The Joint Probability Density Function of the SSC Combiner Output Signal at Two Time Instants in the Presence of Hoyt Fading

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***Abstract* — In this paper, the probability density function of the Switch and Stay Combiner (SSC) output signal to noise ratio (SNR) at one time instant and the joint probability density function of the SSC combiner output signal to noise ratio at two time instants, in the presence of Hoyt fading, are determined in the closed form expressions. The results are shown graphically in several figures and the evaluation of the various parameters influence, such as distribution parameters and decision threshold values, is given.**

Keywords - Probability Density Function; Joint Probability Density Function; Hoyt Fading; Diversity Reception; SSC Combining.

I. INTRODUCTION

The radio wave propagation through wireless communications channels has received a great deal of research interest [1], [2]. The rapid and random fluctuations of the signal envelope and phase in a radio channel are caused with two propagation phenomena: multipath scattering (fast fading) and shadowing (slow fading). In wireless communications the multipath fading is modeled by several distributions such as: Rayleigh, Rice, Nakagami-*m*, Weibull and so on.

Another distribution, which has recently received increased attention in modeling fading channels, is the Hoyt (Nakagami-*q*) distribution. The Hoyt fading model provides a very accurate fit to experimental channel measurements in a various communication applications, like mobile satellite propagation channels [3], and spans the range of the fading figure from the one-sided Gaussian to the Rayleigh distribution [4]. Similarly, the Hoyt distribution can be considered as an accurate fading model for satellite links with strong ionospheric scintillation [5]. Recently, in [6], an ergodic capacity analysis is presented, and in [7] the information outage probability of OSTBC over Hoyt fading channels has been studied. Also in [8] this model has been used in outage analysis of cellular mobile radio systems, while in [9] a capacity analysis of Hoyt fading is provided.

In wireless communication systems, various techniques for reducing fading effect and influence of shadow effect are used. Such techniques are diversity reception, dynamic channel allocation and power control. Upgrading transmission reliability and increasing channel capacity without increasing transmission power and bandwidth is the main goal of diversity techniques.

Diversity reception, based on using multiple antennas at the receiver, space diversity, with two or more branches, is a very efficient method used for improving system's quality of service, so it provides efficient solution for reduction of signal level fluctuations in fading channels. Multiple received copies of signal could be combined on various ways. Among the most popular diversity techniques are: maximal ratio combining (MRC), equal gain combining (EGC), and generalized selection combining (GSC) [1], but their complexity of implementation is relatively high since they require a dedicated communication receiver for each diversity branch. On the other hand, among the simpler diversity combining schemes, the two most popular are selection combining (SC) and switch and stay combining (SSC). Selection combining (SC) and switch and stay combining (SSC) types of diversity systems process only one of the diversity branches, so they are less complicated.

Switch and stay combining (SSC) is an attempt at simplifying the complexity of the system but with loss in performance. In this case, the receiver selects a particular antenna until its quality drops below a predetermined threshold. When this happens, the receiver switches to another antenna and stays with it for the next time slot, regardless of whether or not the channel quality of that antenna is above or below the predetermined threshold. The consideration of SSC systems in the literature has been

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restricted to low-complexity mobile units where the number of diversity antennas is typically limited to two [10-11]. Furthermore, in all these publications, only predetection SSC has thus far been considered wherein the switching of the receiver between the two receiving antennas is based on a comparison of the instantaneous SNR of the connected antenna with a predetermined threshold. This results in a reduction in complexity relative to SC in that the simultaneous and continuous monitoring of both branches SNRs is no longer necessary.

The probability density function (PDF) of the SSC combiner output signal at one time instant and the joint probability density function of the SSC combiner output signal at two time instants in the presence of Rayleigh, Nakagami-m, Weibull and log-normal fading are determined in [12-15], respectively.

In this paper the probability density function of the SSC combiner output signal to noise ratio at one time instant and the joint probability density function of the SSC combiner output signal to noise ratio at two time instants in the presence of Hoyt fading will be determined. The joint probability density function of the SSC combiner output signal to noise ratio at two time instants is important when the decision is based on multiple samples.

The remainder of the document is organized in the following way: Section II introduces the model of the SSC combiner is given and the probability density function of the SSC combiner output signal to noise ratio at one time instant is determined. Subsequently, in Section III, the joint probability density function of the SSC combiner output signal to noise ratio at two time instants is calculated. In fourth section the numerical results are presented.

II. System Model

The use of SSC combiner with great number of branches can minimize the bit error rate (BER) [16]. We determine SSC combiner with two inputs because the gain is the greatest when we use the SSC combiner with two inputs instead of one-channel system. When we enlarge the number of branches the improvement becomes less [16]. The ratio price/complexity is the best for a system with two branches. Because of that it is more economic using SSC combiner with two inputs.

The model of this system is shown in Fig. 1. The signal to noise ratios at the combiner inputs are γ_1 and γ_2, and γ is the combiner output signal to noise ratio.

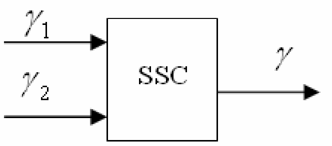

Figure 1. Model of the SSC combiner with two inputs

Let see how the SSC combiner with two inputs works. The probability of the event that the combiner first examines the signal at the first input is P_1, and for the second input is P_2. If the combiner examines first the signal at the first input and if the value of the signal to noise ratio at the first input is above the treshold, γ_T, SSC combiner forwards this signal to the circuit for the decision. If the value of the signal to noise ratio at the first input is below the treshold γ_T, SSC combiner forwards the signal from the other input to the circuit for the decision, regardless if it is above or below the predetermined threshold. If the SSC combiner first examines the signal from the second combiner input it works in the similar way.

The expression for the probability density of the combiner output signal to noise ratio will be determined first for the case $\gamma < \gamma_T$. Based on the work algorithm of the SSC combiner in this case, the probability density is equal, for $\gamma < \gamma_T$:

$$p_\gamma(\gamma) = P_1 \cdot F_{\gamma_1}(\gamma_T) \cdot p_{\gamma_2}(\gamma) + P_2 \cdot F_{\gamma_2}(\gamma_T) \cdot p_{\gamma_1}(\gamma) \quad (1)$$

In the case $\gamma \geq \gamma_T$ the expression for the probability density of the signal to noise ratio at the combiner output is :

$$p_\gamma(\gamma) = P_1 \cdot p_{\gamma_1}(\gamma) + P_1 \cdot F_{\gamma_1}(\gamma_T) \cdot p_{\gamma_2}(\gamma) + \\ + P_2 \cdot p_{\gamma_2}(\gamma) + P_2 \cdot F_{\gamma_2}(\gamma_T) \cdot p_{\gamma_1}(\gamma) \quad (2)$$

where γ_T is the treshold of the decision, and the cumulative probability densities (CDFs) are given by [17]:

$$F_{\gamma_i}(\gamma_T) = \int_0^{\gamma_T} p_{\gamma_i}(x)dx, \qquad i = 1,2 \quad (3)$$

The probabilities P_1 and P_2 are [17]:

$$P_1 = \frac{F_{\gamma_2}(\gamma_T)}{F_{\gamma_1}(\gamma_T) + F_{\gamma_2}(\gamma_T)} \quad (4)$$

$$P_2 = \frac{F_{\gamma_1}(\gamma_T)}{F_{\gamma_1}(\gamma_T) + F_{\gamma_2}(\gamma_T)} \quad (5)$$

The probability densities of the SNRs at the combiner input, γ_1 and γ_2, in the presence of Hoyt fading, are [17]:

$$p_{\gamma_1}(\gamma_1) = \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left(-\frac{(1+q_1^2)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)$$

$$\gamma_1 \geq 0 \quad (6)$$

$$p_{\gamma_2}(\gamma_2) = \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left(-\frac{(1+q_2^2)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right) I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)$$

$$\gamma_2 \geq 0 \quad (7)$$

where q_i are Nakagami-q fading parameters, which range from 0 to 1 and $\bar{\gamma}_i$ are average SNRs for input channels

After putting of the expressions (4)-(7), (10) and (11) into (1), the probability density of the signal to noise ratio at the combiner output γ, is, for $\gamma < \gamma_T$:

$$p_\gamma(\gamma) = P_1 \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{\left(1+q_1^2\right)^2 \gamma_T}{4q_1^2\overline{\gamma}_1}\right)\cdot$$
$$\cdot\frac{\left(1+q_2^2\right)}{2q_2\overline{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_2}{4q_2^2\overline{\gamma}_2}\right) I_0\left(\frac{\left(1-q_2^4\right)\gamma_2}{4q_2^2\overline{\gamma}_2}\right)+$$
$$+P_2 \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{\left(1+q_2^2\right)^2 \gamma_T}{4q_2^2\overline{\gamma}_2}\right)\cdot$$
$$\cdot\frac{\left(1+q_1^2\right)}{2q_1\overline{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_1}{4q_1^2\overline{\gamma}_1}\right) I_0\left(\frac{\left(1-q_1^4\right)\gamma_1}{4q_1^2\overline{\gamma}_1}\right) \quad (8)$$

After putting of the expressions (4)-(7), (10) and (11) into (2), the probability density of the signal to noise ratio at the combiner output γ, is, for $\gamma \geq \gamma_T$:

$$p_\gamma(\gamma) = P_1 \frac{\left(1+q_1^2\right)}{2q_1\overline{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_1}{4q_1^2\overline{\gamma}_1}\right) I_0\left(\frac{\left(1-q_1^4\right)\gamma_1}{4q_1^2\overline{\gamma}_1}\right)+$$
$$+P_1 \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{\left(1+q_1^2\right)^2 \gamma_T}{4q_1^2\overline{\gamma}_1}\right)\cdot$$
$$\cdot\frac{\left(1+q_2^2\right)}{2q_2\overline{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_2}{4q_2^2\overline{\gamma}_2}\right) I_0\left(\frac{\left(1-q_2^4\right)\gamma_2}{4q_2^2\overline{\gamma}_2}\right)+$$
$$+P_2 \frac{\left(1+q_2^2\right)}{2q_2\overline{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_2}{4q_2^2\overline{\gamma}_2}\right) I_0\left(\frac{\left(1-q_2^4\right)\gamma_2}{4q_2^2\overline{\gamma}_2}\right)+$$
$$+P_2 \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{\left(1+q_2^2\right)^2 \gamma_T}{4q_2^2\overline{\gamma}_2}\right)\cdot$$
$$\cdot\frac{\left(1+q_1^2\right)}{2q_1\overline{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_1}{4q_1^2\overline{\gamma}_1}\right) I_0\left(\frac{\left(1-q_1^4\right)\gamma_1}{4q_1^2\overline{\gamma}_1}\right) \quad (9)$$

The cumulative probability densities (CDFs) of the SNRs at the combiner input in the presence of Hoyt fading, after putting of the expressions (6), (7), into (3), are given by:

$$F_{r_1}(\gamma_T) = \int_0^{\gamma_T} \frac{\left(1+q_1^2\right)}{2q_1\overline{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2 x}{4q_1^2\overline{\gamma}_1}\right) I_0\left(\frac{\left(1-q_1^4\right)x}{4q_1^2\overline{\gamma}_1}\right)dx =$$
$$= \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{\left(1+q_1^2\right)^2 \gamma_T}{4q_1^2\overline{\gamma}_1}\right) \quad (10)$$

$$F_{r_2}(\gamma_T) = \int_0^{\gamma_T} \frac{\left(1+q_2^2\right)}{2q_2\overline{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2 x}{4q_2^2\overline{\gamma}_2}\right) I_0\left(\frac{\left(1-q_2^4\right)x}{4q_2^2\overline{\gamma}_2}\right)dx =$$
$$= \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{\left(1+q_2^2\right)^2 \gamma_T}{4q_2^2\overline{\gamma}_2}\right) \quad (11)$$

where $I_e(k, x)$ is Rice's I_e function [18].

After putting of the expressions (10) and (11) into (4) and (5), the probabilities P_1 and P_2 are:

$$P_1 = \frac{\frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{\left(1+q_2^2\right)^2 \gamma_T}{4q_2^2\overline{\gamma}_2}\right)}{\frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{\left(1+q_1^2\right)^2 \gamma_T}{4q_1^2\overline{\gamma}_1}\right)+\frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{\left(1+q_2^2\right)^2 \gamma_T}{4q_2^2\overline{\gamma}_2}\right)} \quad (12)$$

$$P_2 = \frac{\frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{\left(1+q_1^2\right)^2 \gamma_T}{4q_1^2\overline{\gamma}_1}\right)}{\frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{\left(1+q_1^2\right)^2 \gamma_T}{4q_1^2\overline{\gamma}_1}\right)+\frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{\left(1+q_2^2\right)^2 \gamma_T}{4q_2^2\overline{\gamma}_2}\right)} \quad (13)$$

The obtained expressions for the probability density function (PDF) of the output signal to noise ratio after diversity combining can be used to study the moments, the amount of fading, the outage probability and the average bit error rate of proposed system.

III. SYSTEM PERFORMANCES AT TWO TIME INSTANTS

The model of the SSC combiner with two inputs at two time instants considering in this section is shown in Fig. 2. The signal to noise ratios at the inputs are γ_{11} and γ_{21} at the first time moment and they are γ_{12} and γ_{22} at the second time moment.

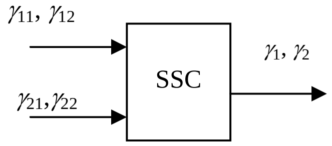

Figure 2. Model of the SSC combiner with two inputs at two time instants

The output signal to noise ratios are γ_1 and γ_2. The indexes for the input signal to noise ratios are: first index is the number of the branch and the other signs time instant observed. For the output signal to noise ratios, the index represents the time instant observed.

The joint probability density function of uncorrelated signals at the input, with Hoyt distribution and same parameters, is [17]:

$$p_{\gamma_1\gamma_2}(\gamma_1,\gamma_2)=\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_2}{4q_1^2\bar{\gamma}_1}\right) \tag{14}$$

Modified Bessel function of the first kind is defined by [19]:

$$I_m(x)=\left(\frac{x}{2}\right)^m\sum_{k=0}^{\infty}\frac{\left(\frac{x^2}{4}\right)^k}{k!\,\Gamma(m+k+1)} \tag{15}$$

Now we have four different cases. The first case is: $\gamma_1<\gamma_T$ and $\gamma_2<\gamma_T$. In this case all signal to noise ratios at the input are below γ_T, i.e.,: $\gamma_{11}<\gamma_T$, $\gamma_{12}<\gamma_T$, $\gamma_{21}<\gamma_T$, and $\gamma_{22}<\gamma_T$.

Let the combiner first examines the signal r_{11}. Because $\gamma_{11}<\gamma_T$, it follows that $\gamma_1=\gamma_{21}$, and since $\gamma_{22}<\gamma_T$ it is $\gamma_2=\gamma_{12}$. The probability of this event is P_1.

When SSC combiner first examines the signal r_{21}, then $\gamma_1=\gamma_{11}$, because $\gamma_{21}<\gamma_T$. Since $\gamma_{12}<\gamma_T$, then it is $\gamma_2=\gamma_{22}$. The probability of this event is P_2. After previous, the joint probability density of the combiner output signal to noise ratios at two time instants, γ_1 and γ_2, is, by using expression (14), for $\gamma_1<\gamma_T$ and $\gamma_2<\gamma_T$:

$$p_{\gamma_1\gamma_2}(\gamma_1,\gamma_2)=P_1\cdot\int_0^{\gamma_T}p_{\gamma_{11}\gamma_{12}}(\gamma_{11},\gamma_2)d\gamma_{11}\int_0^{\gamma_T}p_{\gamma_{22}\gamma_{21}}(\gamma_{22},\gamma_1)d\gamma_{22}+$$

$$+P_2\cdot\int_0^{\gamma_T}p_{\gamma_{21}\gamma_{22}}(\gamma_{21},\gamma_2)d\gamma_{21}\int_0^{\gamma_T}p_{\gamma_{12}\gamma_{11}}(\gamma_{12},\gamma_1)d\gamma_{12}=$$

$$=P_1\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_2}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{\left(1+q_1^2\right)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{\left(1+q_2^2\right)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_1}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{\left(1-q_2^4\right)\gamma_1}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{\left(1+q_2^2\right)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)+$$

$$+P_2\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{\left(1+q_1^2\right)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{\left(1+q_2^2\right)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{\left(1-q_2^4\right)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{\left(1+q_2^2\right)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right) \tag{16}$$

In the similar way we can derive the other joint probability density functions. The joint PDF is, for $\gamma_1\geq\gamma_T$ and $\gamma_2<\gamma_T$:

$$p_{\gamma_1\gamma_2}(\gamma_1,\gamma_2)=P_1\cdot p_{\gamma_{22}}(\gamma_2)\int_0^{\gamma_T}p_{\gamma_{12}\gamma_{11}}(\gamma_{12},\gamma_1)d\gamma_{12}+$$

$$+P_1\cdot\int_0^{\gamma_T}p_{\gamma_{11}\gamma_{12}}(\gamma_{11},\gamma_2)d\gamma_{11}\int_0^{\gamma_T}p_{\gamma_{22}\gamma_{21}}(\gamma_{22},\gamma_1)d\gamma_{22}+$$

$$+P_2\cdot p_{\gamma_{12}}(\gamma_2)\int_0^{\gamma_T}p_{\gamma_{22}\gamma_{21}}(\gamma_{22},\gamma_1)d\gamma_{22}+P_2\cdot\int_0^{\gamma_T}p_{\gamma_{21}\gamma_{22}}(\gamma_{21},\gamma_2)d\gamma_{21}\int_0^{\gamma_T}p_{\gamma_{12}\gamma_{11}}(\gamma_{12},\gamma_1)d\gamma_{12}=$$

$$=P_1\frac{\left(1+q_2^2\right)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{\left(1-q_2^4\right)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)\cdot$$

$$\cdot\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{\left(1+q_1^2\right)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)+$$

$$+P_2\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_2}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{\left(1+q_2^2\right)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_1}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{\left(1-q_2^4\right)\gamma_1}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{\left(1+q_2^2\right)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)+$$

$$+P_1\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_2}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{\left(1+q_1^2\right)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{\left(1+q_2^2\right)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_1}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{\left(1-q_2^4\right)\gamma_1}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{\left(1+q_2^2\right)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)+$$

$$+P_2\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{\left(1+q_1^2\right)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{\left(1+q_2^2\right)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{\left(1-q_2^4\right)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{\left(1+q_2^2\right)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right) \tag{17}$$

for $\gamma_1<\gamma_T$ and $\gamma_2\geq\gamma_T$:

$$p_{\gamma_1\gamma_2}(\gamma_1,\gamma_2)=P_1\cdot\int_0^{\gamma_T}p_{\gamma_{11}}(\gamma_{11})d\gamma_{11}\cdot p_{\gamma_{21}\gamma_{22}}(\gamma_1,\gamma_2)+$$

$$+P_1\cdot\int_0^{\gamma_T}p_{\gamma_{11}\gamma_{12}}(\gamma_{11},\gamma_2)d\gamma_{11}\int_0^{\gamma_T}p_{\gamma_{22}\gamma_{21}}(\gamma_{22},\gamma_1)d\gamma_{22}+$$

$$+P_2\cdot\int_0^{\gamma_T}p_{\gamma_{21}}(\gamma_{21})d\gamma_{21}\cdot p_{\gamma_{11}\gamma_{12}}(\gamma_1,\gamma_2)+P_2\cdot\int_0^{\gamma_T}p_{\gamma_{21}\gamma_{22}}(\gamma_{21},\gamma_2)d\gamma_{21}\int_0^{\gamma_T}p_{\gamma_{12}\gamma_{11}}(\gamma_{12},\gamma_1)d\gamma_{12}=$$

$$=P_1\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{\left(1+q_1^2\right)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{\left(1+q_2^2\right)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_1}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{\left(1-q_2^4\right)\gamma_1}{4q_2^2\bar{\gamma}_2}\right)\frac{\left(1+q_2^2\right)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{\left(1+q_2^2\right)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{\left(1-q_2^4\right)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)+$$

$$+P_2\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{\left(1+q_2^2\right)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)\cdot$$

$$\cdot\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_2}{4q_1^2\bar{\gamma}_1}\right)+$$

$$+P_1\frac{\left(1+q_1^2\right)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{\left(1+q_1^2\right)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{\left(1-q_1^4\right)\gamma_2}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{\left(1+q_1^2\right)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{(1+q_2^2)^2\gamma_1}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_1}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)+$$

$$+P_2\frac{(1+q_1^2)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{(1+q_1^2)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{(1+q_2^2)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right) \tag{18}$$

for $\gamma_1 \geq \gamma_T$ and $\gamma_2 \geq \gamma_T$:

$$p_{\gamma_1\gamma_2}(\gamma_1,\gamma_2)=P_1\cdot p_{\gamma_{11}\gamma_{12}}(\gamma_1,\gamma_2)+P_1\cdot p_{\gamma_{22}}(\gamma_2)\int_0^{\gamma_T}p_{\gamma_{12}\gamma_{11}}(\gamma_{12},\gamma_1)d\gamma_{12}+$$

$$+P_1\cdot\int_0^{\gamma_T}p_{\gamma_{11}}(\gamma_{11})d\gamma_{11}\cdot p_{\gamma_{21}\gamma_{22}}(\gamma_1,\gamma_2)+P_1\cdot\int_0^{\gamma_T}p_{\gamma_{11}\gamma_{12}}(\gamma_{11},\gamma_2)d\gamma_{11}\int_0^{\gamma_T}p_{\gamma_{22}\gamma_{21}}(\gamma_{22},\gamma_1)d\gamma_{22}+$$

$$+P_2\cdot p_{\gamma_{21}\gamma_{22}}(\gamma_1,\gamma_2)+P_2\cdot p_{\gamma_{12}}(\gamma_2)\int_0^{r_T}p_{\gamma_{22}\gamma_{21}}(\gamma_{22},\gamma_1)d\gamma_{22}+$$

$$+P_2\cdot\int_0^{\gamma_T}p_{\gamma_{21}}(\gamma_{21})d\gamma_{21}\cdot p_{\gamma_{11}\gamma_{12}}(\gamma_1,\gamma_2)+P_2\cdot\int_0^{\gamma_T}p_{\gamma_{21}\gamma_{22}}(\gamma_{21},\gamma_2)d\gamma_{21}\int_0^{\gamma_T}p_{\gamma_{12}\gamma_{11}}(\gamma_{12},\gamma_1)d\gamma_{12}=$$

$$=P_1\frac{(1+q_1^2)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{(1+q_1^2)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\frac{(1+q_1^2)}{2q_1\bar{\gamma}_1}$$

$$\exp\left(-\frac{(1+q_1^2)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{(1-q_1^4)\gamma_2}{4q_1^2\bar{\gamma}_1}\right)+$$

$$+P_1\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{(1+q_2^2)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)\cdot$$

$$\cdot\frac{(1+q_1^2)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{(1+q_1^2)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)+$$

$$+P_1\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{(1+q_2^2)^2\gamma_1}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_1}{4q_2^2\bar{\gamma}_2}\right)\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}$$

$$\exp\left(-\frac{(1+q_2^2)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)+$$

$$+P_1\frac{(1+q_1^2)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{(1+q_1^2)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{(1-q_1^4)\gamma_2}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{(1+q_2^2)^2\gamma_1}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_1}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)+$$

$$+P_2\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{(1+q_2^2)^2\gamma_1}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_1}{4q_2^2\bar{\gamma}_2}\right)\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}$$

$$\exp\left(-\frac{(1+q_2^2)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)+$$

$$+P_2\frac{(1+q_1^2)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{(1+q_1^2)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{(1-q_1^4)\gamma_2}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{(1+q_2^2)^2\gamma_1}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_1}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)+$$

$$+P_2\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)\cdot$$

$$\cdot\frac{(1+q_1^2)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{(1+q_1^2)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\frac{(1+q_1^2)}{2q_1\bar{\gamma}_1}$$

$$\exp\left(-\frac{(1+q_1^2)^2\gamma_2}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{(1-q_1^4)\gamma_2}{4q_1^2\bar{\gamma}_1}\right)+$$

$$+P_2\frac{(1+q_1^2)}{2q_1\bar{\gamma}_1}\exp\left(-\frac{(1+q_1^2)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right)I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2\bar{\gamma}_1}\right)\frac{2q_1}{1+q_1^2}I_e\left(\frac{1-q_1^2}{1+q_1^2},\frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)\cdot$$

$$\cdot\frac{(1+q_2^2)}{2q_2\bar{\gamma}_2}\exp\left(-\frac{(1+q_2^2)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right)I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2\bar{\gamma}_2}\right)\frac{2q_2}{1+q_2^2}I_e\left(\frac{1-q_2^2}{1+q_2^2},\frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right) \tag{19}$$

IV. NUMERICAL RESULTS

It is simple to present these expressions grafically using mathematical software, for example “MatLab”. Because of simplicity we supposed that the variances of both signals at the combiner input are equal.

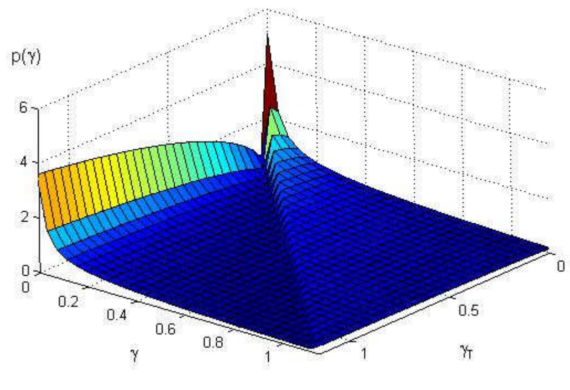

Figure 3. Probability density function of the combiner output signal at one time instant for $\bar{\gamma}_1=\bar{\gamma}_2=1$, $q_1=q_2=0.5$

In the case we observe one time instant, Fig. 3, the probability density function of the combiner output signal to noise ratio is determined as function of input signal to noise ratio γ and the threshold γ_T, for three different variance values for the same distribution parameters in branches of the receiver.

When we observe two time instants, Figs. 4-6, the PDF is given versus input signals at two time instants, γ_1 and γ_2, for different values of the distribution parameters and the threshold γ_T.

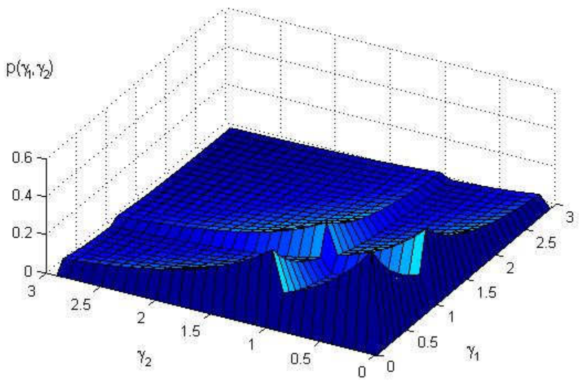

Figure 4. The probability density function of the combiner output signal at two time instants for $\bar{\gamma}_1 = \bar{\gamma}_2 = 1$, $\gamma_T = 1$, $q_1=q_2=0.5$

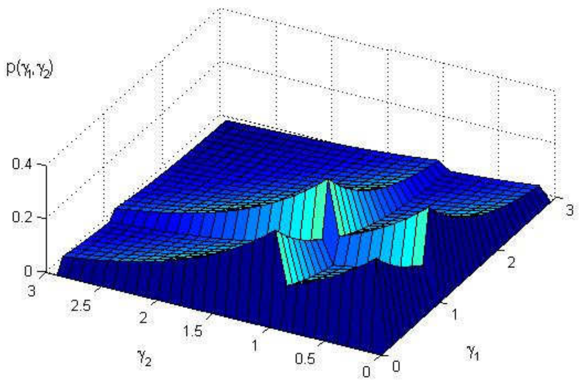

Figure 5. The probability density function of the combiner output signal at two time instants for $\bar{\gamma}_1 = \bar{\gamma}_2 = 1$, $\gamma_T = 1$, $q_1=q_2=0.9$

The bit error probability of digital telecommunication systems in the presence of Hoyt fading can be calculated by the probability density function obtained here. The outage probability also can be calculated using PDF.

The performances of the Switch and Stay Combining/ Selection Combining (SSC/SC) combiner output signal at two time instants in the presence of different types of fading, are determined in our other papers where the results are shown graphically to highlight better performances of the SSC/SC combiner compared to classical SSC and SC combiners at one time instant.

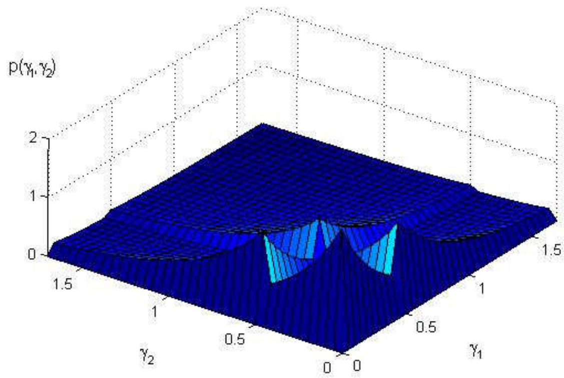

Figure 6. The probability density function of the combiner output signal at two time instants for $\bar{\gamma}_1 = \bar{\gamma}_2 = 0.5$, $\gamma_T = 0.5$, $q_1=q_2=0.5$

V. CONCLUSION

The probability density function of the dual branches SSC combiner output signal at one time instant and the joint probability density function of the SSC combiner output signal at two time instants are determined in closed form. The obtained results are shown graphically for different variance values and decision threshold values.

The bit error probability of digital telecommunication systems in the presence of Hoyt fading can be calculated by the probability density function. The system performances can be significantly improved using the sampling at two time instants. The authors showed in an other work, based on the results obtained in this paper, that the error probability is significantly reduced if the decision making is performed in two time instants. This fact shows that the results obtained in this study are very significant for further research and application in the designing of diversity receivers.

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