Blind Subspace Channel Estimation in MIMO-OFDM Systems with Few Received Blocks

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***Abstract*—In this paper, the blind subspace channel estimation using the block matrix scheme is proposed for multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems. Based on the Toeplitz structure, the block matrix scheme collects a group of the received OFDM symbols into a vector, and then partitions it into a set of equivalent signals. The number of equivalent signals is about N times of OFDM symbols, where N is the size FFT operation. The proposed blind subspace channel estimation can converge within a small amount of OFDM symbols. Besides, the semi-blind channel estimation is also examined by combining few pilot sequences with the subspace method. Simulation results show that the proposed blind and semi-blind algorithms outperform the compared methods.**

Keywords-MIMO-OFDM; blind subspace channel estimation; Toeplitz; AIC; MDL.

I. INTRODUCTION

Wideband wireless communication systems have been extensively studied in recent years for the demands of high data rate and high quality transmission. Orthogonal frequency division multiplexing (OFDM) and multiple-input multiple-output (MIMO) are two key techniques to fulfill those demands appeared in the long-term evolution (LTE) and the future fourth-generation (4G) communication systems [1]-[3]. Channel estimations are necessary for coherent detection in MIMO-OFDM systems. There are in general three categories in channel estimations which are training-based, blind and semi-blind methods, respectively. The training-based method requires extra bandwidth to accommodate the periodic known symbols and thus reduces the spectral efficiency [4][5]. The blind method saves the spectral efficiency by utilizing the statistics of received signals. But, this method requires a large amount of received signals to obtain accurate statistics [6][7]. Semi-blind methods, on the other hand, combine the blind method with few training symbols to solve the ambiguity problem occurred in blind methods [8][9].

In this paper, we discuss the blind and semi-blind subspace channel estimation for MIMO OFDM systems with much fewer received symbols. Blind subspace channel estimation has been widely examined for various precoding OFDM systems. For example, Ali et al. studied the subspace channel estimation for cyclic-prefix (CP)-OFDM, zero-padding (ZP)-OFDM and CP-free OFDM systems, respectively [10]. Li and Roy proposed subspace channel estimation based on exploiting the presence of virtual carriers for single-input single-output OFDM systems [11]. Zeng and Ng investigated in [12] the subspace channel estimation for multi-user and multi-antenna ZP-OFDM systems. Shin et al. extended the work in [11] to MIMO-OFDM systems [13]. The subspace channel estimation often converges in a large amount of received OFDM symbols. To enhance the convergence of the subspace channel estimation, Yu [14] presented the block matrix scheme (BMS) to SIMO CP-free OFDM systems. This approach can obtain a group of equivalent signals which is about N times of OFDM symbols where N is the size of FFT operation. By exploiting the idea from [14], a new block matrix scheme is applied to MIMO CP-OFDM systems, in which the number of equivalent samples is increased and the channel estimation error is lowered.

Notation: Vectors and matrices are denoted by boldface lower and upper case letters, respectively; superscripts of $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and conjugate transpose, respectively; $\mathbf{I}$ denotes an identity matrix; $\mathbf{0}$ denotes a zero vector or matrix with all zero entries; $E[\cdot]$ denotes the statistical expectation; $\|\cdot\|$ denotes the matrix or vector Frobenius norm.

The rest of this paper is organized as follows. In Section II, we introduce the signal model of the MIMO CP-OFDM systems. The subspace channel estimation is briefly described in Section III. In Section IV, the blind and semi-

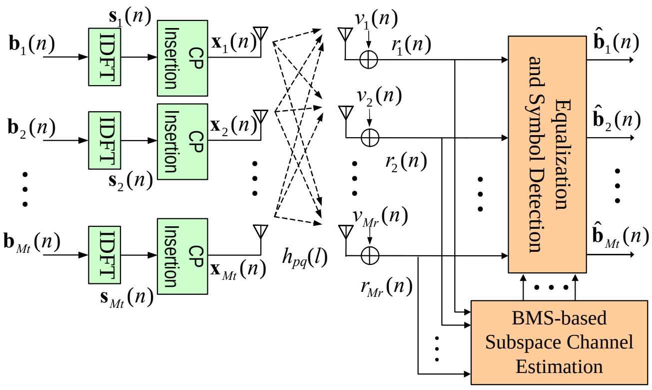

Fig.1. MIMO CP-OFDM block transmission systems. The system has M_t transmit and M_r receive antennas.

blind subspace channel estimations are presented with the assistance of block matrix scheme. The computer simulations are performed in Section V. Finally, conclusion and future work are given in Section VI.

II. System Model

Fig. 1 shows the quasi-synchronous MIMO CP-OFDM system with M_t transmit antennas and M_r receive antennas. Let $\mathbf{b}_q(n)=\left[b_q(n,0),\cdots,b_q(n,N-1)\right]^T$ be the n-th block frequency domain symbol for the q-th transmit antenna. Transmitted symbol $b_q(n,k)$ is assumed to be independent and identically distributed (i.i.d.) complex random variable with zero-mean and variance σ_s^2. After multicarrier modulation implemented by IDFT, the time domain signal vector is given by $\mathbf{s}_q(n)=\mathbf{W}_N^H\mathbf{b}_q(n)=\left[s_q(n,0),\cdots,s_q(n,N-1)\right]^T$ where $\mathbf{W}_N$ is the N-point DFT matrix with the (n,m)-th element $(1/\sqrt{N})\exp(-j2\pi(n-1)(m-1)/N)$. Appending CP components at the front of $\mathbf{s}_q(n)$ yields $\mathbf{x}_q(n)=\left[x_q(n,0),\cdots,x_q(n,Q-1)\right]^T$, where $Q=N+L_c$ and L_c is the length of CP. Denote by $\mathbf{x}(m)=\sum_{n=0}^{\infty}\sum_{j=0}^{Q-1}\mathbf{x}(n,j)\delta\left(m-(nQ+j)\right)$ the transmitted vector among all transmit antennas where $\mathbf{x}(n,j)=\left[x_1(n,j),\cdots,x_{M_t}(n,j)\right]^T$. The discrete-time received signal at the p-th receive antenna is given by

$$r_p(m)=\sum_{q=1}^{M_t}\sum_{l=0}^{L}h_{pq}(l)x_q(m-l)+v_p(m) \tag{1}$$

where $h_{pq}(l)$, l=0,…,L represents the composite channel impulse response between the q-th transmit antenna and the p-th receive antenna with maximum channel order L, and $v_p(m)$ is the additive white Gaussian noise (AWGN) with zero-mean and variance σ_n^2. Noise is assumed to be spatially and temporally white, and be uncorrelated with transmitted symbols. In order to avoid the inter-symbol interference (ISI), we assume that $L\le L_C$. Let $\mathbf{r}(m)=\left[r_1(m),\cdots,r_{Mr}(m)\right]^T$ and stack $\mathbf{r}(m)$, $m=nQ+L_c,\cdots,nQ+Q-1$ as $\mathbf{r}_n$. Then we have

$$\mathbf{r}_n=\left[\mathbf{r}^T(nQ+L_c),\cdots,\mathbf{r}^T(nQ+Q-1)\right]^T=\mathbf{H}_N\mathbf{x}_n+\mathbf{v}_n \tag{2}$$

where $\mathbf{v}_n$ is the noise vector and

$$\mathbf{x}_n=\left[\mathbf{x}^T(n,L_c-L),\cdots,\mathbf{x}^T(n,Q-1)\right]^T\in\mathbb{C}^{(N+L)M_t\times 1}$$

$$\mathbf{H}_N=\begin{bmatrix}\mathbf{h}(L) & \cdots & \mathbf{h}(0) & \cdots & \cdots & \mathbf{0}\\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots\\ \mathbf{0} & \cdots & \cdots & \mathbf{h}(L) & \cdots & \mathbf{h}(0)\end{bmatrix} \tag{3}$$

$$\mathbf{h}(l)=\begin{bmatrix}h_{11}(l) & h_{12}(l) & \cdots & h_{1M_t}(l)\\ h_{21}(l) & h_{22}(l) & \cdots & h_{2M_t}(l)\\ \vdots & \vdots & \vdots & \vdots\\ h_{M_r1}(l) & h_{M_r2}(l) & \cdots & h_{M_rM_t}(l)\end{bmatrix}$$

Note that $\mathbf{H}_N$ is a $NM_r\times(N+L)M_t$ block Toeplitz matrix. To consider a tall and skinny matrix for $\mathbf{H}_N$, we assume that $M_r>M_t$. If it is not the case, the fractionally spaced receiver could be used here. In (2), we assume the channel order is known a priori derived from Akaike information theoretic criterion (AIC) or minimum description length (MDL) [15]. Based on the signal model in (2), we discuss the subspace channel estimation in the next section.

III. Subspace Channel Estimation

With the signal model in (2), various subspace channel estimation techniques have been presented based on different assumptions. We briefly describe the channel estimation in [12] for the purpose of comparison. Let $\mathbf{W}_N$ =[$\mathbf{w}(0)$ **…** $\mathbf{w}(N\text{-}1)$] and define $\mathbf{W}_{CP}$ and $\mathbf{W}_{CP,Mt}$ respectively by

$$\mathbf{W}_{CP}=\left[\mathbf{w}(N-L),\cdots,\mathbf{w}(N-1)\ \ \mathbf{W}_N\right]\in\mathbb{C}^{N\times(N+L)}$$

$$\mathbf{W}_{CP,Mt}=\mathbf{W}_{CP}\otimes\mathbf{I}_{Mt}\in\mathbb{C}^{NM_t\times(N+L)M_t}$$

The signal vector $\mathbf{x}_n$ can be rewritten as

$$\mathbf{x}_n=\mathbf{W}_{CP,Mt}^H\mathbf{b}_n \tag{4}$$

where

$$\mathbf{b}_n=\left[\mathbf{b}^T(n,0),\cdots,\mathbf{b}^T(n,N-1)\right]^T\in\mathbb{C}^{NM_t\times 1}$$

$$\mathbf{b}(n,j)=\left[b_1(n,j),\cdots,b_{Mt}(n,j)\right]^T\in\mathbb{C}^{M_t\times 1}$$

Substituting (4) into (2), $\mathbf{y}_n$ can be expressed by

$$\mathbf{r}_n=\mathbf{H}_N\mathbf{W}_{CP,Mt}^H\mathbf{b}_n+\mathbf{v}_n=\mathbf{H}_W\mathbf{b}_n+\mathbf{v}_n \tag{5}$$

where $\mathbf{H}_W=\mathbf{H}_N\mathbf{W}_{CP,Mt}^H$. In the subspace channel estimation, the channel is identifiable if the matrix $\mathbf{H}_W$ is of full column rank. A necessary and sufficient condition for this full column rank requirement is given in [12], which is stated as follows.

Theorem 1 [12]: In the case of $M_r>M_t$, the matrix $\mathbf{H}_W$ is of full column rank if and only if rank($\mathbf{H}(z)$)=M_t at $z=e^{j2\pi k/N}$, k=0,…,N-1, where $\mathbf{H}(z)=\sum_{n=0}^{L}\mathbf{h}(n)z^{-n}$.

From Theorem 2, we can calculate the signal and noise subspaces from $\mathbf{r}_n$ in (5) if the assumptions of $M_r>M_t$ and rank($\mathbf{H}(e^{j2\pi k/N})$)=$M_t$ are satisfied. To find the noise subspace, the correlation matrix of $\mathbf{r}_n$ is first computed by

$$\mathbf{R}_r=E[\mathbf{r}_n\mathbf{r}_n^H]=\mathbf{H}_W E[\mathbf{b}_n\mathbf{b}_n^H]\mathbf{H}_W^H+\sigma_n^2\mathbf{I}=\sigma_s^2\mathbf{H}_W\mathbf{H}_W^H+\sigma_n^2\mathbf{I} \tag{6}$$

Performing the eigenvalue-eigenvector decomposition (EVD) onto $\mathbf{R}_r$ yields the eigenvectors $\mathbf{U}$. The eigenvectors can be divided into two sets $\mathbf{U}$=[$\mathbf{U}_s$ $\mathbf{U}_n$] according to their eigenvalue spread, where $\mathbf{U}_s\in\mathbb{C}^{NM_r\times(N+L)M_t}$ is the signal subspace spanning the same subspace as $\mathbf{H}_W$, and $\mathbf{U}_n\in\mathbb{C}^{NM_r\times(NM_r-(N+L)M_t)}$ is the noise subspace which is orthogonal to the signal subspace. Let $\mathbf{U}_n=[\mathbf{u}_1,\ldots,\mathbf{u}_{NMr-(N+L)Mt}]$ and $\mathbf{u}_k$ be partitioned into a block vector $\mathbf{u}_k=[\mathbf{u}_k^H(1)\cdots\mathbf{u}_k^H(N)]^H$ where $\mathbf{u}_k(j)$ is a $M_r\times 1$ vector. Then from the subspace orthogonal principle, we have [12]

$$\mathbf{u}_k^H\mathbf{H}_W=\mathbf{0} \text{ or } \mathbf{W}_{CP}^*\mathbf{V}_k^H\mathbf{h}=\mathbf{0} \tag{7}$$

where $\mathbf{h}=\left[\mathbf{h}^T(0),\cdots,\mathbf{h}^T(L)\right]^T$ and $\mathbf{V}_k$ is a $(L+1)M_r\times Q$ matrix

$$\mathbf{V}_k=\begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{u}_k(1) & \cdots & \mathbf{u}_k(N) \\ \vdots & \iddots & \mathbf{u}_k(1) & \iddots & \mathbf{u}_k(N) & \mathbf{0} \\ \mathbf{0} & \iddots & \iddots & \iddots & \iddots & \iddots \\ \mathbf{u}_k(1) & \cdots & \mathbf{u}_k(N) & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}$$

It is shown in [12] that the equations in (7) can determine the channel matrix $\mathbf{h}$ up to an ambiguity matrix. In practical, the correlation matrix in (6) is computed from the sample correlation matrix of $\mathbf{r}_n$. If there are K OFDM symbols available, the sample correlation matrix is given by

$$\hat{\mathbf{R}}_r=(1/K)\sum_{n=1}^{K}\mathbf{r}_n\mathbf{r}_n^H \tag{8}$$

From (8), the eigenvectors $\hat{\mathbf{u}}_k$ and matrix $\hat{\mathbf{V}}_k$ can be obtained. With the constraint that $\mathbf{h}$ has a full column rank, the channel matrix can be estimated by the least square minimization technique

$$\hat{\mathbf{h}}=\arg\min_{\mathbf{h}^H\mathbf{h}=\mathbf{I}}\sum_{k=1}^{NM_r-(N+L)M_t}\left\|\mathbf{W}_{CP}^*\hat{\mathbf{V}}_k^H\mathbf{h}\right\|_F^2 \tag{9}$$

IV. Blind Channel Estimation by Block Matrix Scheme

The estimation performance in (9) is heavily depended on the biasness of the sample correlation matrix. Let $\Delta\hat{\mathbf{R}}_r=\mathbf{R}_r-\hat{\mathbf{R}}_r$ be the bias sample correlation matrix. It is shown in [16] that the norm of the bias matrix is proportional to the dimension of $\mathbf{r}_n$, and inversely proportional to K. Therefore, the subspace channel estimation generally requires a large amount of received blocks to achieve a small perturbation of sample correlation matrix and a low channel estimation error. The block matrix scheme is proposed in this section to improve the subspace channel estimation. With the assistance of block Toeplitz structure in the received signal, the block matrix scheme segments the stacked OFDM symbols into a group of equivalent sub-vectors. The number of equivalent sub-vectors is about Q times of OFDM symbols. Therefore, the biasness of the sample correlation matrix is reduced considerably.

A. Block Matrix Scheme

We first observe that the channel matrix in (2) has a block Toeplitz form. The block matrix scheme is proposed here to increase the number of equivalent samples and then to enhance the performance of channel estimation. By collecting K consecutive received OFDM symbols, the signal vector is given by

$$\tilde{\mathbf{r}}_K=\left[\mathbf{r}^T(0),\mathbf{r}^T(1),\cdots,\mathbf{r}^T(KQ-1)\right]^T=\mathbf{H}_{QK}\tilde{\mathbf{x}}_K+\tilde{\mathbf{v}}_K \tag{10}$$

where $\tilde{\mathbf{x}}_K=[\mathbf{x}^T(-1,Q-L),\cdots\mathbf{x}^T(-1,Q-1),\mathbf{x}_0^T,\mathbf{x}_1^T,\cdots,\mathbf{x}_{K-1}^T]^T$ is a $(QKM_t+LM_t)\times 1$ vector with $\mathbf{x}_{-1}=\mathbf{0}$ and $\mathbf{H}_{QK}$ is a $QKM_r\times(QK+L)M_t$ block Toeplitz matrix which has a similar form to (3). Because of the block Toeplitz structure in $\mathbf{H}_{QK}$, we can select a proper parameter G such that $\mathbf{H}_{QK}$ is expressed by

$$\mathbf{H}_{QK}=\begin{bmatrix} \boxed{\mathbf{H}_G} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \boxed{\mathbf{H}_G} \end{bmatrix} \tag{11}$$

where $\mathbf{H}_G$ is also a block Toeplitz matrix with dimensions $GM_r\times(G+L)M_t$. Using (10) and (11), a sub-vector $\tilde{\mathbf{r}}_{K,g}$ of $GM_r\times 1$ is defined by $\tilde{\mathbf{r}}_{K,g}=\left[\mathbf{r}^T(g),\cdots,\mathbf{r}^T(g+G-1)\right]^T$, which is obtained as

$$\tilde{\mathbf{r}}_{K,g}=\mathbf{H}_G\tilde{\mathbf{x}}_{K,g}+\tilde{\mathbf{v}}_{K,g},\quad g=0,\cdots,KQ-G \tag{12}$$

where $\tilde{\mathbf{x}}_{K,g}=\left[\mathbf{x}(g-L),\cdots,\mathbf{x}(g+G-1)\right]^T$ and $\tilde{\mathbf{v}}_{K,g}$ contains the noise components.

From the sub-vectors in (12), the subspace channel estimation can be performed if the channel matrix $\mathbf{H}_G$ has a full column rank. A tall and skinny matrix is only a necessary but not a sufficient condition for the block Toeplitz matrix to be of full column rank. That is $GM_r>(G+L)M_t$ can not guarantee that $\mathbf{H}_G$ has a full column rank. More precisely, a necessary and sufficient condition for this requirement has been presented in [17].

Theorem 2 [17]: Assume that $\mathbf{h}(0)$, $\mathbf{h}(L)$ and $\mathbf{H}(z)$ have a full column rank for all z. The block Toeplitz matrix $\mathbf{H}_G$ has a full column rank if and only if G is no less than the degree of orthogonal complement polynomial matrix of $\mathbf{H}(z)$.

If the assumptions in Theorem 2 are satisfied such that $\mathbf{H}_G$ has a full column rank, the subspace channel estimation is developed as follows. We first show that the symbols in $\tilde{\mathbf{x}}_{K,g}$ are uncorrelated. Since $\mathbf{s}_q(n)=\mathbf{W}_N^H\mathbf{b}_q(n)$, we find that $\mathbf{s}_q(n)$ is also an i.i.d. random vector because of $E[\mathbf{s}_q(n)\mathbf{s}_q^H(n)]=\mathbf{W}_N^HE[\mathbf{b}_q(n)\mathbf{b}_q^H(n)]\mathbf{W}_N=\sigma_s^2\mathbf{I}$. Denote by $\mathbf{R}_G=E[\tilde{\mathbf{r}}_{K,g}\tilde{\mathbf{r}}_{K,g}^H]$ the correlation matrix of $\tilde{\mathbf{r}}_{K,g}$. If we properly choose the parameter G such that the symbols in $\tilde{\mathbf{x}}_{K,g}$ are uncorrelated, the noise subspace can be computed from the EVD of $\mathbf{R}_G=E[\tilde{\mathbf{r}}_{K,g}\tilde{\mathbf{r}}_{K,g}^H]$

$$\mathbf{R}_G=\sigma_s^2\mathbf{H}_G\mathbf{H}_G^H+\sigma_n^2\mathbf{I}=\mathbf{E}_s\Sigma_s\mathbf{E}_s^H+\sigma_n^2\mathbf{E}_n\mathbf{E}_n^H \tag{13}$$

where $\mathbf{\Sigma}_s$ is a diagonal matrix consisting of $(G+L)M_t$ eigenvalues larger than σ_n^2, $\mathbf{E}_s$ is the signal subspace which equals the range space of $\mathbf{H}_G$, and $\mathbf{E}_n$ is the noise subspace. Using the orthogonal property between signal subspace and

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noise subspace, we have $\mathbf{E}_n^H \mathbf{H}_G = \mathbf{0}$. Let $\mathbf{q}_i$ be the i-th column of $\mathbf{E}_n$ and partition $\mathbf{q}_i$ into a $G\times 1$ block vector

$$\mathbf{q}_i = \left[\mathbf{q}_i^T(0), \mathbf{q}_i^T(1), \cdots, \mathbf{q}_i^T(G-1)\right]^T \tag{14}$$

where $\mathbf{q}_i(j) \in C^{M_r}$, $j = 0, \cdots, G-1$. Exploiting the block Toeplitz structure of $\mathbf{H}_G$, $\mathbf{E}_n^H \mathbf{H}_G = \mathbf{0}$ is rewritten by

$$\mathbf{Q}_i^H \mathbf{h} = 0, \quad i = 1, \cdots, GM_r - (G+L)M_t \tag{15}$$

where $\mathbf{Q}_i \in C^{(L+1)M_r \times (G+L)}$

$$\mathbf{Q}_i = \begin{pmatrix} \mathbf{q}_i(G-1) & \cdots & \mathbf{q}_i(0) & \cdots & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \mathbf{q}_i(G-1) & \cdots & \mathbf{q}_i(0) \end{pmatrix} \tag{16}$$

A theorem is given in [17] that (15) can determine the channel matrix $\mathbf{h}$ up to an ambiguity matrix.

Theorem 3 [17]: Let $\mathbf{h}'$ be a matrix that has the same size as that of $\mathbf{h}$, $\mathbf{H}'_G$ be constructed from $\mathbf{h}'$ in the same form as $\mathbf{H}_G$ is constructed from $\mathbf{h}$. Assume that $\mathbf{h}(0)$, $\mathbf{h}(L)$ and $\mathbf{H}(z)$ have a full column rank for all z, and G is no less than the degree of orthogonal complement polynomial matrix of $\mathbf{H}(z)$. Then $\mathbf{h}'$ is equal to $\mathbf{h}\boldsymbol{\Omega}$ where $\boldsymbol{\Omega}$ is an $M_t \times M_t$ invertible matrix if and only if $\mathbf{h}'$ has a full column rank and $span(\mathbf{H}'_G)$ is equal to $span(\mathbf{H}_G)$.

In a finite sample scenario, we use the sample correlation matrix $\hat{\mathbf{R}}_G$ instead of the ensemble average correlation matrix $\mathbf{R}_G$ to compute the noise subspace. Due to the biasness of the sample correlation matrix, the homogeneous equations in (15) will not be satisfied. The constrained least square optimization criterion is adopted to find the channel matrix

$$\hat{\mathbf{h}} = [\hat{\mathbf{h}}_1 \cdots \hat{\mathbf{h}}_{Mt}] = \arg\min_{\|\mathbf{h}_i\|=1} \mathbf{h}_i^H \mathbf{Q}\mathbf{Q}^H \mathbf{h}_i \tag{17}$$

where $\mathbf{Q} = \left[\mathbf{Q}_1 \cdots \mathbf{Q}_{GM_r-(G+L)M_t}\right]$. The estimates of $\mathbf{h}$ in (17) are the eigenvectors associated with the M_t smallest eigenvalues of the matrix $\mathbf{Q}\mathbf{Q}^H$. From Theorem 3, $\hat{\mathbf{h}}$ differs from $\mathbf{h}$ by an ambiguity matrix $\boldsymbol{\Omega}$. In the blind channel estimation, the assistance of pilot sequences is a practical way to solve the ambiguity and alleviate the phase rotation in the symbol detection.

B. Semi-blind Approach

The semi-blind estimation technique estimates the channels by combining the blind method with the pilot information [18]. From (2), $\mathbf{r}_n$ can be rewritten as

$$\mathbf{r}_n = \mathbf{H}_{cir}\mathbf{s}_n + \mathbf{v}_n$$

where $\mathbf{H}_{cir}$ is a $NM_r \times NM_t$ block circular matrix, and $\mathbf{s}_n = (\mathbf{W}_N^H \otimes \mathbf{I}_{Mt})\mathbf{b}_n$. Performing DFT operation onto $\mathbf{r}_n$ yields $\mathbf{y}_n = DFT(\mathbf{r}_n) = [\mathbf{y}^T(n,0) \cdots \mathbf{y}^T(n, N-1)]^T$ where

$$\mathbf{y}(n,k) = \mathbf{H}(k)\mathbf{b}(n,k) + \boldsymbol{\eta}(n,k) \tag{18}$$

, $\mathbf{H}(k) = \sum_{l=0}^{L} \mathbf{h}(l) e^{-j2\pi lk/N}$ is a $M_r \times M_t$ matrix and $\boldsymbol{\eta}(n,k)$ is the noise. Assume that there are A OFDM symbols and each one contains B pilots at $k_1, k_2, \ldots, k_B$ subcarriers. Define $\mathbf{Y}(n)$ and $\mathbf{B}(n)$ and $\boldsymbol{\eta}(n)$ respectively by

$$\begin{aligned} \mathbf{Y}(n) &= \left[\mathbf{y}(n,k_1), \mathbf{y}(n,k_2), \cdots, \mathbf{y}(n,k_B)\right]^T \\ \mathbf{B}(n) &= \left[\mathbf{b}(n,k_1), \mathbf{b}(n,k_2), \cdots, \mathbf{b}(n,k_B)\right]^T \\ \boldsymbol{\eta}(n) &= \left[\boldsymbol{\eta}(n,k_1), \boldsymbol{\eta}(n,k_2), \cdots, \boldsymbol{\eta}(n,k_B)\right]^T \end{aligned} \tag{19}$$

Then from (18) and (19), $\mathbf{Y}(n)$ is given by

$$\mathbf{Y}(n) = \sum_{l=0}^{L} \boldsymbol{\Phi}^l \mathbf{B}(n)\mathbf{h}^T(l) + \boldsymbol{\eta}(n) = \mathbf{D}(n)\tilde{\mathbf{h}} + \boldsymbol{\eta}(n) \tag{20}$$

where $\boldsymbol{\Phi} = diag(e^{-j2\pi k_1/N}, \cdots, e^{-j2\pi k_B/N})$, $\tilde{\mathbf{h}} = [\mathbf{h}(0), \cdots, \mathbf{h}(L)]^T$, $\mathbf{D}(n) = [\mathbf{B}(n), \boldsymbol{\Phi}^1\mathbf{B}(n), \cdots, \boldsymbol{\Phi}^L\mathbf{B}(n)]$. Stacking $\mathbf{Y}(n)$ for $n=n_1, n_2, \ldots, n_A$ produces

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}(n_1) \\ \vdots \\ \mathbf{Y}(n_A) \end{pmatrix} = \begin{pmatrix} \mathbf{D}(n_1) \\ \vdots \\ \mathbf{D}(n_A) \end{pmatrix} \tilde{\mathbf{h}} + \begin{pmatrix} \boldsymbol{\eta}(n_1) \\ \vdots \\ \boldsymbol{\eta}(n_A) \end{pmatrix} = \mathbf{D}\tilde{\mathbf{h}} + \boldsymbol{\eta} \tag{21}$$

To solve the ambiguity problem in the subspace channel estimation, we integrate linear equations in (15) and (21) together. Denote by $vec(.)$ the vectorization of a matrix by stacking its columns in order. Let $\mathbf{y}_v = vec(\mathbf{Y})$, $\tilde{\mathbf{h}}_v = vec(\tilde{\mathbf{h}})$, $\mathbf{h}_v = vec(\mathbf{h})$, and $\boldsymbol{\eta}_v = vec(\boldsymbol{\eta})$. Then (15) and (21) become

$$\begin{aligned} \mathbf{Q}_{Mt}^H \mathbf{h}_v &= 0, \\ \mathbf{y}_v &= \mathbf{D}_{Mr}\tilde{\mathbf{h}}_v + \boldsymbol{\eta}_v \end{aligned} \tag{22}$$

where $\mathbf{Q}_{Mt} = \mathbf{I}_{Mt} \otimes \mathbf{Q}$ and $\mathbf{D}_{Mr} = \mathbf{I}_{Mr} \otimes \mathbf{D}$. We further observe that $\tilde{\mathbf{h}}_v$ and $\mathbf{h}_v$ have the same components with different arrangement. After carefully simplification, we obtain $\tilde{\mathbf{h}}_v = \mathbf{P}_v \mathbf{h}_v$ where $\mathbf{P}_v$ is a $M_r M_t(L+1) \times M_r M_t(L+1)$ permutation matrix.

$$\mathbf{P}_v(x,y) = \begin{cases} 1, & \begin{array}{l} x = (r-1)M_t(L+1) + lM_t + t, \\ y = (t-1)M_r(L+1) + lM_r + r \\ l = 0 \cdots L, t = 1 \cdots M_t, r = 1 \cdots M_r \end{array} \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

Thus the semi-blind approach can find the estimates of $\mathbf{h}$ by the following minimization criterion,

$$\hat{\mathbf{h}}_v = \arg\min_{\mathbf{h}_v} \left\|\mathbf{y}_v - \mathbf{D}_{Mr}\mathbf{P}_v\mathbf{h}_v\right\|^2 + \alpha\left\|\mathbf{Q}_{Mt}^H\mathbf{h}_v\right\|^2 \tag{24}$$

where α is a weighting constant. The solution of the problem in (24) is given by

$$\hat{\mathbf{h}}_v = (\mathbf{P}_v^H\mathbf{D}_{Mr}^H\mathbf{D}_{Mr}\mathbf{P}_v + \alpha\mathbf{Q}_{Mt}\mathbf{Q}_{Mt}^H)^{-1}\mathbf{P}_v^H\mathbf{D}_{Mr}^H\mathbf{y}_v \tag{25}$$

C. Discussions

The parameter G needs to be selected properly such that $\mathbf{H}_G$ is of full column rank. Two possible considerations are examined as follows. Firstly, the symbols in $\tilde{\mathbf{x}}_{K,g}$ are required to be uncorrelated explained in Sec IV.A. Since $\mathbf{x}(n,i)= \mathbf{x}(n,i+N)$ for $i=0,\ldots,L_c$, those identical components will not appear in $\tilde{\mathbf{x}}_{K,g}$ at the same time if $G+L\leq N$. Secondly, from Theorem 2, the necessary and sufficient condition for $\mathbf{H}_G$ has a full column rank is that G is no less than the degree of orthogonal complement polynomial matrix of $\mathbf{H}(z)$. We can show that a more practical selection to ensure that G is sufficiently large to satisfy this requirement is $G\geq M_rL$. Therefore, the selectable range of G is given by $M_rL\leq G\leq N-L$.

Besides, comparing with $\hat{\mathbf{R}}_r$ in (8), the sample correlation matrix for the proposed method is calculated by

$$\hat{\mathbf{R}}_G = \frac{1}{KQ-G+1}\sum_{g=0}^{KQ-G}\tilde{\mathbf{r}}_{K,g}\tilde{\mathbf{r}}_{K,g}^H \tag{26}$$

Therefore, $\hat{\mathbf{R}}_G$ is averaged by (KQ-G+1) equivalent signals with dimension of $GM_r\times1$, while $\hat{\mathbf{R}}_r$ is averaged by K OFDM symbols with dimension of $NM_r\times1$. According to the analysis in [16], the biasness of the sample correlation matrix for the proposed method is reduced considerably.

V. Computer Simulations

Computer simulations are given here to verify the performances of the proposed channel estimations (CE). The number of subcarriers is set N=64 and number of CP=16. The 16QAM modulation scheme is applied. The input signal-to-noise ratio (SNR) is defined as the bit SNR at single receive antenna. The independent Rayleigh channel with exponentially decaying power delay profile of channel order L=5 is used in simulations. In the semi-blind approach, we use A=2, B=8 and $\alpha=100$. The normalized root mean-squared error (NRMSE) between the estimated and true channels is given by

$$NRMSE = \sqrt{\frac{1}{N_m M_r M_t (L+1)}\sum_{p=1}^{N_m}\frac{\|\hat{\mathbf{h}}(p)-\mathbf{h}(p)\|_F^2}{\|\mathbf{h}(p)\|_F^2}} \tag{34}$$

where the subscript p refers to the p-th simulation run and N_m denotes the number of Monte Carlo runs.

We first examine in Fig. 2 the influences of block matrix size G varied from 12 to 80 for the BMS-based channel estimators. The selectable range of G suggested in Sec IV is $10\leq G\leq 59$. It is observed from Fig. 2 that the selection of G is very robust for the proposed BMS-based methods even G is larger than 59. When $G\geq 60$, there are a portion of equivalent signals suffer from the correlated transmitted signals due to CP components. However, the amount of the correlated signals is small such that it has insignificantly influence on the channel estimation. Besides, the blind method uses pilot sequences to correct the ambiguity matrix while the semi-blind method integrates the subspace information with pilot sequences in calculation of channel estimation. Therefore, the semi-blind method outperforms the blind one.

The RNMSE versus the input SNR for the compared channel estimation methods is plotted in Fig. 3. With the selection of G=64, the proposed methods produce almost Q=80 times of equivalent signals while the method in [14] has only 17 times of equivalent signals. Therefore, the proposed methods outperform the other two channel estimation methods. Furthermore, combining with 16 pilot signals for each transmit antenna, the semi-blind method obtains the lowest MSE values. Fig. 4 shows the RNMSE versus the number of OFDM symbols. As the number of OFDM symbols increases, the proposed BMS-based methods decrease the RNMSE on a steeper slope than the methods in [12] and [14]. Especially the semi-blind method converges to the lowest MSE after about K>50.

With the estimated channels in Figs. 3 and 4, we examine the BER performances of the minimum mean-square error (MMSE) equalizers. For the sake of comparison, the BER with real channel coefficients is also plotted. The BERs versus the input SNR are discussed in Fig. 5. Since the proposed BMS-based methods produce a lower estimation error than the compared methods, the equalizers with the former methods outperform those with the latter ones. The semi-blind method almost achieves the same BER performance as the equalizer with real channels. Finally, the BERs versus the number of OFDM symbols are examined in Fig. 6. The proposed BMS-based methods converge faster than the other two methods. Interestingly, the semi-blind method reaches the error floor of the BER at about K=50 in which the lowest MSE is also met.

VI. Conclusion and Future Work

We proposed in this paper the blind and semi-blind subspace channel estimation for MIMO CP-OFDM systems. Inspired by the block Toeplitz structure, the block matrix scheme is first presented to increase the number of equivalent signals. The block matrix scheme decreases the biasness of the correlation matrix, noise subspace and then the channel estimation. The identifiability of the proposed channel estimation is further studied, where the estimated channels differ from the true channels by an invertible matrix. With the assistance of few pilot sequences, the semi-blind method combining the subspace method with pilot information is provided at the end. Computer simulations verify the superiority of the proposed blind and semi-blind CE over the compared ones.

We will extend the work in this paper to the OFDM systems with the virtual carriers (VCs). The VCs are often properly distributed on the dedicated band with zero values in OFDM systems for shaping the transmission spectrum and alleviating the adjacent channel interference (ACI). With the existence of VCs, the property that the symbols in $\tilde{\mathbf{x}}_{K,g}$ are uncorrelated is not hold. Additional work will be required to make the block matrix scheme applicable to OFDM systems with VCs.

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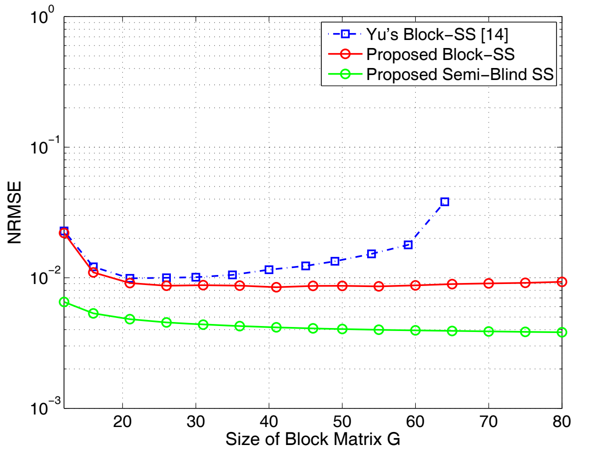

Fig. 2. NRMSE vs. Parameter *G* for different BMS-based channel estimation methods with *SNR*=20dB, *K*=400.

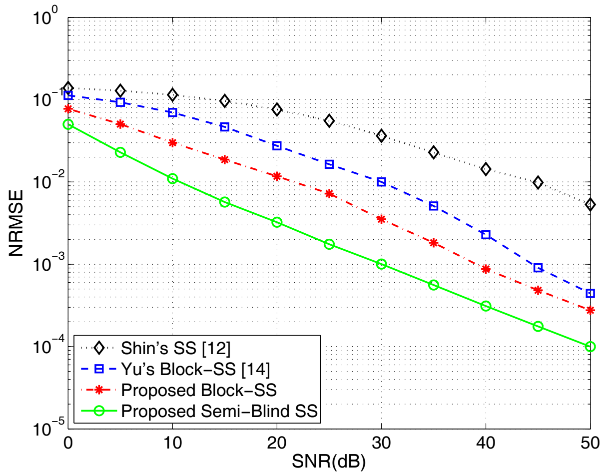

Fig. 3. NRMSE vs. SNR for compared channel estimation methods with *G*=64, *K*=200.

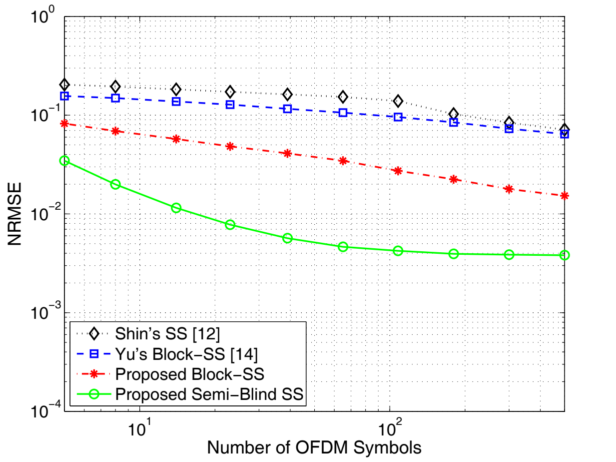

Fig. 4. NRMSE vs. the number of OFDM symbols for compared channel estimation methods with *G*=64, *SNR*=20dB.

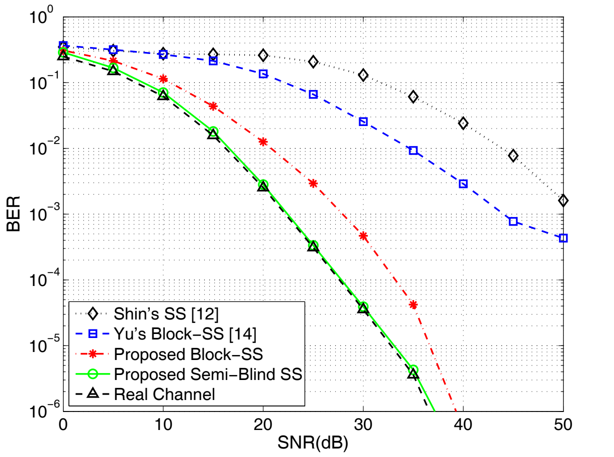

Fig. 5. BER vs. SNR for compared channel estimation methods with G=64, K=200..

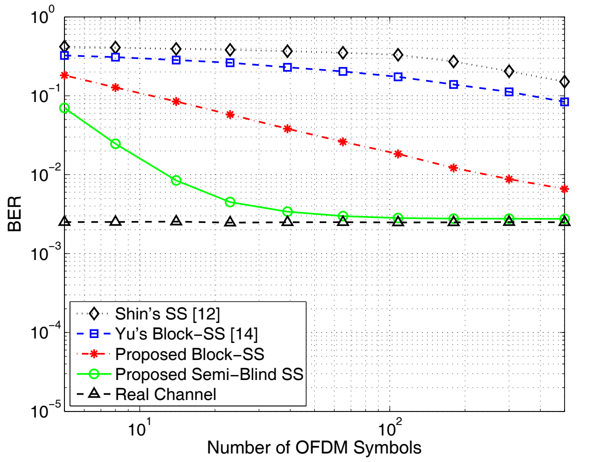

Fig. 6. BER vs. the number of OFDM symbols for compared channel estimation methods with G=64, SNR=20dB.