Different Criteria of Selection for Quantized Feedback of Minimum-Distance Based MIMO Precoder

Ancuta Moldovan*, Ghadir Madi†, Baptiste Vrigneau†, Tudor Palade* and Rodolphe Vauzelle†
*Technical University of Cluj-Napoca, Communications Department
400027 Cluj-Napoca, Romania
Email: firstname.lastname@com.utcluj.ro
†University of Poitiers, XLIM-SIC Department
86962 Futuroscope Cedex, France
Email: firstname.lastname@univ-poitiers.fr

***Abstract*—The achievable performance of a multi-antenna communication system relies on the amount of information describing the channel available at the transmitter and at the receiver. In this paper, we propose different criteria for selecting the optimal precoding matrix from the codebook, in a limited feedback spatial multiplexing system. The distortion function considered to design the codebook aims to maximize the minimum Euclidean distance between signal points at the receiver side. The proposed approaches are compared considering the bit error rate performance, under the constraint of preserving the same rate on the feedback channel. Since the signal adaptation must be performed in real-time, the computational complexity involved in performing the selection is also evaluated for each method. The simulation results show that the proposed solutions are promising methods for quantization, as they ensure very small loss compared to the ideal case.**

***Index Terms*—channel state information; limited feedback; MIMO precoders; precoder selection.**

I. Introduction

Exploiting propagation diversity, by using multiple antennas at the transmitter and receiver, promises high-capacity and high-quality wireless communication links. In most of the multiple-input multiple-output (MIMO) applications, the channel state information (CSI) at the receiver is needed to perform the equalization and the detection tasks; furthermore, it is known that some degrees of channel knowledge at the transmitter can further boost the system performance through a channel-aware precoding operation of the data streams.

Assuming perfect CSI is available at the transmitter side, different solutions, that optimize pertinent criteria, were proposed in the literature: minimum mean square error (MMSE) [1], signal-to-noise (SNR) maximization at the receiver side ($\max$-SNR) [2], capacity maximization [3] or maximization of the received minimal symbol vector distance ($\max - d_{\min}$) [4]. The limitation of these methods is the fact that obtaining accurate CSI at the transmitter (CSIT) requires to assign a substantial amount of resources on the feedback channel, which may reduce the efficiency of the communication system.

In order to meet the bandwidth requirements on the feedback channel, an efficient quantization of the CSI is mandatory. In [5], Love et al. present a general overview of feedback quantization in various transmitter adaptation schemes. Among these, the quantization of the beamforming vector leads to a significant advancement in feedback techniques. The solution in [6] assumes that the receiver chooses the precoding matrix from a finite cardinality codebook, designed off-line and known at both sides of the wireless communication link. The challenges associated with this quantization scheme are the design of the codebook and the criterion to select the optimal precoding matrix from the codebook. The framework used for the limited feedback beamforming is related to the Grassmann sub-space packing problem, approach that was shown to ensure the outage minimization, the SNR and the rate maximization.

Based on these insights, in [7], we have made the first steps for the design of a new quantization scheme for the $\max - d_{\min}$ precoder. The reason why we have opt for the $\max - d_{\min}$ precoder is the fact that, under the assumption of perfect CSI, it achieves good performance in terms of bit error rate (BER), providing a significant gain of SNR compared to others precoders [8]. For the quantized feedback, the codebook design method and the selection of the optimal precoding matrix are based on the maximization of the minimum Euclidean distance. The proposed method seams to give good results in MIMO systems with two transmit antennas.

In this paper, based on the codebook designed in [7], we propose different functions for the selection of the optimal precoding matrix from the codebook, for a maximum likelihood (ML) detection. We present results obtained by evaluating the BER in a (2,2) MIMO system, while taking into account the feedback rate and the computational complexity of the proposed solutions.

The remainder of the paper is structured as follows. In Section II, the system model and the $\max - d_{\min}$ precoder are presented. Section III introduces the codebook design method. In Section IV, different selection criteria are presented and

 ISBN: 978-1-61208-203-5

evaluated , considering the same rate on the feedback channel. Section V resumes the conclusions and states the future work.

II. System model and the $\max -d_{\min}$ precoder

We consider a MIMO system with n_T transmit and n_R receive antennas operating over an i.i.d. Rayleigh flat-fading channel and b independent data streams are to be transmitted ($b \leq \min(n_T, n_R)$). Assuming perfect knowledge of the channel state information at both sides of the wireless link, a precoder and a decoder matrices, $\mathbf{F}(n_T \times b)$ and $\mathbf{G}(b \times n_R)$, can be designed, so that the basic system model is:

$$\mathbf{y} = \mathbf{GHFs} + \mathbf{Gn} \tag{1}$$

where $\mathbf{H}$ is the $n_R \times n_T$ channel matrix, $\mathbf{s}$ is the $b \times 1$ vector of transmitted symbols and $\mathbf{n}$ is the $n_R \times 1$ additive white Gaussian noise (AWGN) vector.

By using the following decomposition $\mathbf{F} = \mathbf{F}_v \mathbf{F}_d$ and $\mathbf{G} = \mathbf{G}_v \mathbf{G}_d$, the input-output relation (1) can be rewritten as:

$$\mathbf{y} = \mathbf{G}_d \mathbf{H}_v \mathbf{F}_d \mathbf{s} + \mathbf{G}_d \mathbf{n}_v \tag{2}$$

where $\mathbf{H}_v = \mathbf{G}_v \mathbf{H} \mathbf{F}_v = \mathrm{diag}(\sigma_1, ..., \sigma_b)$ is the $b \times b$ eigen-mode channel matrix, with σ_i representing each sub-channel gain, in a decreasing order; $\mathbf{n}_v = \mathbf{G}_v \mathbf{n}$ is the $b \times 1$ additive noise vector on the channel eigen-mode; the unitary matrices $\mathbf{F}_v$ and $\mathbf{G}_v$ are chosen so as to diagonalize the channel and to reduce the dimension to b.

As only the ML detection is considered at the reception, the decoder $\mathbf{G}_d$ has no impact on the performance and it is considered to be $\mathbf{G}_d = \mathbf{I}_b$, where $\mathbf{I}_b$ is the $b \times b$ identity matrix. Regarding the transmit precoder, the optimization under the $\max -d_{\min}$ criterion gives the matrix $\mathbf{F}_d$:

$$\mathbf{F}_d = \arg\max_{\mathbf{F}_d} \min_{\mathbf{s}_k, \mathbf{s}_l \in C^b, \mathbf{s}_k \neq \mathbf{s}_l} ||\mathbf{H}_v \mathbf{F}_d (\mathbf{s}_k - \mathbf{s}_l)|| \tag{3}$$

where $\mathbf{s}_l$ and $\mathbf{s}_k$ are 2 symbols vectors whose entries are elements of the received constellation C.

The solution of (3) is difficult since it involves the computation of the minimum distance. A very exploitable solution was given in [4] for two independent data streams, b = 2 and a 4-QAM, with a spectral efficiency of η = 4bits/s/Hz. If we consider the 2-dimensional virtual channel $\mathbf{H}_v = \mathrm{diag}(\sigma_1, \sigma_2)$, it can be totally defined by two parameters: a positive real parameter $\rho = \sqrt{\sigma_1^2 + \sigma_2^2}$ which is the channel gain and $\gamma = \arctan(\sigma_2/\sigma_1)$ the channel angle, $\pi/4 \geq \gamma > 0$.

The precoding solution is SNR-independent and is based on the value of the channel angle:

- if $0 \leq \gamma \leq \gamma_0$

$$F_d^{d_{\min}} = F_{r1} = \sqrt{E_T} \begin{bmatrix} \sqrt{\frac{3+\sqrt{3}}{6}} & \sqrt{\frac{3-\sqrt{3}}{6}} e^{\frac{i\pi}{12}} \\ 0 & 0 \end{bmatrix} \tag{4}$$

- if $\gamma_0 \leq \gamma \leq \pi/4$

$$F_d^{d_{\min}} = F_{octa} = \sqrt{\frac{E_T}{2}} \begin{bmatrix} \cos\psi & 0 \\ 0 & \sin\psi \end{bmatrix} \begin{bmatrix} 1 & e^{\frac{i\pi}{4}} \\ -1 & e^{\frac{i\pi}{4}} \end{bmatrix} \tag{5}$$

where $\psi = \arctan \frac{\sqrt{2}-1}{\cos\gamma}$ is related to the power allocation and $\gamma_0 = \arctan \sqrt{\frac{3\sqrt{3}-2\sqrt{6}+2\sqrt{2}-3}{3\sqrt{3}-2\sqrt{6}+1}} \simeq 17.28^0$ is a constant threshold, computed by considering that the two forms of the precoder provide the same $d_{\min}$.

A power constraint has to be satisfied, the average transmit power being limited to E_T, so $\mathrm{trace}(\mathbf{F}_d \mathbf{F}_d^*) = E_T$.

III. Codebook design based on channel statistics

The approach is related to the Grassmannian line packing technique, presented in [6], for the design of the beamforming codebook $\mathcal{F}_v$=$\{\widetilde{\mathbf{F}}_{v_1}, ..., \widetilde{\mathbf{F}}_{v_N}\}$, where N is the cardinality of the codebook and $n = \log_2$N is the number of feedback bits. The method used for the design of $\mathcal{F}_v$ can be applied only for an uncorrelated Rayleigh fading channel and it is independent on the channel realization and on the number of receive antennas. In order to eliminate these limitations, in [7] we have presented a new limited feedback scheme that involves the quantization of the $\max -d_{\min}$ precoder. The codebook is empirically design from simulation, as presented in what follows.

Design criterion: Using a sufficiently large number of channel realizations, for each element $\widetilde{\mathbf{F}}_{v_i}$ from $\mathcal{F}_v$, $i = 1$:N, we determine $\mathbf{F}_d$ that maximizes $d_{\min}$:

$$\arg\max_{\mathbf{F}_d} \min_{\mathbf{s}_k, \mathbf{s}_l \in C^b, \mathbf{s}_k \neq \mathbf{s}_l} ||\mathbf{G}_v \mathbf{H} \widetilde{\mathbf{F}}_{v_i} \mathbf{F}_d (\mathbf{s}_k - \mathbf{s}_l)|| \tag{6}$$

The product given by each pair $(\widetilde{\mathbf{F}}_{v_i}, \mathbf{F}_d)$ represents an entry in the new codebook $\mathcal{F}$=$\{\widetilde{\mathbf{F}}_1, ..., \widetilde{\mathbf{F}}_N\}$. The elements are dependent on the configuration of the MIMO system thanks to the design of the $\max -d_{\min}$ precoder. For certain channel realizations, meaning values of the channel angle γ above the threshold γ_0, the new entries are also dependent on the matrix describing the MIMO channel. Since the parameter that connects the matrix $\mathbf{H}$ and the quantized values of the precoder is the channel angle, to each element in $\mathcal{F}$ we associate a value of γ related to the $\mathbf{F}_d$ precoding matrix. All the quantized values of γ are contained in a second codebook $\mathcal{A} = \{\widetilde{\gamma}_1, ..., \widetilde{\gamma}_N\}$.

IV. Selection criteria

In this section, we are providing different design approaches for the selection of the optimal precoding matrix from the codebook. Numerical results are presented assuming a (2,2) MIMO system, over which two independent data streams are transmitted and a 4-QAM modulation.

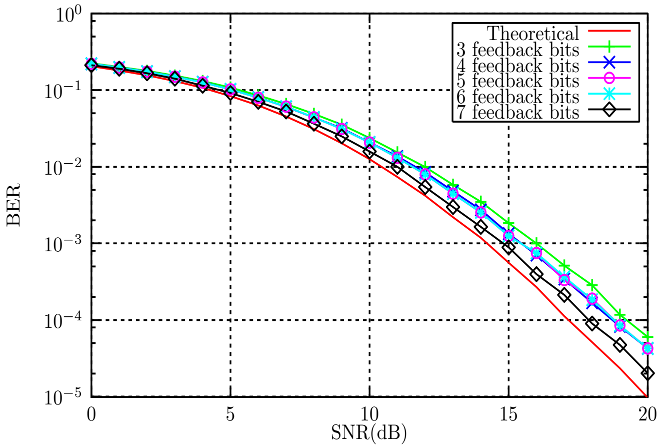

Figure 1: Performance of the $d_{\min}$ criterion dependent on the number of feedback bits

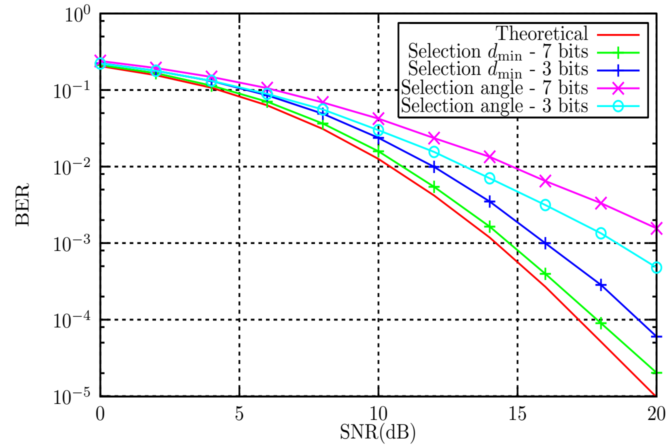

Figure 2: Performance of the $d_{\min}$ and the angle selection criteria dependent on the number of feedback bits

A. $d_{\min}$ selection

The selection criterion maximizes the minimum Euclidean distance between signal points on the received constellation. For the given codebook $\mathcal{F}$, the receiver encodes as follows:

$$\arg\max_{\widetilde{\mathbf{F}}} \min_{\mathbf{s}_k,\mathbf{s}_l \in C^b, \mathbf{s}_k \neq \mathbf{s}_l} ||\mathbf{G}_v \mathbf{H} \widetilde{\mathbf{F}}_i (\mathbf{s}_k - \mathbf{s}_l)|| \tag{7}$$

In Figure 1 are depicted the BER results for the proposed limited feedback scheme. The results for the perfect feedback are also plotted for comparison. For a BER of 10^{-4} and considering 7 feedback bits, the loss relatively to the ideal case is 0.7 dB. Reducing the feedback rate at 3 bits leads to a significant degradation of the BER performance, of about 2 dB, which is considered to be unacceptable.

The selection provides good performance with a sufficient number of feedback bits, but the computation of $d_{\min}$ depends on both the modulation's order and on the channel's statistics. Moreover, in equation (7) it is necessary to consider all possible error vectors in searching for the optimal precoding matrix. In [4] it was shown that for a 4-QAM modulation there is a number of 14 difference vectors that must be considered, but it increases significantly with the modulation's order. Since the selection of the precoder must be performed in real-time, it is necessary to reduce the computational complexity. In what follows, we will consider other selection functions mainly based on the parameters involved in the singular value decomposition (SVD) of the channel matrix.

B. Angle selection

The channel angle is a parameter that can be used to relate the current channel realization with the codebook entries. The new criterion is based on the channel angle and it is intended to minimize the difference between the actual channel angle γ, and the quantized values $\widetilde{\gamma}_i$, from the dictionary $\mathcal{A}$. The function returns the index k of the selected codebook entry, that will be used to determine the precoding matrix $\widetilde{\mathbf{F}}_k$ from the associated codebook $\mathcal{F}$.

$$k = \min_{i=1:N} |\gamma - \widetilde{\gamma}_i| \tag{8}$$

One advantage of the criterion is the fact that it avoids the computation of the minimum distance, but it requires the diagonalization of the channel matrix. Since traditional SVD algorithms involve costly arithmetic operations [9], increased efficiency can be obtained by making use of hardware oriented arithmetic techniques, based on the CORDIC (COrdinate Rotation Digital Computer) arithmetic [10].

The simulation results depicted in Figure 2 show a significant performance degradation, in terms of BER, when the angle selection is applied. Based on the codebook construction, we assumed that the BER depreciation is caused by values of $\gamma \leq \gamma_0$. The next step was to determine to what extent these values influence the BER. So, using a set of 10^5 samples of the channel matrix, we estimated the cumulative distribution function (cdf) for a (2,2) MIMO system. From the results in Figure 3, the probability that the channel angle is bellow the threshold is $P(\gamma \leq \gamma_0) \simeq 0.44$ This means that, in almost half of the cases so there is no connection between the codebook entries and the actual channel realization, so for this region, the quantization scheme is inefficient.Moreover, in this case, the higher the number of codebook entries, the lower is the BER performance since the quantized values of the precoder are independent on the channel matrix.

In order to highlight the influence of the $\max - d_{\min}$ precoder forms on the system's performance, we partition the channel space based on the values of the channel angle. In Figure 4, the BER plots are computed separately, considering either channel matrices with $\gamma \leq \gamma_0$, or with $\gamma \geq \gamma_0$ and a feedback rate of 3 bits. The optimal BER values, for the same channel statistics, are also plotted for comparison. The

 ISBN: 978-1-61208-203-5

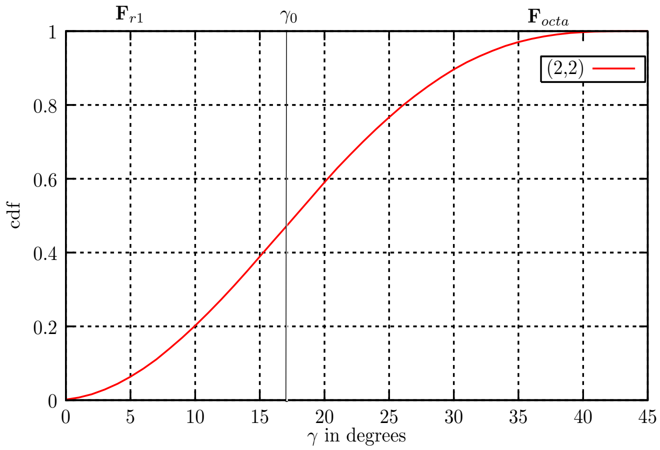

Figure 3: Cumulative distribution function for γ in a (2,2) MIMO system

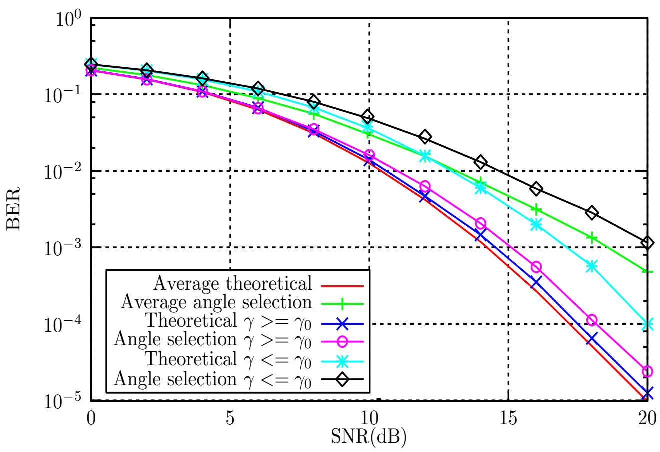

Figure 4: Performance of the angle selection criterion based on the value of the channel angle - 3 feedback bits

results lend credence to our observation that the angle based selection is to be used only for channel matrices with γ above the threshold.

C. Hybrid selection

Based on the results obtained with the previous two selection criteria, we propose a new function that combines them, depending on the value of the channel angle. So, for values of the channel angle bellow the threshold the $d_{\min}$ criterion is used to select the optimum precoder from the codebook, while for values of the channel angle above the threshold, the selection is based on γ.

The results obtained with the hybrid selection are depicted in Figure 5. It can be observed that, with a 3-bits feedback rate, the performance is comparable with the one obtained with the $d_{\min}$ criterion and the same number of bits on the feedback channel. In what concerns the complexity, the channel diagonalization is needed, but for more than half of the scenarios the $d_{\min}$ computation is avoided.

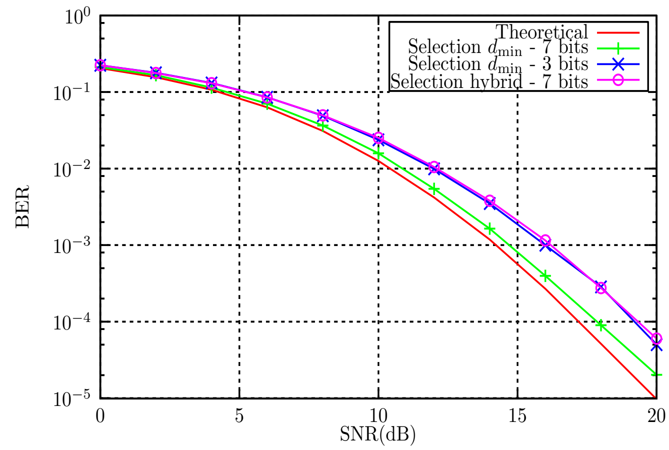

Figure 5: Performance of the hybrid and the $d_{\min}$ selection criteria

D. $\max -\sigma_1$ selection

In the hybrid selection, we focus on channel matrices with the angle above the threshold. In what follows, we propose a selection criteria that takes into account also the channel matrices corresponding to the $\mathbf{F}_{r1}$ form of the $\max -d_{\min}$ precoder. Since the $P(\gamma \leq \gamma_0) \simeq 0.44$ it means that in almost half of the channel realization the precoder uses only the first virtual sub-channel to transmit the data symbols. Based on this observation, we propose a quantization scheme that emphasizes the use of the first singular value.

The same approach as in Section III is considered for the codebook generation, but the criterion is intended to maximize the first singular value $\max -\sigma_1$ from the virtual channel matrix given by:

$$\sigma_1(\arg\max_{\mathbf{F}_d}(\mathbf{G}_v \mathbf{H} \widetilde{\mathbf{F}}_{v_i} \mathbf{F}_d)) \tag{9}$$

Regarding the selection of the optimal precoder from the codebook, the same criterion that maximizes the first singular value σ_1 is applied. Compared to the $d_{\min}$ criterion, the new function does not require considering the difference vectors set involved in the computation of the minimum distance. The BER results are depicted in Figure 6. For a BER of 10^{-4}, with a 7 bits feedback rate, the loss relative to the $d_{\min}$ based quantization scheme is around 0.6 dB. The rate reduction at 3 feedback bits adds no extra SNR loss.

In the attempt to reduce the complexity due to the computation of the minimum distance, while keeping a low rate on the feedback channel, we have introduced different alternative methods for selecting the optimal precoder from the codebook. The new functions are related to the design of the $\max -d_{\min}$ precoder and are dependent on the singular value decomposition of the channel matrix. For channel realizations characterized by a value of the channel angle higher than the threshold we have proposed a solution based on γ. Another

 ISBN: 978-1-61208-203-5

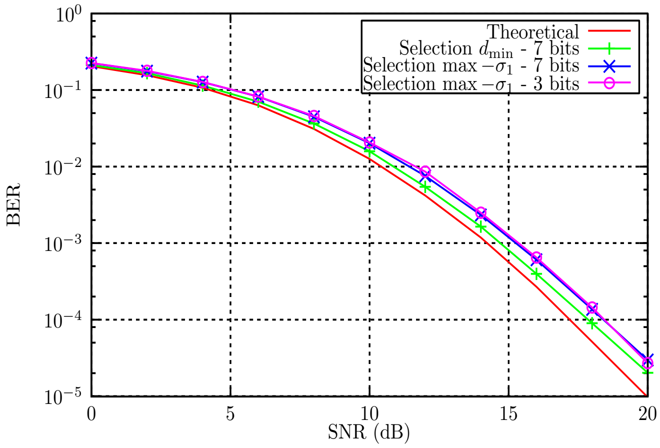

Figure 6: Performance of the $d_{\min}$ and the $\max(\sigma_1)$ selection criteria

solution relies on emphasising the use of the first sub-channel from the virtual channel matrix to transmit the data.

The proposed distortion functions ensure wireless connections with different performance in terms of reliability, depending also on the number of feedback bits. When choosing a certain selection method one must take into account the complexity of the involved computations. It must be stated that for larger MIMO systems is even more important to consider the complexity.

V. CONCLUSION

In this paper, we propose a quantization scheme for uncorrelated Rayleigh fading channels, applied to the $\max - d_{\min}$ precoder. The codebook design method and the selection of the optimal precoder from the codebook are based on the maximization of the minimum Euclidean distance on the received constellation. If the two forms of the $\max - d_{\min}$ precoder are considered in the development of the quantization scheme, the simulation results have shown that, with a reduced feedback rate, 3 bits, the method ensures a very small loss compared to the ideal case.

REFERENCES

[1] A. Scaglione, S. Barbarossa, G. Giannakis and H. Sampath, ”Optimal designs for space-time linear precoders and decoders”,*IEEE Transactions on Signal Processing* vol. 50, no. 5, pp. 1051-1064, May 2002.

[2] P. Stoica and G. Ganesan, ”Maximum-SNR spatial-temporal formatting design for MIMO channels”,*IEEE Transactions on Signal Processing* vol. 50, no. 12, pp. 3036-3042, 2002.

[3] I.E. Telatar”, ”Capacity of multi-antenna Gaussian channels”,*European Transactions on Telecommunications* vol. 10, pp. 585-595, 1999.

[4] L. Collin, O. Berder, P. Rostaing and G. Burel, ”Optimal minimum distance based precoder for MIMO spatial multiplexing systems”,*IEEE Transactions on Signal Processing* vol. 52, no. 3, pp. 617-627, 2004.

[5] D.J. Love, R.W. Hearth, V.K.N. Lau, D. Gesbert, B.D. Rao and M. Andrews, ”An overview of limited feedback in wireless communications systems”,*IEEE Journal on Selected Areas in Communications* vol. 28, no. 8, pp. 1341 - 1365, October 2008.

[6] D.J. Love and R.W. Hearth, ”Limited feedback unitary precoding for spatial multiplexing systems”,*IEEE Transactions on Signal Processing* vol. 51, no. 8, pp. 2967-2976, August 2005.

[7] A. Moldovan, G. Madi, B. Vrigneau, T. Palade and R. Vauzelle, ”SVD algorithms and quantization applied to MIMO $\max - d_{\min}$ based precoder”,*Proceedings of EURASIP Workshop on Signal Processing and Applied Mathematics for Electronics and Communications, SPAMEC 2011*, pp. 5-8, 2011.

[8] B. Vrigneau, J. Letessier, P. Rostaing, L. Collin and G. Burrel, ”Statistical comparison between $\max - d_{\min}$, $\max$-SNR and MMSE precoders”, *40th Asilomar Conference on Signals, Systems and Computers*, Oct. 2006.

[9] J.R. Cavallaro and F.T. Luk, “CORDIC Arithmetic for an SVD Processor,” *in Proc. 8th Symp Computation Arithmetic*, pp. 113-120, May 1987.

[10] J.E. Volder, “The CORDIC trigonometric computing technique,” *IRE Transactions on Electronic Computers* vol. 8, no. 3 pp. 330-334, Sep. 1959.