Modeling and Performance Evaluation of Small Cell Wireless Networks with Base Station Channels Breakdowns

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***Abstract*—The ever-increasing number of customers and the need for higher data rates and multimedia services require the deployment of Small Cell Networks. This paper considers modeling, performance evaluation and reliability of Small Cell Wireless Networks, taking into account the retrial phenomenon, finite number of customers served in a cell and channels breakdowns. The aim of this paper is to give a detailed performance and reliability analysis of these next-generation networks considering different breakdowns disciplines using Generalized Stochastic Petri nets formalism. The novelty of this investigation consists in the consideration of two breakdowns disciplines: channels breakdowns and base station (synchronous) breakdowns. For the first one, each channel is an independent working unit, and it can fail independently of other channels state. For the second one, the breakdowns are synchronous, hence all the channels of the base station fail down simultaneously. Hence, we show how this high level model allows us to cope with the complexity of such finite-source retrial networks, under the different breakdowns disciplines and how several steady-state performance and reliability indices can be derived. Through numerical examples, we discuss the effect of the network parameters on performance.**

Keywords-Small Cell Networks; Retrial phenomenon; Channels breakdowns; Base station breakdowns; Generalized Stochastic Petri nets; Performance and reliability indices.

I. INTRODUCTION

The ever-increasing number of customers and the need for higher data rates and multimedia services lead to stringent requirements on the bit rate/km2 that next-generation cellular wireless networks are expected to deliver. A promising approach to solving this problem is through the deployment of Small Cell Networks (SCNs), which represent a novel networking paradigm based on the idea of deploying short-range, low-power, and low-cost base stations (BSs) operating in conjunction with the main macro-cellular network infrastructure. Small Cells operate in licensed and unlicensed spectrum that have a range of 10 meter to 200 meters, compared to a mobile Macrocell which might have a range of a few kilometers. The use of SCNs is envisioned to enable next-generation networks to provide high data rates, allow offloading traffic from the macro cell and provide dedicated capacity to homes, enterprises, or urban hotspots. SCNs encompass a broad variety of cell types, such as micro, pico, femto cells, as well as advanced wireless relays and distributed antennas. Regarding compatible technologies, Small Cells are available for a wide range of air interfaces including GSM, CDMA2000, TD-SCDMA, W-CDMA, LTE and WiMax.

This paper considers modeling, performance evaluation and reliability of small cell wireless networks, taking into account the repeated calls of customers and channels breakdowns. Models with repeated calls (or retrial phenomenon) arise in various practical areas as telecommunication, computer networks and cellular mobile networks [1], [2], [3]. These models are based on the fact that, when servers are all busy or unavailable, customers attempting to get a service are not put in a queue but will try again to reach the servers after a random delay. Significant references reveal the non-negligible impact of repeated calls on the network performances. These repeated calls arise due to a blocking in a network with limited capacity resources or are due to impatience of customers. For a recent summary of the fundamental methods, results and applications on this topic, the reader is referred to [4], [5], [6].

To this end, we observe a wireless network where a supported area is divided into small cells, each of them is served by a base station having a limited number of channels which could be subject to breakdowns. These random breakdowns may have a heavy influence on the network quality of service. On the other hand, the number of mobiles (or customers) served in a cell is also small, such that models with a finite number of sources should be considered. These three aspects, customer retrial, finite number of sources and breakdowns of the base station channels, will be dealt with in this paper.

Although the reliability study is of great importance, there are only few works that take into consideration retrial phenomenon involving the unreliability of the servers, as it can be seen in the recent classified bibliography of Artalejo [6]. Moreover, most studies deal with single unreliable server retrial queueing systems [7], [8] or an infinite customers source [9].

Regarding finite-source retrial systems with unreliable multiple servers, we have found some few papers as [10]

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in which the servers are asymmetric (heterogenous) and the models are analyzed by queueing theory, and our recent paper [11] where retrial systems with servers breakdowns policy were analyzed using Generalized stochastic Petri nets (GSPNs) formalism. However, the several breakdowns mechanisms considered in the literature, can be classified as *servers breakdowns*.

In this paper, we propose the applicability of GSPNs for modeling and performance evaluation of Small Cell Networks (SCNs) with unreliable base station channels. The novelty of this investigation consists in the consideration of two breakdowns policies: servers (channels) breakdowns and station breakdowns. For the first one, each channel is an independent working unit, and it can fail independently of other channels state. For the second one, the breakdowns are synchronous, hence all the channels of the station fail down simultaneously (base station breakdown). This phenomenon occurs in practice, for example, when a system consists of several interconnected machines that are inseparable, or when all the machines are run by a single operator which may be fails at any time. In such situations, the whole station has to be treated as a single entity.

The paper is organized as follows: First, we give an overview of syntax and semantics of GSPNs formalism. In Section 3, we present the mathematical model describing the customers behavior in SCNs. Next, the GSPNs models for the different breakdowns disciplines are developed. In Section 5, we show how several steady-state performance and reliability indices can be derived. Then, based on numerical examples, we validate the proposed models with respect to the reliable case and we discuss the effect of the network parameters on performance. Finally, we give a conclusion.

II. Syntax and Semantics of Generalized Stochastic Petri Nets

In this section, we developed briefly the basics concepts of Generalized Stochastic Petri Nets formalism (GSPNs), that the readers are needed to better understand the proposed models describing small cell networks.

Generalized stochastic Petri nets [12], [13] are mathematical and graphical models, that are well suited for representing and analyzing stochastic and concurrent systems with synchronization characteristics.

A GSPN is a directed graph that consists of two kinds of nodes, called places (drawn as circles) and transitions that are partitioned into two different classes: timed transitions (represented by means of rectangles) describe the execution of time consuming activities and can fire only after a random delay characterized by a negative exponential probability distribution, and immediate transitions (represented by thin bars), which model logic activities as synchronization, have priority over timed transitions and fire in zero time once they are enabled. Formally, a GSPN can be defined as a seven-tuple $(P, T, I, O, Inh, M_0, W)$ where:

- P is the set of places;
- T is the set of timed and immediate transitions;
- $I, Inh : P \times T \to IN$ are the input and inhibitor functions, which provides the multiplicities of the input and inhibitor arcs from places to transitions (IN is the set of natural numbers);
- $O : T \times P \to IN$ is the output function which provides the multiplicities of the output arcs from transitions to places;
- $M_0 : P \to IN$ is the initial marking, which describes the initial state of the system;
- $W : T \to IR^+$ is a function that associates rates of negative exponential distribution to timed transitions and weights to immediate transitions.

An inhibitor arc is represented by a line terminating with a rounded head. The presence of a token in the inhibitor place inhibit the firing of the transition.

The system state is described by means of markings. A marking is a mapping from P to IN, which gives the number of tokens in each place. A transition is said to be enabled in a given marking, if and only if each of its normal input places contains at least as many tokens as the multiplicity of the connecting arc, and each of its inhibitor input places contains fewer tokens than the multiplicity of the corresponding inhibitor arc.

The firing of an enabled transition creates a new marking (state) of the net. The set of all markings reachable from initial marking M_0 is called the *reachability set*. The *reachability graph* is the associated graph obtained by representing each marking by a vertex and placing a directed edge from vertex M_i to vertex M_j, if marking M_j can be obtained by the firing of some transition enabled in marking M_i. This graph consists of *tangible markings* enabling only timed transitions and *vanishing markings* in which at least one immediate transition is enabled. Since the process spends zero time in the vanishing markings, they are eliminated from the reachability graph by merging them with their successor tangible markings [12]. This elimination results in a *tangible reachability graph*, which is isomorphic to a continuous time Markov chain (CTMC). The solution of this CTMC at steady-state is the stationary probability vector π which can be expressed as the solution of the linear system of equilibrium equations $\pi.Q = 0$ with the normalization condition $\sum_i \pi_i = 1$, where π_i denotes the steady-state probability that the process is in state M_i and Q is the infinitesimal generator. Having the probabilities vector π, we can compute several stationary performance indices of the system.

III. Mathematical Model

The mathematical model describing the customers behavior in small cell wireless networks can be viewed as a retrial model consisting of a multiserver service station, an imaginary waiting space called *orbit* and a finite source of K homogeneous customers. Each customer can be in one of the following states: free, under service or in orbit at any time. The probability that any particular customer generates a primary request for service in any interval $(t, t+dt)$ is $\lambda dt + o(dt)$ as $dt \to 0$ if the customer is free at time t. The service station consists of c identical (symmetric) servers subject to breakdowns and repairs. Each server can be in operational (up) or non-operational (down) state, and it can be idle or busy. If one of the servers is *up and idle* at the moment of the arrival of a call, then the call starts to be served immediately, the customer moves into the under service state and the server moves into busy state. The service times are independent and identically exponentially distributed with parameter μ. After service completion, the server becomes idle. Otherwise, if all the servers are busy or down at the arrival of a call, the customer joins the orbit and starts generation of a flow of repeated calls exponentially distributed with rate ν until it finds one operational free server.

A server can fail during the interval $(t, t+dt)$ with probability $\delta dt + o(dt)$ as $dt \to 0$ if it is idle, and with probability $\gamma dt + o(dt)$ if it is busy. In the literature, three breakdowns disciplines were defined:

- The **active breakdowns discipline**[10], [9] when $\delta = 0$ and $\gamma > 0$. This means that a server can fail only in busy state.
- The **independent breakdowns discipline**[10] when $\delta > 0$, $\gamma > 0$ and $\delta = \gamma$. In this case, a server can fail in busy or free state with the same probability.
- The **dependent breakdowns discipline** which we have proposed recently in [11]. In this case, the failure probability depends on the server state. Hence, the rates $\delta > 0$ and $\gamma > 0$ could be equal or not.

If the server fails in busy state, the interrupted customer returns to the orbit to resume service later. The repair time of a server is exponentially distributed with a finite mean $1/\tau$. We assume that the repairman follows FIFO discipline to fix up the servers breakdowns, repairs one server at a time and after repair, the server is as good as new.

The several breakdowns mechanisms studied in the literature, can be classified as *servers breakdowns*. In this paper, we introduce the *station breakdowns policy*, which describes systems with synchronous breakdowns of all servers of the station.

IV. GSPN Models of Small Cell Networks with Channels and Base Station Breakdowns

In the following, we present the GSPN models describing Small Cell Wireless Networks with different breakdowns disciplines.

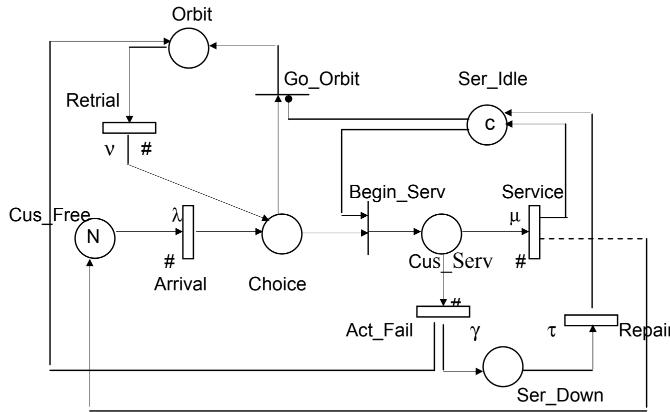

Figure 1. GSPN model of small cell wireless networks with retrials and active channels breakdowns

A. Retrial Networks with Active Channels Breakdowns

Figure 1 shows the GSPN describing a wireless network small cell with retrials and active breakdowns of channels. In this model, the place Cus_Free contains the free customers, the place $Orbit$ represents the orbit, Cus_Serv contains customers under service, Ser_Idle represents the operational free servers (channels) and the place Ser_Down contains non-operational (failed) servers.

The initial marking of the net is: $\{K, 0, 0, c, 0, 0\}$ which represents the fact that all customers are initially free, the c channels are operational free and the orbit is empty.

The firing of the transition $Arrival$ indicates the arrival of a primary call. The service semantics of this transition is ∞-servers (represented by the symbol $\#$ placed next to transition) because all free customers are able to generate primary calls. At the arrival of a primary or repeated call to the place $Choice$, if the place Ser_Idle contains at least one operational free channel, the immediate transition $Begin\_Serv$ fires. This firing represents the fact that the customer starts to be served and the server moves into busy state. However, the immediate transition Go_Orbit fires at the arrival of a call who finds no operational free channel i.e. Ser_Idle is empty. Hence, the customer joins immediately the place $Orbit$. Once in orbit, it starts generation of a flow of repeated calls exponentially distributed with rate ν. The firing of transition $Retrial$ represents the arrival of a repeated call from orbit.

When the timed transition $Service$ fires, the customer under service returns to free state (to the place Cus_Free) and the channel becomes idle and ready to serve another customer. The service semantics of transition $Service$ is ∞-servers because several channels can work simultaneously.

A channel can fail during a service period. This is represented by the fact that the transition Act_Fail fires before $Service$ (application of race policy). Thus, the interrupted customer joins the orbit and the failed channel joins the

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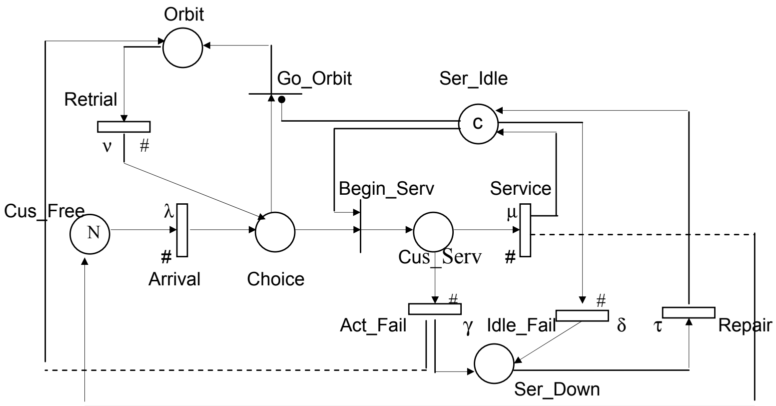

Figure 2. GSPN model of small cell wireless networks with retrials and dependent channels breakdowns

place Ser_Down where it will be immediately repaired. The firing of transition $Repair$ represents the end of the repair time. The repairman repairs one server at a time. Thus, the service semantics of this transition $Repair$ is *single*.

B. Retrial Networks with Dependent Channels Breakdowns

In the model describing dependent breakdowns of channels, depicted in Figure 2, during a service period, a channel can fail, which is represented by the firing of transition Act_Fail. In this case, the interrupted customer joins the orbit and the failed channel joins the place Ser_Down to be repaired. On the other hand, a channel can also fail if it is idle (in the place Ser_Idle). This is represented by the firing of transition $Idle\_Fail$ which has an ∞-servers semantics because several idle channels can also fail in the same time. The transitions Act_Fail and $Idle\_Fail$ representing the breakdown during busy and idle state respectively, may have the same ($\delta = \gamma > 0$) or different rates which correspond to independent and dependent breakdowns disciplines respectively.

C. Retrial Networks with Base Station Breakdowns

In models considering base station breakdowns, all the channels fail down simultaneously and they also return to the operational state at the same time, when the base station is repaired. Hence, the corresponding GSPN models vary slightly from the previous ones. In fact, the models are the same as those given in Figure 1 and Figure 2, in which the multiplicity of the arcs connecting the place Cus_Serv to transition Act_Fail, Act_Fail to the places Ser_Down and $Orbit$, Ser_Down to the transition $Repair$ and the transition $Repair$ to place Ser_Idle equals c (rather that 1), because the c active channels fail down at the same time. The firing of transition Act_Fail will move c tokens in the place Ser_Down, which represents the breakdown of all base station. At the end of the reparation period (after a mean delay of $1/\gamma$), c tokens corresponding to the c channels will be deposited in Ser_Idle. Moreover, in Figure 2, to model the possibility that all channels of the station fail down when being in idle state, we should modify the multiplicity of the arcs connecting the place Ser_Idle to transition $Idle\_Fail$ and $Idle\_Fail$ to place Ser_Down (c rather than 1). The service semantics of the transitions Act_Fail and $Idle\_Fail$ is *single-server semantics*, because the base station is a single unit. Hence, the symbols $\#$ should be omitted from these two transitions.

V. Performance and Reliability Analysis

The aim of this study is twofold. Firstly, we have to verify the correctness of our models and their ergodicity. Next, we derive the formulas of the most important steady-state performance indices.

The primordial qualitative property we have to verify is the *boundness* of the proposed models. This property ensures that each place of the net is bounded and so the model state space is finite. The second important qualitative property is the *liveness*. A transition t is live if from any reachable marking, there is a reachable marking enabling t. Thus, t is live implies that the activity modeled by this transition can always take place from any state. In the proposed models, all transitions are live. Finally, another interesting qualitative property we had to check is the presence of *home states*.

The proposed GSPN models are bounded, live and the initial marking is a home state. Thus, the underlying continuous time Markov chains are ergodic. Hence, the steady-state probability distribution vector π exists and is unique. Once this probability vector is computed, several performance and reliability indices of small cell wireless networks with retrial phenomenon and different breakdowns disciplines can be derived as follows. In these formulas, $M_i(p)$ denotes the number of tokens in place p in marking M_i, A the set of reachable tangible markings, and $A(t)$ is the set of tangible markings reachable by transition t and $E(t)$ is the set of markings where the transition t is enabled.

- `Mean number of busy channels` (n_s): This corresponds to the mean number of tokens in the place Cus_Serv which is also the mean number of customers under service.

$$n_s = \sum_{i:M_i \in A} M_i(Cus\_Serv).\pi_i$$

- `Mean number of customers in orbit` (n_o): This correspond to the mean number of tokens in the place $Orbit$ which models the orbit.

$$n_o = \sum_{i:M_i \in A} M_i(Orbit).\pi_i$$

- `Mean number of operational free channels` (n_d): This represents the average number of tokens in the place Ser_Idle.

$$n_d = \sum_{i:M_i \in A} M_i(Ser\_Idle).\pi_i$$

- `Mean number of failed channels` (n_f): This represents the mean number of tokens in the place Ser_Down.

$$n_f = \sum_{i:M_i \in A} M_i(Ser\_Down).\pi_i = s - (n_s + n_d)$$

- `Mean rate of generation of primary calls` $(\overline{\lambda})$: This represents the throughput of the transition $Arrival$.

$$\overline{\lambda} = \sum_{i:M_i \in A(Arrival)} \lambda.M_i(Cus\_Free).\pi_i$$

- `Mean rate of generation of repeated calls` $(\overline{\nu})$: This represents the retrial frequency of customers in orbit. It corresponds to the throughput of the transition $Retrial$.

$$\overline{\nu} = \sum_{i:M_i \in A(Retrial)} \nu.M_i(Orbit).\pi_i$$

- `Mean rate of service` $(\overline{\mu})$: This represents the throughput of the transition $Service$.

$$\overline{\mu} = \sum_{i:M_i \in A(Service)} \mu.M_i(Cus\_Serv).\pi_i$$

- `Mean rate of repair` $(\overline{\tau})$: This represents the throughput of the transition $Repair$.

$$\overline{\tau} = \sum_{i:M_i \in A(Repair)} \tau.M_i(Ser\_Down).\pi_i$$

- `Blocking probability of a primary call` (B_p):

$$B_p = \frac{\sum_{j:M_j \in A} \sum_{i=1}^{K-s} i.\lambda.Prob[M_j(Cus\_Free) = i \& M_j(Ser\_Idle) = 0]}{\overline{\lambda}}$$

- `Blocking probability of a repeated call` (B_r):

$$B_r = \frac{\sum_{j:M_j \in A} \sum_{i=1}^{K-s} i.\nu.Prob[M_j(Orbit) = i \& M_j(Ser\_Idle) = 0]}{\overline{\nu}}$$

- `Blocking probability` (B):

$$B = B_p + B_r$$

- `Utilization of` s `channels` (U_s): $(1 \leq s \leq c)$ This corresponds to the probability that s servers are busy :

$$U_s = \sum_{i:M_i(Cus\_Serv) \geq s} \pi_i$$

- `Availability of` s `channels` (A_s): $(1 \leq s \leq c)$ This corresponds to the probability that s servers are operational and idle.

$$A_s = \sum_{i:M_i(Ser\_Idle) \geq s} \pi_i$$

- `Failure probability of` s `channels` (F_s): $(1 \leq s \leq c)$ This corresponds to the probability that s servers are failed:

$$F_s = \sum_{i:M_i(Ser\_Down) \geq s} \pi_i$$

- `Utilization of the repairman` (U_r): This corresponds to the probability that at least one server is failed:

$$U_r = F_1 = \sum_{i:M_i(Ser\_Down) \geq 1} \pi_i$$

- `Failure frequency of busy channels` $(\overline{\gamma})$): This represents the throughput of the transition $Failure$ (or $Fail\_Act$) for active breakdowns case and dependent breakdowns case respectively.

$$\overline{\gamma} = \begin{cases} \sum_{i:M_i \in A(Failure)} \gamma.M_i(Cus\_Serv).\pi_i, \\ \text{in active breakdowns,} \\ \sum_{i:M_i \in A(Fail\_Act)} \gamma.M_i(Cus\_Serv).\pi_i, \\ \text{in dependent breakdowns.} \end{cases}$$

- `Failure frequency of idle channels` $(\overline{\delta})$): This represents the throughput of the transition $Fail\_Idle$.

$$\overline{\delta} = \sum_{i:M_i \in A(Fail\_Idle)} \delta.M_i(Ser\_Idle).\pi_i$$

- `Mean waiting time` $(\overline{W})$: The mean waiting time $\overline{W}$ of the customers in the steady state, can be easily obtained with the help of Little's formula:

$$\overline{W} = n_o / \overline{\lambda}$$

- `Mean response time` $(\overline{R})$:

$$\overline{R} = (n_o + n_s) / \overline{\lambda}$$

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Table I
VALIDATION OF RESULTS IN RELIABLE CASE

	Reliable model	Active breakdowns of servers	Active breakdowns of station	Dependent breakdowns of servers	Dependent breakdowns of station
Population size	20	20	20	20	20
Number of servers	4	4	4	4	4
Primary call generation rate	0.1	0.1	0.1	0.1	0.1
Service rate	1	1	1	1	1
Retrial rate	1.2	1.2	1.2	1.2	1.2
Server's failure rate	-	1e-25	1e-25	1e-25	1e-25
Server's repair rate	-	1e+25	1e+25	1e+25	1e+25
Mean number of busy servers	1.800748	1.800764	1.800764	1.800763	1.800763
Mean number of customers in orbit	0.191771	0.191786	0.191786	0.191785	0.191785
Mean rate of customers arrivals	1.800748	1.800745	1.800745	1.800745	1.800745
Mean response time	1.106495	1.1065036	1.1065036	1.1065031	1.1065031

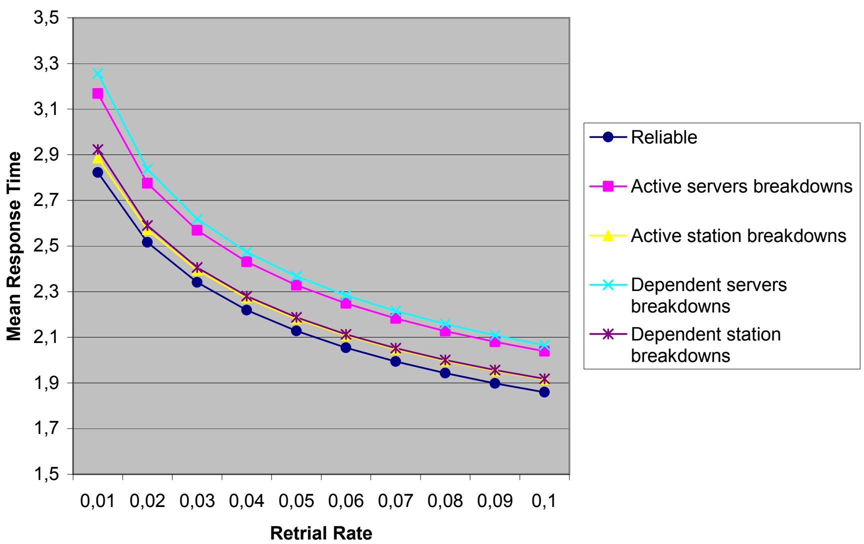

Figure 3. Effect of Retrial Rate on the Mean Response Time with $K = 50$, $c = 4$, $\lambda = 0.4$, $\mu = 5$, $\gamma = 0.02$, $\delta = 0.01$, $\tau = 0.5$

VI. VALIDATION OF MODELS

In this section, we consider some numerical results, to validate the proposed models. To this aim, the results obtained in the reliable case were compared to those obtained by the Pascal program given in the book of Falin and Templeton [4], since if the failure rate in non-reliable models is very low and repair rate is very high, the measures should approach the corresponding ones in reliable models.

In Table 1, we can see that the corresponding performance measures for the model with active channels breakdowns, the model with active station breakdowns, the model with dependent channels breakdowns and the model with dependent station breakdowns are very close to the reliable case. In fact, the derived results are the same up to the 4th decimal digit.

In Figures 3 and 4, the mean response time is splotted versus the retrial rate ν and channels number c respectively. We have presented five curves which correspond to the reliable case, the active and independent channels and station breakdowns disciplines. From Figure 3, we can see how much the increase of retrial rate affects the mean response time which decreases in reliable case and for the different breakdowns disciplines. The numerical results agree with the intuition that the mean response time is better (lower) in the reliable case for all values of the retrial rate. It is also shown from this figure that among the four breakdowns disciplines, the model with active breakdowns of base station gives the best mean response times particularly when the retrial rate is smaller, but when the repeated calls arrive more frequently, the two station breakdowns disciplines (active and dependent) are very close.

In Figure 4, it is demonstrated that the channels number of the base station has a significant influence on the mean response time. We can also see that a small change in

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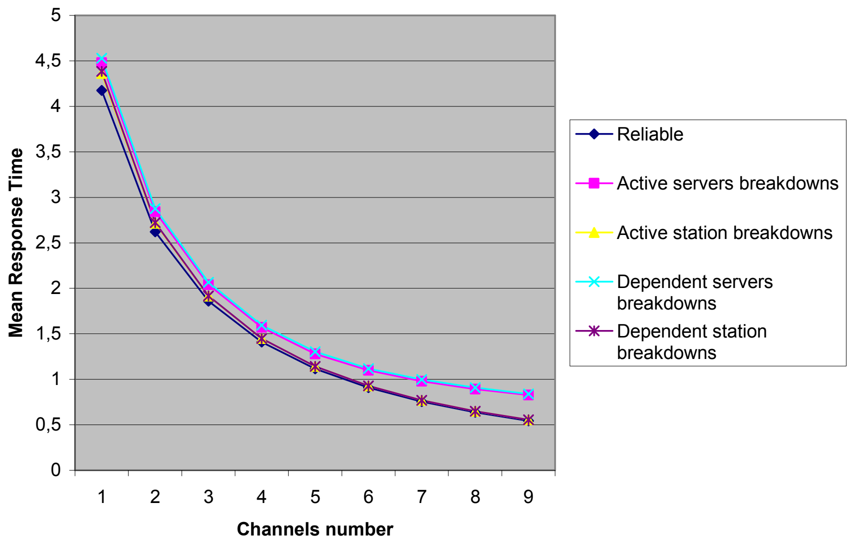

Figure 4. Effect of Channels Number on the Mean Response Time with $K = 50$, $\lambda = 0.4$, $\mu = 5$, $\nu = 1$, $\gamma = 0.02$, $\delta = 0.01$, $\tau = 0.5$

the number of channels, particularly from 1 to 3, produces big difference in the mean response time in reliable and unreliable cases ($\approx -55\%$). However, after a certain value the decrease is not considerable. On the other hand, we can observe that the models with base station breakdowns give the best results, and the worst response time is obtained in dependent breakdowns of channels discipline.

VII. Conclusion

The exponentially increasing demand for wireless data services requires a massive network densification. A promising solution to this problem is the concept of Small Cell Networks, which is founded on the idea of a very dense deployment of short-range, low-cost and low-power base stations.

This paper aims at modeling, performance evaluation and reliability of Small Cell Wireless Networks, taking into account the repeated calls of blocked customers, the finite number of customers served in a cell and the breakdowns of base station channels. Hence, we showed how the behavior of customers in Small Cell Wireless Networks with different breakdowns disciplines can be intuitively described using Generalized Stochastic Petri nets formalism and how several performance and reliability indices can be derived.

References

[1] J. Roszik, J. Sztrik and C. Kim, *Retrial queues in the performance modeling of cellular mobile networks using MOSEL*, Inter. Journal of Simulation: Systems, Science and Technology, vol. 1-2, pp. 38-47, 2005.

[2] J. R. Artalejo and M. J. Lopez-Herrero, *Cellular mobile networks with repeated calls operating in random environment*,Computers & operations research, vol. 37, no7, pp. 1158-1166, 2010.

[3] V. D. Tien, *A new computational algorithm for retrial queues to cellular mobile systems with guard channels*, Computers & Industrial Engineering, vol. 59, pp. 865-872, 2010.

[4] G. I. Falin and J. G. C. Templeton, *Retrial Queues*, Chapman and Hall, London, 1997.

[5] J. R. Artalejo and A. Gómez-Corral, *Retrial Queueing Systems: A Computational Approach*, Springer, Berlin, 2008.

[6] J. R. Artalejo, *Accessible bibliography on retrial queues: Progress in 2000-2009*, Mathematical and Computer Modelling, vol. 51, pp. 1071-1081, 2010.

[7] J. Sztrik and D. Efrosinin, *Tool supported reliability analysis of finite-source retrial queues*, Automation and Remote Control, vol. 71, pp. 1388-1393, 2010.

[8] J. Sztrik and C. S. Kim, *Tool supported performability investigations of heterogeneous finite-source retrial queues*, Annales Univ. Sci. Budapest., Sect. Comp., vol. 32, pp. 201-220, 2010.

[9] J. Wang, J. Cao and Q. Li, *Reliability analysis of the retrial queue with server breakdowns and repairs*, Queueing Systems, vol. 38, pp. 363-380, 2001.

[10] J. Roszik and J. Sztrik, *Performance analysis of finite-source retrial queues with nonreliable heterogenous servers*, Journal of Mathematical Sciences, vol. 146, pp. 6033-6038, 2007.

[11] N. Gharbi and C. Dutheillet, *An algorithmic approach for analysis of finite-source retrial systems with unreliable servers*, Computers and Mathematics with Applications, vol. 62, pp. 2535-2546, 2011.

[12] M. Ajmone Marsan, G. Balbo, G. Conte, S. Donatelli, and G. Franceschinis, *Modelling with Generalized Stochastic Petri Nets*, John Wiley & Sons, New York, 1995.

[13] M. Diaz, *Les réseaux de Petri - Modèles Fondamentaux*, Paris, Hermès Science Publications, 2001.

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