An Application of Second-Order Reed-Muller Codes for Multiple Target Localization in Wireless Sensor Networks

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***Abstract*—Compressive sensing is a new signal processing technique for efficient reconstruction of an n-dimensional signal from m ($m \ll n$) measurements. Most of compressive sensing researches are based on randomization, while research on deterministic sampling is essential for practical implementation. In this paper, we study $m \times n$ deterministic binary sensing matrices using second-order Reed-Muller codes, which satisfy a statistically restricted isometry property with reduced complexity for an application of multiple target localization in wireless sensor networks. We formulate multiple target locations as a sparse matrix, then exploit received signal strength information to recover noisy information using the deterministic sensing matrices and greedy algorithms to locate each target. The simulation results show that our scheme also achieves high accuracy in terms of localization errors when compared to traditional approaches with the random sensing matrices.**

***Keywords*—wireless sensor network; multiple target localization; compressive sensing; deterministic sensing matrix; Reed-Muller codes.**

I. Introduction

Wireless sensor networks can play a key role in target tracking, monitoring, environmental sensing and some other applications. Many approaches for network localization based on local sensor information have been developed with low-cost, low-power and small size constraints [1]–[8]. In this paper, we consider a scenario that the nodes are randomly deployed in a large area, and we determine multiple target locations based on the Received Signal Strength (RSS) information from their neighbors. Some of existing localization algorithms for this scenario are inefficient, since they require a large number of data between transmitter and receiver [9]–[14], [18]. Fortunately, the compressive sensing can help us to overcome these problems. The goal of compressive sensing is to recover an unknown signal vector $\mathbf{x} \in \mathbb{R}^n$ from linear measurement $\mathbf{y}$ obtained by

$$\mathbf{y} = \mathbf{\Phi x}, \tag{1}$$

where $\mathbf{\Phi} = \{\mathbf{\Phi}_i\}_{i=1}^m \in \mathbb{R}^{m\times n}$ is the sensing matrix. The most concern is when the number of measurement m is much smaller than n, i.e., $m \ll n$. In this case, finding an exact solution $\mathbf{x}$ based on the measurement $\mathbf{y}$ is an ill-posed problem since the system of equations is under-determined. However, we can deal with it by finding an approximation of $\mathbf{x}$ by solving this problem as

$$\min_{\mathbf{x}} ||\mathbf{x}||_0 \qquad \text{subject to} \quad \mathbf{\Phi x} = \mathbf{y}, \tag{2}$$

where $||\mathbf{x}||_0 = |\text{supp}(\mathbf{x})|$, and a vector is called k-sparse if it has at most k nonzeros elements. The compressive sensing technique guarantees exact recovery of the original signal $\mathbf{x}$ with high probability if the sensing matrices satisfy the Restricted Isometry Property (RIP). That is, for a fixed k, there exists a small number $\delta_k \in (0, 1)$ such that

$$(1-\delta_k)||\mathbf{x}_k||_2^2 \le ||\mathbf{\Phi x}_k||_2^2 \le (1+\delta_k)||\mathbf{x}_k||_2^2 \tag{3}$$

for any k-sparse $\mathbf{x}_k$. Hence, the problem (2) can be solved either by using greedy algorithms such as Basis Pursuit (BP) [15], Orthogonal Matching Pursuit (OMP) [16], [17], or replaced by solving for sparse signal via l_1 minimization as

$$\min_{\mathbf{x}} ||\mathbf{x}||_1 \qquad \text{subject to} \quad \mathbf{\Phi x} = \mathbf{y}. \tag{4}$$

In traditional compressive sensing approach, researchers have used random projection for the sensing matrices $\mathbf{\Phi}$ to obtain the measurement $\mathbf{y}$, since the RIP can be satisfied with some random matrices with their entries following Gaussian process, Bernoulli process, etc. [19], [20]. Thus, a k-sparse signal $\mathbf{x} \in \mathbb{R}^n$ can be exactly reconstructed from m measurements. However, random matrices have many drawbacks: significant space requirement for storage, no efficient algorithm to verify the RIP, hard to deployment in many applications, to name a few. To this end, designing deterministic sensing matrices is essential for practical implementation. Recently, many advantages of deterministic sensing matrices have been shown. The most of these advantages is their fast and efficient reconstruction nature. In [21], Calderbank constructed some statistical RIP conditions such as Statistical Restricted Isometry Property (StRIP) and Uniqueness-guaranteed Statistical Restricted Isometry Property (UStRIP). These are weaker versions of the RIP that allow to construct deterministic sensing matrices. In [22], DeVore gave a generalization of construction via algebraic curves over finite fields. The author constructed binary sensing matrices of size $p^2 \times 2^{p+1}$ by using polynomials over finite field $\mathbb{F}_p$. This idea has been developed in many researches [23]. By choosing appropriate algebraic curves, these deterministic sensing matrices were better than DeVore's one. In [24], an application of coding theory in compressive sensing was presented, where a fast reconstruction algorithm for deterministic compressive sensing using second-order Reed-Muller codes was proposed. The matrix $\mathbf{\Phi}$ is said to satisfy the StRIP (k, δ) if

$$\mathbf{Pr}\left\{ \Big| ||\mathbf{\Phi x}||^2 - ||\mathbf{x}||^2 \Big| \le \delta ||\mathbf{x}||^2 \right\} \ge 1 - \epsilon, \tag{5}$$

holds with probability exceeding $1-\delta$, and we assume that $\mathbf{x}$ distributes uniformly among k-sparse vectors. They showed that if $\mathbf{\Phi}$ satisfies the StRIP respect to the parameters ϵ and δ, high probability reconstruction is also guaranteed. The deterministic sensing matrices formed by Reed-Muller codes, Bose-Chaudhuri-Hocquenghem (BCH) codes and some others can achieve this StRIP condition.

In this paper, we study the construction of deterministic sensing matrices formed by second-order Reed-Muller codes and how to apply this theory to multiple target localization in wireless sensor networks. We formulate each target location as a sparse vector in the discrete spatial domain. Then, we measure the RSS information from the targets and apply the construction of deterministic sensing matrix formed by second-order Reed-Muller codes as the measurement matrix. These matrices satisfy the StRIP, so that the approximated solution of (2) can be obtained by using a recovery algorithm in the last step.

The organization of the paper is as follows. In Section II, we explain a motivation of developing real-valued second-order Reed-Muller codes for deterministic sensing matrices in compressive sensing. In Section III, we formulate the multiple target localization problem as an application of compressive sensing by using the sensing matrices dealt in Section II. Numerical results are considered in Section IV, followed by concluding remarks in Section V.

II. Real-valued second-order Reed-Muller codes in deterministic sensing matrix construction

A. Main construction

Recall that for any two binary vector $\mathbf{a}=(a_0,\cdots,a_{p-1})$ and $\mathbf{b}=(b_0,\cdots,b_{p-1})$ in $\mathbb{Z}_2^p$, the inner product is defined as

$$\mathbf{a}\cdot\mathbf{b}=\mathbf{a}^T\mathbf{b}=\sum_{i=1}^{p-1}a_ib_i \mod 2, \tag{6}$$

where $(\cdot)^T$ denote the transpose operation. The second-order Reed-Muller code is given as follows.

$$\phi_{\mathbf{P},\mathbf{b}}(\mathbf{a})=\frac{(-1)^{w(\mathbf{b})}}{\sqrt{2^p}}i^{(2\mathbf{b}+\mathbf{P}\mathbf{a})^T\mathbf{a}} \tag{7}$$

where $\mathbf{P}$ is a $p\times p$ binary symmetric matrix, $\mathbf{b}$ is a $p\times 1$ binary vector in $\mathbb{Z}_p^2$ and $w(\mathbf{b})$ is the weight of $\mathbf{b}$, i.e., number of bit-1 entries. For given matrix $\mathbf{P}$ and vector $\mathbf{b}$, the second-order Reed-Muller code is a $2^p\times 1$ vector. For implementation purposes, the matrices $\mathbf{P}$ are set as all-zero matrices or the matrices with zero-diagonals. Thus, there is only $2^{p(p-1)/2}$ matrices $\mathbf{P}$ satisfying this condition, which are $\{\mathbf{P_1},\cdots,\mathbf{P_{2^{p(p-1)/2}}}\}$ and the functions $\{\phi_{\mathbf{P},\mathbf{b}}(\mathbf{a})\}$ are real-valued. The set

$$\mathcal{F}_{\mathbf{P}}=\{\phi_{\mathbf{P},\mathbf{b}}|\mathbf{b}\in\mathbb{Z}_2^p\} \tag{8}$$

forms a basis of $\mathbb{Z}_2^p$. The inner product on $\mathcal{F}_{\mathbf{P}}$ is defined as follows. For any two vectors $\phi_{\mathbf{P},\mathbf{b}}$ and $\phi_{\mathbf{P}',\mathbf{b}'}$ in $\mathcal{F}_{\mathbf{P}}$

$$\langle\phi_{\mathbf{P},\mathbf{b}},\phi_{\mathbf{P}',\mathbf{b}'}\rangle=\begin{cases}\frac{1}{\sqrt{2^q}} & 2^q \quad \text{times},\\ 0 & 2^p-2^q \quad \text{times},\end{cases} \tag{9}$$

where $q=\text{rank}(\mathbf{P}-\mathbf{P}')$. The deterministic sensing matrix in [25] has the form

$$\mathbf{\Phi}_{RM}=\begin{bmatrix}\mathbf{U_{P_1}} & \mathbf{U_{P_2}} & \cdots & \mathbf{U_{P_{2^{p(p-1)/2}}}}\end{bmatrix}_{2^p\times 2^{p(p+1)/2}} \tag{10}$$

where $\mathbf{U_{P_i}}$ is unitary matrix correspoding to $\mathcal{F}_{P_i}$, $i=1,\cdots,2^{p(p-1)/2}$. Note that if we set $m=2^p$ and $n=2^{p(p+1)/2}$, we get an $m\times n$ sensing matrix $\mathbf{\Phi}_{RM}$. The reconstruction problem using this matrix is to reconstruct the k-sparse vector $\mathbf{x}$ from the data $\mathbf{y}$ given by

$$\mathbf{y}=\mathbf{\Phi}_{RM}\mathbf{x}. \tag{11}$$

In [26], the Delsarte-Goethals sets $DG(p,r)$ provide some restricted conditions for set of matrices $\mathbf{P}$. The set $DG(p,r)$ is a collection of $p\times p$ binary symmetric matrices with property that for any distinct matrices $\mathbf{P},\mathbf{Q}\in DG(p,r)$, the rank of $\mathbf{P}+\mathbf{Q}$ is greater or equal to $p-2r$. This implies that these sets are nested

$$DG(p,0)\subset DG(p,1)\subset\cdots\subset DG\left(p,\frac{p-1}{2}\right). \tag{12}$$

The set $DG(p,0)$ is called Kerdock set [27]. Setting $\mathbf{P}$ to range over $DG(p,(p-1)/2)$, the sensing matrices made from the matrices $\mathbf{P}$ are the matrices of size $2^p\times 2^{p(r+2)}$.

B. Matrices with construction guarantee

Since the deterministic designs are based on the implemented and practical aspects, we focus on the sensing matrices whose entries are ± 1 by removing the normalization factor of $1/\sqrt{2^p}$ in $\mathbf{\Phi}_{RM}$

$$\hat{\mathbf{\Phi}}=\sqrt{m}\mathbf{\Phi}_{RM}. \tag{13}$$

Let us denote $\mu(\mathbf{A})$ as the largest magnitude of entries $\mathbf{A}$.

$$\mu(\mathbf{A})=\max_{k,j}|\mathbf{A}_{k,j}|. \tag{14}$$

Thus $\mu(\hat{\mathbf{\Phi}})=1$. For a fixed signal $\mathbf{x}\in\mathbb{R}^n$ where $||x||_0=k$ the recovery in (2) is exact with high probability for the number of observations $m\geq C\cdot k\cdot\log n$ where C is a known small constant. We have

$$\hat{\mathbf{\Phi}}^*\hat{\mathbf{\Phi}}=2^{p(p+1)/2}\mathbf{I}=n\mathbf{I},$$

where $(\cdot)^*$ denotes conjugate operation. For a small value of $\delta\in(0,1)$, the eigeinvalues of $\hat{\mathbf{\Phi}}^*\hat{\mathbf{\Phi}}$ are close to n with high probability. Thus, $||\frac{1}{n}\hat{\mathbf{\Phi}}^*\hat{\mathbf{\Phi}}-\mathbf{I}||_2\leq 1/n$, and we have

$$\mathbf{Pr}\left\{\left|||\hat{\mathbf{\Phi}}\mathbf{x}||^2-||\mathbf{x}||^2\right|\leq\delta||\mathbf{x}||^2\right\}\geq 1-\frac{1}{n}, \tag{15}$$

Hence, the matrix $\hat{\mathbf{\Phi}}$ satisfies the StRIP with sparsity k and $\epsilon=\frac{1}{n}$. We can find some further information on binary symmetric matrices formed by second-order Reed-Muller codes in [25], [28], [29].

C. Examples

Let $p = 2$, then $\mathbb{Z}_2^2 = \left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}$. There is only $2^{2(2-1)/2} = 2$ binary symmetric matrices $\mathbf{P}$ of size 2×2 satisfying the condition. These are

$$\mathbf{P}_1 = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, \mathbf{P}_2 = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \tag{16}$$

Thus, the corresponding unitary matrices $\mathbf{U}_{\mathbf{P}_1}$ and $\mathbf{U}_{\mathbf{P}_2}$ are

$$\mathbf{U}_{\mathbf{P_1}} = \frac{1}{2}\begin{bmatrix}1 & -1 & -1 & 1\\1 & 1 & -1 & -1\\1 & -1 & 1 & -1\\1 & 1 & 1 & 1\end{bmatrix}, \mathbf{U}_{\mathbf{P_2}} = \frac{1}{2}\begin{bmatrix}1 & -1 & -1 & 1\\1 & 1 & -1 & -1\\1 & -1 & 1 & -1\\-1 & -1 & -1 & -1\end{bmatrix}$$

Hence, we get the deterministic sensing matrix $\hat{\mathbf{\Phi}}$ as

$$\hat{\mathbf{\Phi}} = \begin{bmatrix}1 & -1 & -1 & 1 & 1 & -1 & -1 & 1\\1 & 1 & -1 & -1 & 1 & 1 & -1 & -1\\1 & -1 & 1 & -1 & 1 & -1 & 1 & -1\\1 & 1 & 1 & 1 & -1 & -1 & -1 & -1\end{bmatrix}_{2^2 \times 2^3}.$$

III. An application on multiple target localization in Wireless Sensor Networks

A. Problem formulation

Consider an area which is divided into a discrete grid with n points. Denote k as the number of targets which are located in this area. Each target is $n \times 1$ vector whose elements are zeros, except 1 at the index of grid point where target is located. With k targets, we get a matrix of target locations over the grid as

$$\boldsymbol{\theta} = [\boldsymbol{\theta}_1 \quad \boldsymbol{\theta}_2 \quad \cdots \quad \boldsymbol{\theta}_k]_{n \times k}. \tag{17}$$

We take m measurements respect to each target under matrix Ψ, which will be explained in the next subsection. Then the RSS signals are given by

$$\underbrace{\begin{bmatrix}\mathbf{x}_1\\\mathbf{x}_2\\\vdots\\\mathbf{x}_k\end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix}\Psi_1 & 0 & \cdots & 0\\0 & \Psi_2 & \cdots & 0\\\vdots & \ddots & \ddots & 0\\0 & \cdots & \cdots & \Psi_k\end{bmatrix}}_{\mathbf{\Psi}} \underbrace{\begin{bmatrix}\boldsymbol{\theta}_1\\\boldsymbol{\theta}_2\\\vdots\\\boldsymbol{\theta}_k\end{bmatrix}}_{\boldsymbol{\theta}} \tag{18}$$

We can describe the compression procedure as follows.

$$\underbrace{\begin{bmatrix}\mathbf{y}_1\\\mathbf{y}_2\\\vdots\\\mathbf{y}_k\end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix}\hat{\Phi}_1 & 0 & \cdots & 0\\0 & \hat{\Phi}_2 & \cdots & 0\\\vdots & \ddots & \ddots & 0\\0 & \cdots & \cdots & \hat{\Phi}_k\end{bmatrix}}_{\hat{\mathbf{\Phi}}} \underbrace{\begin{bmatrix}\mathbf{x}_1\\\mathbf{x}_2\\\vdots\\\mathbf{x}_k\end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix}\mathbf{n}_1\\\mathbf{n}_2\\\vdots\\\mathbf{n}_k\end{bmatrix}}_{\mathbf{n}}. \tag{19}$$

Our goal is to find all the locations of these targets with an accurate, fast and efficient algorithm with a small value of m.

B. Localization process

The matrix form of (19) is

$$\mathbf{y} = \hat{\mathbf{\Phi}}\mathbf{\Psi}\boldsymbol{\theta} + \mathbf{n}. \tag{20}$$

These matrices $\mathbf{\Psi}, \hat{\mathbf{\Phi}}$ are generated as follows.

- RSS matrix $\mathbf{\Psi} = \text{diag}\{\Psi_1, \cdots, \Psi_k\}$ is made from using the radio propagation channel model

$$\begin{aligned}\{\Psi_i\}_{uv} &= PL(d_{uv})\\ &= PL(d_0) - 10n_p \log_{10}\left(\frac{d_{uv}}{d_0}\right), u, v = 1, \cdots, n,\end{aligned} \tag{21}$$

  where $d_0 = 1$m is the reference distance, d_{uv} is the real distance between transmitter and receiver in meters, $PL(d_0)$ is computed using the free space path loss equation, and n_p is the path loss component.

- In the sensing matrix $\hat{\mathbf{\Phi}} = \text{diag}\{\Phi_1, \cdots, \Phi_k\}$, each $\Phi_i (i = 1, \cdots, k)$ is generated by the matrices satisfying the StRIP, as we have discussed in the previous section.

According to the compressive sensing theory, the localization problem is stated as the recovery of a sparse signal $\mathbf{x}_i \quad (i = 1, \cdots, k)$ from measurement $\mathbf{y}_i$, which is equivalent to reconstruct target location $\boldsymbol{\theta}_i$ from $\mathbf{y}_i$. Assume that $||\mathbf{n}||_2 \le \epsilon$ where ϵ is a small positive constant. Since each signal $\mathbf{x}_i$ is represented by a sparsity basis, each sparse vector $\boldsymbol{\theta}_i$ can be found either by solving the following l_1-minimization problem

$$\hat{\boldsymbol{\theta}} = \arg\min ||\boldsymbol{\theta}||_1 \quad \text{subject to} \quad \mathbf{y} = \mathbf{\Theta}\boldsymbol{\theta}, \tag{22}$$

where $\mathbf{\Theta} = \hat{\mathbf{\Phi}}\mathbf{\Psi}$, or by solving the convex optimization program by calling the OMP algorithm as

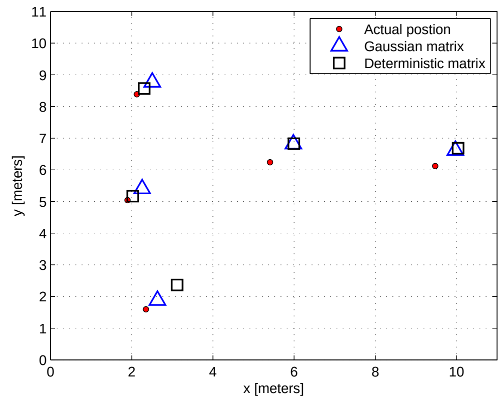
$$\hat{\boldsymbol{\theta}} = \text{OMP}(\mathbf{y}, \mathbf{\Theta}, \epsilon). \tag{23}$$

The main idea of the OMP algorithm is to find the columns of the matrix $\mathbf{\Theta}$ whose linear combination is close to $\mathbf{y}$. The OMP is more simpler and faster than other alternatives. In this paper, we have applied this algorithm to simulate and to generate sensing matrices $\hat{\mathbf{\Phi}}$. To improve the performance and to find the exact target locations in the grid points, a threshold has been defined to select the largest component in the location vector of n components with minimum overal distance error.

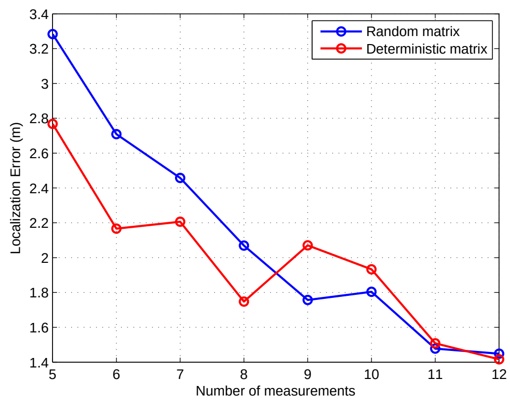
IV. Numerical Results

In this section, we examine the performance of multiple target localization using compressive sensing under random matrices and deterministic matrices formed by second-order Reed-Muller codes in an indoors environment. We randomly deployed M sensors in an area with the size of 10m×10m with N grid points, and placed the targets by randomly selecting k grid points in uniform manners. We added Gaussian noises with zero mean and standard deviation of 0.05 to the observation $\mathbf{y}$. We used the Average Localization Error (ALE) to quantify the localization accuracy, which is defined as

$$\text{ALE}(\mathbf{p}) = \frac{1}{k}\sqrt{||\mathbf{p} - \mathbf{p}^*||_2^2}, \tag{24}$$

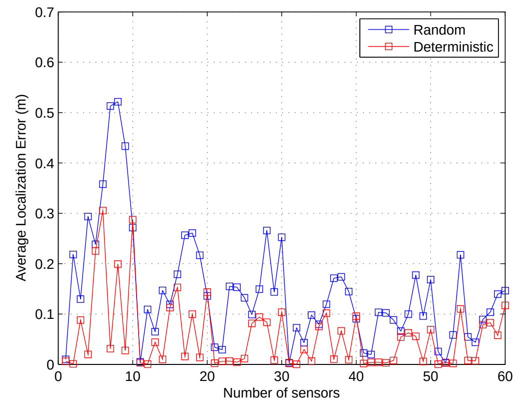


(a) Position recovery performance with $M = 8$.

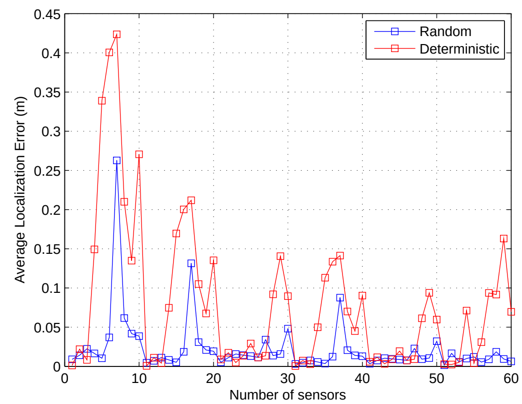


(b) Average localization error respect to 5 targets.

Fig. 1. Localization of 5 targets under random sensing matrix and deterministic sensing matrix.



(a) 8×64 sensing matrix.



(b) 32×64 sensing matrix.

Fig. 2. Average localization error.

where $\mathbf{p}$ is the actual point and $\mathbf{p}^*$ is the estimated point. Each run is collected 100 times. In the simulations, each RSS information $\Psi_i (i = 1, \cdots, k)$ was obtained by

$$\{\Psi_i\}_{uv}(d) = -46.2 - 10n_p \log_{10}(d), u, v = 1, \cdots, n, \quad (25)$$

Each location was observed over 100 simulations.

Figure 1 shows the position recovery performance. The area is divided by 64×64 grid points. According to the compressive sensing approach, with 5 targets, the number of RSS measurements required was at least $M > 2k = 10$ in the random i.i.d. Gaussian matrix case and 8 in the proposed deterministic matrix for exact solution recovery. Figure 1(a) shows the position recovery for 5 targets with $M = 8$. It shows that the proposed scheme achieves good performance as the traditional scheme does. Note that only deterministic sensing matrices are practically feasible when considering actual implementation. Figure 1(b) shows the ALE versus the number of measurements. The ALE of the traditional random approach was more dramatically decreased than the proposed one when the number of measurement increased.

Figure 2 shows the ALE with M randomly selected measurements in a given set of sensors. M was to set at 8 and 32 in this simulation. In the case of $M = 8$, the ALE of the proposed scheme is smaller than the traditional case as seen in Figure 2(a). However, when $M = 32$ we observe the opposite results as in Figure 2(b). Since M is large, the independence among columns of the deterministic sensing matrices is not guaranteed, while the RIP holds with random matrices in this case. Thus, the perfect reconstruction by the proposed method may be not guaranteed, and performance by the random matrices is better than the proposed one when the number of measurements becomes large.

For classical approach, each sensor must be recorded n measurements, which brings large communication cost, especially in large-scale networks. Based on the idea of reducing cost in compressive sensing theory and the advantages of deterministic sensing matrices formed by the second-order Reed-Muller codes on recovery, our method reduces the overall communication bandwidth requirement per sensor, and acchieve high localization accuracy. However, the trade-off between high level of accuracy and low computational cost should be considered as well.

V. Concluding Remarks

In this paper, we presented an approach for multiple target localization in wireless sensor networks using deterministic sensing matrices. We begin with problem formulation and present a localization method from sparse measurement based

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on compressive sensing theory. Constructing a sparse measurement matrix is one of the most difficult part during this process. We investigated second-order Reed-Muller codes and applied them to form the measurement matrices in our problem. A key advantage of compressive sensing with these matrices is that it admits a fast reconstruction algorithm, especially for basis pursuit, and depends only on number of measurements m and sparsity k, not depends on the signal length n, in addition to their deterministic structure. Numerical results show that these matrices also guarantee to recover approximated solutions as the traditional schemes do, especially when the signal vectors are very sparse. We expect that this type of matrices will be useful for various localization applications in wireless sensor networks.

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REFERENCES

[1] Y. Shang, W. Ruml, Y. Zhang, and M. Fromherz, "Localization from connectivity in sensor networks," *IEEE Trans. Parallel Distributed Syst.*, vol. 15, no. 11, pp. 961-974, Nov. 2004.

[2] N. Patwari, J. N. Ash, and S. Kyperountas, "Locating the nodes: Cooperative localization in wireless sensor networks," *IEEE Sig. Proc. Mag.*, vol. 22, no. 4, pp. 54-69, July 2005.

[3] I. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: A survey," *Computer Networks*, vol. 38, no. 4, pp. 393-422, 2002.

[4] J. Liu, Y. Zhang, and F. Zhao, "Robust distributed node localization with error management," *Proc. ACM MobiHoc 2006*, pp. 250-261, Florence, Italy, May 2006.

[5] X. Ji and H. Zha, "Sensor positioning in wireless ad-hoc sensor networks with multidimensional scaling," *Proc. IEEE INFOCOM 2004*, pp. 2652-2661, Hong Kong, China, Mar. 2004.

[6] N. Patwari and A. O. Hero III, "Manifold learning algorithms for localization in wireless sensor networks," *Proc. IEEE ICASSP 2004*, vol. 3, pp. 857-860, Montreal, Canada, May 2004.

[7] N. Patwari, A. O. Hero III, M. Perkins, N. S. Correal, and R. J. Odea, "Relative location estimation in wireless sensor networks," *IEEE Trans. Sig. Proc.*, vol. 51, no. 8, pp. 2137-2148, Aug. 2003.

[8] G. Mao, B. Fidan, and B. D. O. Anderson, "Wireless sensor network localization techniques," *Computer Networks*, vol. 51, no. 10, pp. 2529-2553, July 2007.

[9] L. Doherty, L. E. Ghaoui, and S. J. Pister, "Convex position estimation in wireless sensor networks," *Proc. IEEE INFOCOM 2002*, vol. 3, pp. 1655-1663, San Francisco, USA, Apr. 2002.

[10] P. Biswas, T. C. Liang, K. C. Toh, Y. Ye, and T. C. Wang, "Semidefinite programming approaches for sensor network localization with noisy distance measurements," *IEEE Trans. Auto. Sci. Eng.*, vol. 3, no. 4, pp. 360-371, Oct. 2006.

[11] Q. Shi, C. He, L. Jiang, and J. Luo, "Sensor network localization via nondifferentiable optimization," *Proc. IEEE GLOBECOM 2008*, New Orleans, USA, Dec. 2008.

[12] R. Zetik, S. Jovanoska, and R. Thomä, "Simple method for localisation of multiple tag-free targets using UWB sensor network," *Proc. IEEE ICUWB 2011*, Bologna, Italy, Sept. 2011.

[13] A. Kushki, K. N. Plataniotis, and A. N. Venetsanopoulos, "Kernel-based positioning in wireless local area networks," *IEEE Trans. Mobile Comp.*, vol. 6, no. 6, pp. 689-705, June 2007.

[14] V. Cevher, P. Boufounos, R. G. Baraniuk, A. C. Gilbert, and M. J. Strauss, "Near-optimal Bayesian localization via incoherence and sparsity," *Proc. IPSN 2009*, San Francisco, USA, Apr. 2009.

[15] W. Dai and O. Milenkovic, "Subspace pursuit for compressive sensing signal reconstruction," *IEEE Trans. Info. Theory*, vol. 55, no. 5, pp. 2230-2249, May 2009.

[16] D. Needell and J. Tropp, "CoSaMP: Iterative signal recovery from incomplete and inaccurate samples," *Appl. Comput. Harmon. Anal.*, vol. 26, no. 3, pp. 301-321, 2009.

[17] J. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. Info. Theory*, vol. 53, no. 12, pp. 4655-4666, Dec. 2007.

[18] V. Cevher, M. F. Duarte, and R. G. Baraniuk, "Distributed target localization via spatial sparsity," *Proc. EUSIPCO 2008*, Lausanne, Switzerland, Aug. 2008.

[19] E. Candès, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Info. Theory*, vol. 52, no. 2, pp. 489-509, Feb. 2006.

[20] D. Donoho, "Compressed sensing," *IEEE Trans. Info. Theory*, vol. 52, no. 4, pp. 1289-1306, Apr. 2006.

[21] R. Calderbank, S. Howard, and S. Jafarpour, "Construction of a large class of deterministic sensing matrices that satisfy a statistical isometry property," *IEEE Trans. Info. Theory*, vol. 4, no. 2, pp. 358-374, Apr. 2010.

[22] R. DeVore, "Deterministic constructions of compressed sensing matrices," *Jour. Complexity*, vol. 23, no. 4-6, pp. 918-925, 2007.

[23] S. Li, F. Gao, G. Ge, and S. Zhang, "Deterministic construction of compressive sensing matrices via algebraic curves," *IEEE Trans. Info. Theory*, vol. 58, no. 8, pp. 5035-5041, Aug. 2012.

[24] M. A. Tsfasman and S. G. Vladu, *Algebraic-Geometric Codes*, in *Math. Appl. (Soviet Series)*, vol. 58, Kluwer Academic Publishers, 1991.

[25] S. Howard, A. Calderbank, and J. Searle, "A fast reconstruction algorithm for deterministic compressive sensing using second order Reed-Muller codes," *Proc. IEEE CISS 2008*, pp. 11-15, Princeton, USA, Mar. 2008.

[26] P. Delsarte and J. M. Gothals, "Alternating bilinear forms over GF(q)," *Jour. Combinatorial Theory*, vol. 19, pp. 26-50, 1975.

[27] A. Kerdock, "A class of low-rate nonlinear binary codes," *Info. & Control*, vol. 20, pp. 182-187, 1972.

[28] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes*, North-Holland Publishing, 1977.

[29] A. R. Hammons, P. V. Kumar, A. R. Calderbank, N. J. A. Sloane, and P. Sole, "The $\mathbb{Z}_4$-linearity of Kerdock codes, Preparata, Goethals, and related codes," *IEEE Trans. Info. Theory*, vol. 40, no. 2, pp. 301-319, Mar. 1994.