is used to optimize the relay amplifying matrix. Since cooperation is employed, the optimum relay amplifying matrix can be non-diagonal. On the other hand, it will be a diagonal matrix if a non-cooperative relay network is required as in [6]. The diagonal relay amplifying matrix under total power constraint at the destination node is designed for the case where the direct link between a source and a destination is considered weak and can be negligible. Using the derived optimum relay amplifying matrix, effects of three different kinds of channel uncertainty both cooperative case and noncooperative case on BER was studied.

The remaining paper is organized into four sections. Section II describes the system model and information transmission scheme. Section III presents a cooperative distributed MMSE relay scheme based on the AF strategy. Three different imperfect channel information cases for both cooperative and noncooperative distributed wireless networks are considered. Section IV presents simulation results. Finally, Section V concludes the paper.

Notation: Matrices and vectors are denoted, respectively, by uppercase and lowercase boldface characters (e.g., $\mathbf{A}$ and $\mathbf{a}$). The transpose, complex conjugate, inverse, and trace of $\mathbf{A}$ are denoted, respectively, by $\mathbf{A}^T$, $\mathbf{A}^*$, $\mathbf{A}^{-1}$, and $tr(\mathbf{A})$. The Hermitian of $\mathbf{A}$ is denoted by $\mathbf{A}^H$, i.e., the complex conjugate and transpose of $\mathbf{A}$. An $n \times n$ identity matrix is denoted by $\mathbf{I}_N$. $\mathbf{A} \in \mathbf{C}^{m \times n}$ is an $m \times n$ complex matrix. The expectation operator is $E[\ \cdot\ ]$. Notations $|a|$, $||\mathbf{a}||$, and $||\mathbf{A}||_F$ denote the absolute value of a for any scalar, 2-norm of $\mathbf{a}$, and Frobenius-norm of $\mathbf{A}$, respectively. An $n \times n$ diagonal matrix is denoted by $\text{diag}\big(a_1,\ \cdots,\ a_N\big)$. An $\arg\min_{\mathbf{A}} J(\mathbf{A})$ denotes the value of $\mathbf{A}$ at which $J(\mathbf{A})$ is minimized.

II. System Model

Figure 1 shows a wireless relay network with N cooperative distributed relay nodes between a source node and a destination node. As shown in Fig. 1, there are two stages for information transmission where a source node transmits a signal symbol s in Stage I and the relay nodes retransmit their received signals to a destination node in Stage II using the amplifying matrix. Certain channel and uncertain channel conditions are described in the remaining of this section.

A. Certain Channel Condition

In a certain channel condition, all the relay nodes have perfect information of their local channel coefficients from the source node to the relay nodes and from the relay nodes to the destination node. Let $\mathbf{h}_s \in \mathbf{C}^{N\times 1}$ denote the perfect channel coefficient complex column vector from the source node to the relay nodes as

$$\mathbf{h}_s = [h_{s,1}, h_{s,2},\ \cdots,\ h_{s,N}]^T$$

where $h_{s,i},\ i = 1, \cdots, N$, is the i-th element of $\mathbf{h}_s$, representing the channel coefficient from the source node to the i-th relay node. The received signal complex column vector $\mathbf{r} \in \mathbf{C}^{N\times 1}$ at the relay nodes is written as

$$\mathbf{r} = \mathbf{h}_s s + \mathbf{v}_s \tag{1}$$

where $\mathbf{v}_s \in \mathbf{C}^{N\times 1}$ is a zero mean complex additive white Gaussian noise column vector with covariance matrix $\sigma^2_{v_s}\mathbf{I}_N$. Each channel $h_{s,i}$ is assumed to be independent identically distributed with a zero mean circular complex Gaussian of unit variance and quasi-static Rayleigh fading so that they stay fixed during an observation interval. For the cooperation of relay nodes, it is assumed that all the relay nodes exchange their received signals each other with negligible errors. The amplified signal complex column vector $\mathbf{x} \in \mathbf{C}^{N\times 1}$ at the relay nodes is given by

$$\mathbf{x} = \mathbf{F}\mathbf{r} \tag{2}$$

where $\mathbf{F} \in \mathbf{C}^{N\times N}$ is an optimum relay amplifying matrix at the relay nodes. Let $\mathbf{h}_y \in \mathbf{C}^{1\times N}$ denote the perfect channel coefficient complex row vector from the relay nodes to the destination node as

$$\mathbf{h}_y = [h_{y,1}, h_{y,2},\ \cdots,\ h_{y,N}]$$

where $h_{y,i},\ i = 1, \cdots, N$, is the i-th element of $\mathbf{h}_y$, representing the channel coefficient from the i-th relay node to the destination node. Each channel $h_{y,i}$ is also assumed to be independent identically distributed with a zero mean circular complex Gaussian of unit variance and quasi-static Rayleigh fading. The received complex signal $y \in \mathbf{C}^{1\times 1}$ at the destination node can be represented as

$$y = \mathbf{h}_y\mathbf{x} + v_y \tag{3}$$

where $v_y \in \mathbf{C}^{1\times 1}$ is a zero-mean complex additive white Gaussian noise variable with variance $\sigma^2_{v_y}$. Substituting (1) and (2) into (3), the received complex signal $y \in \mathbf{C}^{1\times 1}$ at the destination node can be written as

$$y = \mathbf{h}_y\mathbf{F}\mathbf{h}_s s + \mathbf{h}_y\mathbf{F}\mathbf{v}_s + v_y. \tag{4}$$

B. Uncertain Channel Condition

In reality, due to channel estimation errors, the estimates of the imperfect channel coefficient complex vectors $\bar{\mathbf{h}}_s$ and $\bar{\mathbf{h}}_y$ should be used instead of perfect $\mathbf{h}_s$ and $\mathbf{h}_y$, respectively, under uncertain channel conditions.

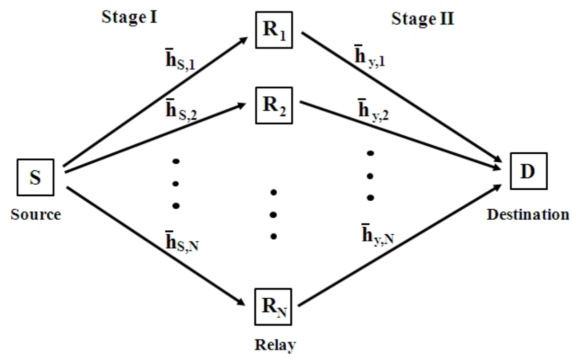

Fig. 2. Wireless relay network with one-source-one-destination pair and N relay nodes under channel uncertainty for both source-relay links and relay-destination links ($\bar{\mathbf{h}}_s,\ \bar{\mathbf{h}}_y$).

As shown Fig. 2, the estimated channel coefficient vectors can be written, respectively, as

$$\bar{\mathbf{h}}_s = \mathbf{h}_s - \boldsymbol{\phi}_s \quad \text{or} \quad \bar{\mathbf{h}}_s = \mathbf{h}_s + \boldsymbol{\phi}_s \tag{5}$$

and

$$\bar{\mathbf{h}}_y = \mathbf{h}_y - \boldsymbol{\phi}_y \quad \text{or} \quad \bar{\mathbf{h}}_y = \mathbf{h}_y + \boldsymbol{\phi}_y \tag{6}$$

where $\boldsymbol{\phi}_s \in \mathbf{C}^{N\times 1}$ and $\boldsymbol{\phi}_y \in \mathbf{C}^{1\times N}$ are, respectively, the channel estimation error column and row vectors consisting of complex independent identically distributed zero mean Gaussian random variables with covariance matrix $\sigma^2_{\phi_s}\mathbf{I}_N$ and $\sigma^2_{\phi_y}\mathbf{I}_N$. The estimated channel coefficient vectors are applied for both source-relay links and relay-destination links. Therefore, the received signal complex column vector $\mathbf{r}_1 \in \mathbf{C}^{N\times 1}$ at the relay nodes is represented as

$$\mathbf{r}_1 = \bar{\mathbf{h}}_s s + \boldsymbol{\phi}_s s + \mathbf{v}_s \tag{7}$$

where $\mathbf{v}_s \in \mathbf{C}^{N\times 1}$ is a zero-mean complex additive white Gaussian noise vector with covariance matrix $\sigma^2_{v_s}\mathbf{I}_N$. The subscript 1 denotes case 1 where $\bar{\mathbf{h}}_s$ and $\bar{\mathbf{h}}_y$ are used. The processed signal complex column vector $\mathbf{x}_1 \in \mathbf{C}^{N\times 1}$ at the relay node outputs is written as

$$\mathbf{x}_1 = \mathbf{F}_1\mathbf{r}_1 \tag{8}$$

where $\mathbf{F}_1 \in \mathbf{C}^{N\times N}$ is a relay amplifying matrix employed by the relay nodes to improve performance at the destination node. The received complex signal $y_1 \in \mathbf{C}^{1\times 1}$ at the destination node can be written as

$$y_1 = \mathbf{h}_y\mathbf{x}_1 + v_y = \bar{\mathbf{h}}_y\mathbf{x}_1 + \boldsymbol{\phi}_y\mathbf{x}_1 + v_y \tag{9}$$

where $v_y \in \mathbf{C}^{1\times 1}$ is a zero mean complex additive white Gaussian noise with variance $\sigma^2_{v_y}$. Substituting (7) into (8) and using (9), the received complex signal $y_1 \in \mathbf{C}^{1\times 1}$ at the destination node can be written as

$$\begin{aligned} y_1 = {} & \bar{\mathbf{h}}_y\mathbf{F}_1\bar{\mathbf{h}}_s s + \boldsymbol{\phi}_y\mathbf{F}_1\bar{\mathbf{h}}_s s + \bar{\mathbf{h}}_y\mathbf{F}_1\boldsymbol{\phi}_s s + \boldsymbol{\phi}_y\mathbf{F}_1\boldsymbol{\phi}_s \\ & + \bar{\mathbf{h}}_y\mathbf{F}_1\mathbf{v}_s + \boldsymbol{\phi}_y\mathbf{F}_1\mathbf{v}_s + v_y. \end{aligned} \tag{10}$$

In the next section, the optimum relay amplifying matrix $\mathbf{F}$ will be determined using the MMSE criteria for the channel uncertainty conditions. Note that in [6], Khajehnouri et. al. considered a noncooperative distributed wireless relay network. Hence, the relay amplifying matrix was a diagonal matrix in [6]. However, the relay amplifying matrix in this current paper would be non-diagonal because cooperation is made among the relay nodes by exchanging their received signals.

III. Cooperative Distributed MMSE Relay Scheme Under Channel Uncertainty

The MMSE will try to minimize the mean square error between the signal component at the destination node and the transmitted signal s at the source node. Total power constraint at the relay nodes will be included in the minimization. Assume both channel estimation vectors for both source-relay and relay-source links are not perfect, i.e., ($\bar{\mathbf{h}}_s,\ \bar{\mathbf{h}}_y$). The optimum relay amplifying matrix to minimize mean square error between the signal component at the destination node and the transmitted signal at the source node in (10) can be found from

$$\begin{aligned} \mathbf{F}_1^{\dagger} &= \arg\min_{F_1} J(\mathbf{F}_1) \\ \text{s.t.} \quad & E\big[||\mathbf{x}_1||_2^2\big] = \mathrm{P}_1 \end{aligned} \tag{11}$$

under the relay power constraint where the objective function $J(\mathbf{F}_1)$ is written using (10) as

$$\begin{aligned} J(\mathbf{F}_1) &= E\big[\big|\mathbf{h}_y\mathbf{x}_1 - s\big|^2\big] = E\big[\big|\bar{\mathbf{h}}_y\mathbf{x}_1 + \boldsymbol{\phi}_y\mathbf{x}_1 - s\big|^2\big] \\ &= E\big[\big|\bar{\mathbf{h}}_y\mathbf{F}_1\bar{\mathbf{h}}_s s + \boldsymbol{\phi}_y\mathbf{F}_1\bar{\mathbf{h}}_s s + \bar{\mathbf{h}}_y\mathbf{F}_1\boldsymbol{\phi}_s s + \boldsymbol{\phi}_y\mathbf{F}_1\boldsymbol{\phi}_s s \\ &\qquad + \bar{\mathbf{h}}_y\mathbf{F}_1\mathbf{v}_s + \boldsymbol{\phi}_y\mathbf{F}_1\mathbf{v}_s - s\big|^2\big] \\ &= tr\big(\sigma_s^2\bar{\mathbf{h}}_y\mathbf{F}_1\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\mathbf{F}_1^H\bar{\mathbf{h}}_y^H + \sigma_s^2\sigma^2_{\phi_s}\sigma^2_{\phi_y}\mathbf{F}_1\mathbf{F}_1^H \\ &\quad + \sigma_s^2\sigma^2_{\phi_y}\mathbf{F}_1\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\mathbf{F}_1^H + \sigma_s^2\sigma^2_{\phi_s}\bar{\mathbf{h}}_y\mathbf{F}_1\mathbf{F}_1^H\bar{\mathbf{h}}_y^H \\ &\quad - \sigma_s^2\bar{\mathbf{h}}_s^H\mathbf{F}_1^H\bar{\mathbf{h}}_y^H + \sigma^2_{v_s}\bar{\mathbf{h}}_y\mathbf{F}_1\mathbf{F}_1^H\bar{\mathbf{h}}_y^H \\ &\quad + \sigma^2_{v_s}\sigma^2_{\phi_y}\mathbf{F}_1\mathbf{F}_1^H - \sigma_s^2\bar{\mathbf{h}}_y\mathbf{F}_1\bar{\mathbf{h}}_s\big) + \sigma_s^2 \end{aligned} \tag{12}$$

where $E[\boldsymbol{\phi}_y] = E[\boldsymbol{\phi}_s] = \mathbf{0}$ is used in (12). The total power constraint at the relay nodes can be represented from (8) as

$$\mathrm{P}_1 = E\big[||\mathbf{x}_1||_2^2\big] = \sigma_s^2||\mathbf{F}_1\bar{\mathbf{h}}_s||_2^2 + \big(\sigma_s^2\sigma^2_{\phi_s} + \sigma^2_{v_s}\big)||\mathbf{F}_1||_F^2. \tag{13}$$

Because the total power at the relay nodes is constrained P_1, the constrained optimization in [12] is to be performed as

$$\begin{aligned} L(\mathbf{F}_1, \lambda_1) &= J(\mathbf{F}_1) + \lambda_1\big(E\big[||\mathbf{x}_1||_2^2\big] - \mathrm{P}_1\big) \\ &= tr\big(\sigma_s^2\bar{\mathbf{h}}_y\mathbf{F}_1\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\mathbf{F}_1^H\bar{\mathbf{h}}_y^H + \sigma_s^2\sigma^2_{\phi_s}\sigma^2_{\phi_y}\mathbf{F}_1\mathbf{F}_1^H \\ &\quad + \sigma_s^2\sigma^2_{\phi_y}\mathbf{F}_1\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\mathbf{F}_1^H + \sigma_s^2\sigma^2_{\phi_s}\bar{\mathbf{h}}_y\mathbf{F}_1\mathbf{F}_1^H\bar{\mathbf{h}}_y^H \end{aligned}$$

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$$
\begin{aligned}
&-\sigma_s^2\bar{\mathbf{h}}_s^H\mathbf{F}_1^H\bar{\mathbf{h}}_y^H+\sigma_{v_s}^2\bar{\mathbf{h}}_y\mathbf{F}_1\mathbf{F}_1^H\bar{\mathbf{h}}_y^H\\
&+\sigma_{v_s}^2\sigma_{\phi_y}^2\mathbf{F}_1\mathbf{F}_1^H-\sigma_s^2\bar{\mathbf{h}}_y\mathbf{F}_1\bar{\mathbf{h}}_s\big)+\sigma_s^2\\
&+\lambda_1\big(\sigma_s^2||\mathbf{F}_1\bar{\mathbf{h}}_s||_2^2+\big(\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2\big)||\mathbf{F}_1||_F^2-\mathrm{P}_1\big)
\end{aligned}
\tag{14}
$$

where λ_1 is a Lagrangian multiplier. Take the derivative of (14) in terms of complex conjugate of $\mathbf{F}_1$, i.e., $\mathbf{F}_1^*$, and λ_1, respectively, using the properties of the derivative matrices [13], [14]. Then,

$$
\begin{aligned}
\frac{\partial L(\mathbf{F}_1,\lambda_1)}{\partial\mathbf{F}_1^*}&=\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_y\mathbf{F}_1\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\sigma_s^2+\mathbf{F}_1\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\sigma_s^2\sigma_{\phi_y}^2+\mathbf{F}_1\sigma_s^2\sigma_{\phi_y}^2\\
&+\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_y\mathbf{F}_1\sigma_{v_s}^2-\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_s^H\sigma_s^2+\mathbf{F}_1\sigma_s^2\sigma_{\phi_s}^2\sigma_{\phi_y}^2\\
&+\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_y\mathbf{F}_1\sigma_s^2\sigma_{\phi_s}^2+\lambda_1\mathbf{F}_1\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\sigma_s^2\\
&+\lambda_1\mathbf{F}_1\sigma_s^2\sigma_{\phi_s}^2+\lambda_1\mathbf{F}_1\sigma_{v_s}^2=\mathbf{0}
\end{aligned}
\tag{15}
$$

and

$$
\frac{\partial L(\mathbf{F}_1,\lambda_1)}{\partial\lambda_1}=\sigma_s^2||\mathbf{F}_1\bar{\mathbf{h}}_s||_2^2+\big(\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2\big)||\mathbf{F}_1||_F^2-\mathrm{P}_1=0.
\tag{16}
$$

Therefore, the optimal amplifying relay matrix $\mathbf{F}_1^\dagger$ can be written as

$$
\begin{aligned}
\mathbf{F}_1^\dagger=\sigma_s^2\big(\mathbf{h}_y^H\mathbf{h}_y+\sigma_{\phi_y}^2\mathbf{I}_N+\lambda_1\mathbf{I}_N\big)^{-1}\big(\mathbf{h}_y^H\bar{\mathbf{h}}_s^H\big)\times\\
\big(\bar{\mathbf{h}}_s\bar{\mathbf{h}}_s^H\sigma_s^2+\sigma_s^2\sigma_{\phi_s}^2\mathbf{I}_N+\sigma_{v_s}^2\mathbf{I}_N\big)^{-1}.
\end{aligned}
\tag{17}
$$

After applying the matrix inversion lemma [15] $(\mathbf{A}+\mathbf{BTD})^{-1}=\mathbf{A}^{-1}-\mathbf{A}^{-1}\mathbf{B}\big(\mathbf{T}^{-1}+\mathbf{DA}^{-1}\mathbf{B}\big)^{-1}\mathbf{DA}^{-1}$, the optimal amplifying relay matrix $\mathbf{F}_1^\dagger$ can be written as

$$
\mathbf{F}_1^\dagger=\frac{\sigma_s^2\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_s^H}{\Big(\lambda_1+\sigma_{\phi_y}^2+\big|\big|\bar{\mathbf{h}}_y\big|\big|_2^2\Big)\Big(\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2+\sigma_s^2\big|\big|\bar{\mathbf{h}}_s\big|\big|_2^2\Big)}.
\tag{18}
$$

Substituting (18) into (13), the total power P_1 at the relay nodes can be written as

$$
\mathrm{P}_1=\frac{\sigma_s^6||\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_s^H\bar{\mathbf{h}}_s|_2^2+\sigma_s^4\big(\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2\big)\big|\big|\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_s^H\big|\big|_F^2}{\Big[\Big(\lambda_1+\sigma_{\phi_y}^2+||\bar{\mathbf{h}}_y||_2^2\Big)\Big(\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2+\sigma_s^2||\bar{\mathbf{h}}_s||_2^2\Big)\Big]^2}.
\tag{19}
$$

Using (19), the Lagrangian multiplier λ_1 can be written as

$$
\begin{aligned}
\lambda_1=&\frac{\pm\sigma_s^2\sqrt{\mathrm{P}_1^{-1}\big(\sigma_s^2\big|\big|\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_s^H\bar{\mathbf{h}}_s\big|\big|_2^2+\big(\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2\big)\big|\big|\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_s^H\big|\big|_F^2\big)}}{\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2+\sigma_s^2\big|\big|\bar{\mathbf{h}}_s\big|\big|_2^2}\\
&-\Big(\big|\big|\bar{\mathbf{h}}_y\big|\big|_2^2+\sigma_{\phi_y}^2\Big).
\end{aligned}
\tag{20}
$$

The same BER performance is accomplished regardless of the sign of λ_1. Hence, only the positive sign of λ_1 is considered, i.e., $\bar{\lambda_1}$, where $\bar{\omega}=max(0,\ \omega)$. Substituting (20) into (18), the optimum relay amplifying matrix $\mathbf{F}_1^\dagger$ can be written as

$$
\mathbf{F}_1^\dagger=\frac{\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_s^H\sqrt{\mathrm{P}_1}}{\sqrt{\sigma_s^2||\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_s^H\bar{\mathbf{h}}_s||_2^2+\big(\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2\big)||\bar{\mathbf{h}}_y^H\bar{\mathbf{h}}_s^H||_F^2}}.
\tag{21}
$$

Note that $\mathbf{F}_1^\dagger$ does not depend on $\sigma_{\phi_y}^2$, but $\bar{\mathbf{h}}_y$ does. The MMSE can be computed from $J(\mathbf{F}_1)$ in (14) by replacing $\mathbf{F}_1$ with $\mathbf{F}_1^\dagger$.

From (14) and (17), the constrained optimal solutions $\bar{\lambda}_i$ and $\mathbf{F}_i^\dagger,\ i=2,\ 3$, of other two special cases under channel uncertainty and power constraints at the relay nodes are derived, respectively, as

$$
\begin{aligned}
\bar{\lambda}_2=&\frac{\sigma_s^2\sqrt{\mathrm{P}_2^{-1}\big(\sigma_s^2\big|\big|\mathbf{h}_y^H\bar{\mathbf{h}}_s^H\bar{\mathbf{h}}_s\big|\big|_2^2+\big(\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2\big)\big|\big|\mathbf{h}_y^H\bar{\mathbf{h}}_s^H\big|\big|_F^2\big)}}{\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2+\sigma_s^2\big|\big|\bar{\mathbf{h}}_s\big|\big|_2^2}\\
&-\big|\big|\mathbf{h}_y\big|\big|_2^2
\end{aligned}
\tag{22}
$$

$$
\mathbf{F}_2^\dagger=\frac{\mathbf{h}_y^H\bar{\mathbf{h}}_s^H\sqrt{\mathrm{P}_2}}{\sqrt{\sigma_s^2\big\|\mathbf{h}_y^H\bar{\mathbf{h}}_s^H\bar{\mathbf{h}}_s\big\|_2^2+\big(\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2\big)\big\|\mathbf{h}_y^H\bar{\mathbf{h}}_s^H\big\|_F^2}}
\tag{23}
$$

for only source-relay links when $\sigma_{\phi_y}^2=0$, and

$$
\begin{aligned}
\bar{\lambda}_3=&\frac{\sigma_s^2\sqrt{\mathrm{P}_3^{-1}\big(\sigma_s^2\big|\big|\bar{\mathbf{h}}_y^H\mathbf{h}_s^H\mathbf{h}_s\big|\big|_2^2+\sigma_{v_s}^2\big|\big|\bar{\mathbf{h}}_y^H\mathbf{h}_s^H\big|\big|_F^2\big)}}{\sigma_{v_s}^2+\sigma_s^2\big|\big|\mathbf{h}_s\big|\big|_2^2}\\
&-\big(\big|\big|\bar{\mathbf{h}}_y\big|\big|_2^2+\sigma_{\phi_y}^2\big)
\end{aligned}
\tag{24}
$$

$$
\mathbf{F}_3^\dagger=\frac{\bar{\mathbf{h}}_y^H\mathbf{h}_s^H\sqrt{\mathrm{P}_3}}{\sqrt{\sigma_s^2||\bar{\mathbf{h}}_y^H\mathbf{h}_s^H\mathbf{h}_s||_2^2+\sigma_{v_s}^2||\bar{\mathbf{h}}_y^H\mathbf{h}_s^H||_F^2}}
\tag{25}
$$

for only relay-destination links when $\sigma_{\phi_s}^2=0$.

For performance comparisons, the diagonal relay amplifying matrices under total power constraint at relay nodes are derived in the rest of this section. Hence, the diagonal relay amplifying matrix $\mathbf{F}_4^\dagger$ corresponding to both imperfect channel estimation vectors $\bar{\mathbf{h}}_s^H$ and $\bar{\mathbf{h}}_y^H$ in noncooperative wireless networks can be written as

$$
\mathbf{F}_4^\dagger=\mathrm{diag}\big(f_1,\ f_2,\ \cdots,\ f_N\big),\ i=1,\ \cdots,\ N
\tag{26}
$$

where

$$
f_i=\frac{\bar{h}_{y,i}^*\bar{h}_{s,i}^*\sqrt{\mathrm{P}_4}}{\sqrt{\sum_{i=1}^N|\bar{h}_{y,i}|^2|\bar{h}_{s,i}|^2\big(\sigma_s^2|\bar{h}_{s,i}|^2+\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2\big)}}.
\tag{27}
$$

Similarly, the diagonal relay amplifying matrices $\mathbf{F}_5^\dagger$ and $\mathbf{F}_6^\dagger$ corresponding to only imperfect channel estimation column vector $\bar{\mathbf{h}}_s^H$ and only imperfect channel estimation row vector $\bar{\mathbf{h}}_y^H$ in noncooperative wireless networks can be written, respectively, as

$$
\mathbf{F}_5^\dagger=\mathrm{diag}\big(f_1,\ f_2,\ \cdots,\ f_N\big),\ i=1,\ \cdots,\ N
\tag{28}
$$

where

$$
f_i=\frac{h_{y,i}^*\bar{h}_{s,i}^*\sqrt{\mathrm{P}_5}}{\sqrt{\sum_{i=1}^N|h_{y,i}|^2|\bar{h}_{s,i}|^2(\sigma_s^2|\bar{h}_{s,i}|^2+\sigma_s^2\sigma_{\phi_s}^2+\sigma_{v_s}^2)}}
\tag{29}
$$

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and

$$\mathbf{F}_6^{\dagger} = \mathrm{diag}\big(f_1,\ f_2,\ \cdots,\ f_{N}\big),\ i = 1,\ \cdots,\ N \qquad (30)$$

where

$$f_i = \frac{\bar{h}_{y,i}^{*} h_{s,i}^{*} \sqrt{\mathrm{P}_6}}{\sqrt{\sum_{i=1}^{N} |\bar{h}_{y,i}|^2 |h_{s,i}|^2 (\sigma_s^2 |h_{s,i}|^2 + \sigma_{v_s}^2)}}. \qquad (31)$$

IV. Simulation Results

The Monte-Carlo simulation results is performed to evaluate BER performance of the cooperative or noncooperative distributed MMSE relay scheme with imperfect channel information. All simulations are performed for one-source-one-destination pair $N = 2$ cooperative or noncooperative distributed relay nodes. It is assumed that all relay nodes are located at equidistance between the source node and the destination node. The perfect channel coefficient vectors $\mathbf{h}_s$ and $\mathbf{h}_y$ are generated from independent Gaussian random variables with zero mean and unity variance. All nodes with only one antenna have the same noise power, i.e., $\sigma_{v_s}^2 = \sigma_{v_y}^2$. It is also assumed that the transmitted signal at the source node is modulated by 4-ary quadrature amplitude modulation (4QAM). Perfect/imperfect channel coefficients with unity power are used. And, the total power constraints are set to $\mathrm{P}_1 = \mathrm{P}_2 = \mathrm{P}_3 = \mathrm{P}_4 = \mathrm{P}_5 = \mathrm{P}_6 = 1$.

For three different channel uncertainty conditions, i.e., 5%, 10%, and 20% of channel uncertainty are, respectively, modeled as additive white Gaussian noise and added to the perfect channel coefficient values, i.e., the variances of the channel estimation errors are chosen to satisfy $10\log_{10}(\sigma_{\phi_s}^2/\sigma_{h_s}^2) = 10\log_{10}(\sigma_{\phi_y}^2/\sigma_{h_y}^2) = -13$ dB, -10 dB, and -7 dB whenever the links are imperfect. For comparison, the simulation results with perfect channel information are also included, i.e., $\sigma_{\phi_s}^2 = \sigma_{\phi_y}^2 = 0$.

TABLE I
RELAY STRATEGIES AND CHANNEL UNCERTAINTY CONDITIONS IN FIG. 3 THROUGH FIG. 6 WITH N=2 RELAY NODES

Fig. No.	Cooperative/noncooperative	Channel uncertainty
Fig. 3	cooperative	$(\bar{\mathbf{h}}_s, \bar{\mathbf{h}}_y)$
Fig. 4	cooperative	$(\bar{\mathbf{h}}_s, \mathbf{h}_y)$ or $(\mathbf{h}_s, \bar{\mathbf{h}}_y)$
Fig. 5	noncooperative	$(\bar{\mathbf{h}}_s, \bar{\mathbf{h}}_y)$
Fig. 6	noncooperative	$(\bar{\mathbf{h}}_s, \mathbf{h}_y)$ or $(\mathbf{h}_s, \bar{\mathbf{h}}_y)$

Figures 3 through 6 show the BER versus SNR for the cases listed in TABLE 1, where SNR$\triangleq \sigma_s^2\sigma_h^2/\sigma_v^2 = \sigma_s^2/\sigma_v^2$ represents the signal-to-noise power ratio. For example, Fig. 3 shows BER versus SNR for $N = 2$ cooperative distributed relay networks under a $(\bar{\mathbf{h}}_s, \bar{\mathbf{h}}_y)$ environment, i.e., both imperfect links $\bar{\mathbf{h}}_s$ and $\bar{\mathbf{h}}_y$. Also, both perfect channel case $(\mathbf{h}_s, \mathbf{h}_y)$ is included for comparisons. Due to the effect of imperfect channel information on both source-relay and relay-destination links in Fig. 3, the worst BER is observed, compared to the other cases in Figs. 4-6. In particular, it is observed that increasing the variances of the channel estimation errors, worse BER can be observed.

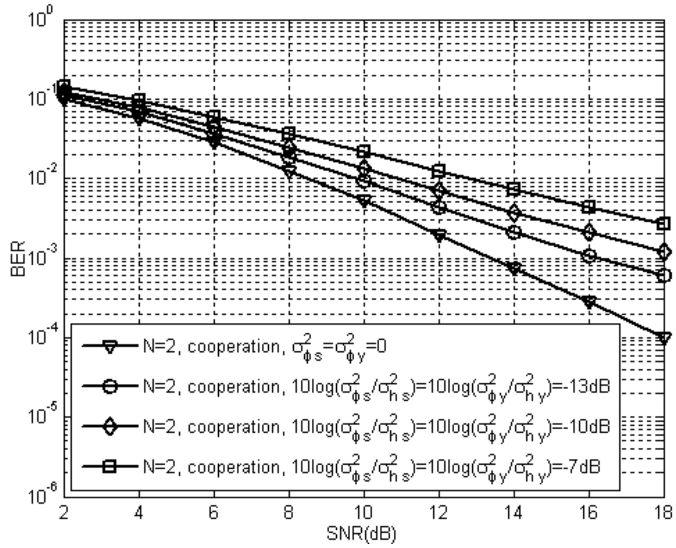

Fig. 3. BER performance of $N = 2$ cooperative distributed relay networks under certain/uncertain channel conditions using both imperfect channel vectors $(\bar{\mathbf{h}}_s, \bar{\mathbf{h}}_y)$.

It is observed from Fig. 5 that the case $(\mathbf{h}_s, \bar{\mathbf{h}}_y)$ shows a better performance than that of the $(\bar{\mathbf{h}}_s, \mathbf{h}_y)$ case because the

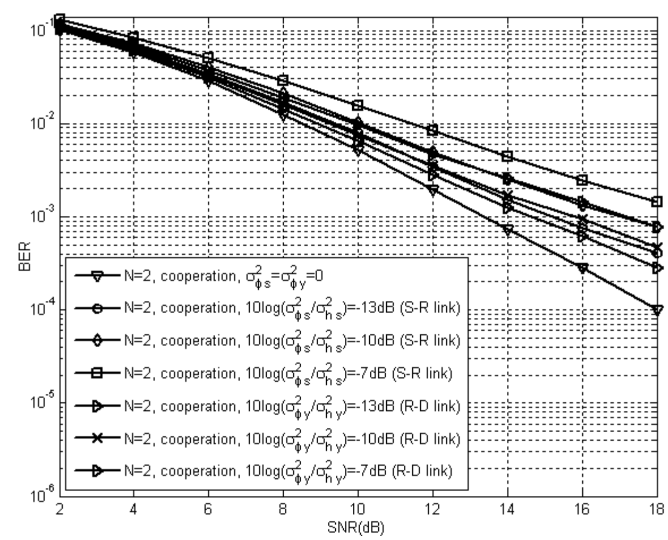

Fig. 4. BER performance of $N = 2$ cooperative distributed relay networks under the cases of $(\bar{\mathbf{h}}_s, \mathbf{h}_y)$ or $(\mathbf{h}_s, \bar{\mathbf{h}}_y)$.

diagonal relay amplifying matrix $\mathbf{F}_3^{\dagger}$ in (24) is independent of the variance of the channel estimation error ϕ_y.

As observed from Fig. 3 and Fig. 5, the cooperative distributed relay scheme provides a better BER performance than the noncooperative one, e.g., 2 dB at BER = 10^{-3}. As with the case of the cooperative channel uncertainty, BER of the noncooperative relay under the $(\mathbf{h}_s, \bar{\mathbf{h}}_y)$ case shows a

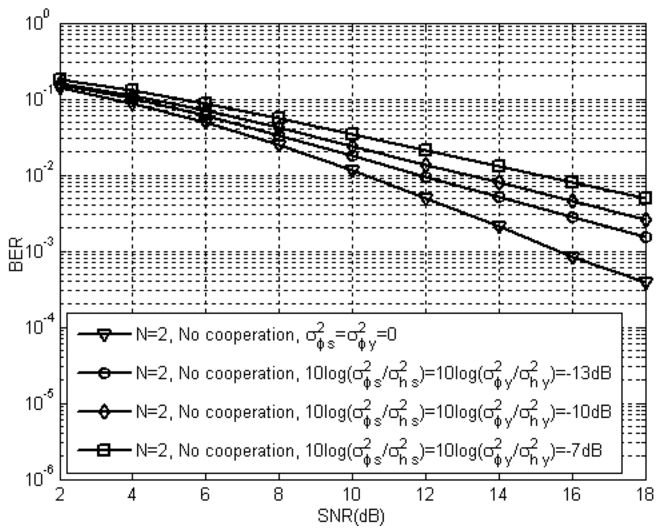

Fig. 5. BER performance of $N = 2$ noncooperative distributed relay networks under certain/uncertain channel conditions using both imperfect channel vectors $(\bar{\mathbf{h}}_s, \bar{\mathbf{h}}_y)$.

better performance than that of the $(\bar{\mathbf{h}}_s, \mathbf{h}_y)$ case because the diagonal relay amplifying matrix $\mathbf{F}_6^\dagger$ in (31) is independent of the variance of the channel estimation error ϕ_y. Finally, it

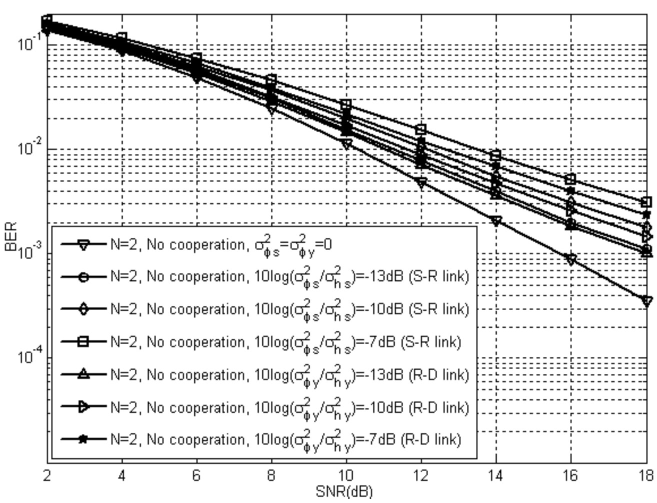

Fig. 6. BER performance of $N = 2$ noncooperative distributed relay networks under the cases of $(\bar{\mathbf{h}}_s, \mathbf{h}_y)$ or $(\mathbf{h}_s, \bar{\mathbf{h}}_y)$.

is observed in general from Figs. 4 and 6 that the effect of the noncooperative channel uncertainty is larger than that of the cooperative one. And if both links are uncertain, then the performance is worse than the other cases.

V. Conclusion

This paper considered distributed MMSE AF relay schemes in wireless relay networks under channel uncertainty and power constraints at relay nodes. It used one-source-one-destination pair and N-relay cooperative as well as noncooperative relay networks. Under either certain or uncertain channel condition, BER performance of cooperative distributed relay network is better than that of noncooperative case. Under channel uncertainty, it has been observed that increasing variances of the channel estimation error results in loss of diversity order. In addition, BER corresponding to the channel uncertainty before relays shows a worse performance than that of the channel uncertainty after relays.

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