Bit Error Rate Analysis of MIMO Schemes in LTE Systems

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***Abstract*— In this paper, a Bit Error Rate (BER) analysis is presented for Multiple-Input Multiple-Output (MIMO) schemes in the 3GPP Long Term Evolution (LTE) system. Analytical expressions for the average BER of the system are derived over flat Rayleigh fading channels for two different MIMO schemes as defined in LTE, assuming M-ary quadrature amplitude modulation (M-QAM) schemes and are evaluated numerically. Monte-Carlo simulation results of the LTE system are also provided to verify the accuracy of the mathematical analysis. It is shown that the results obtained from Monte-Carlo simulations match closely with those obtained from the derived mathematical formulas.**

***Keywords*- Performance Analysis, MIMO, LTE, M-QAM Modulation.**

I. INTRODUCTION

To increase the capacity and speed of wireless communication systems, a new wireless data networks has been emerged and has been standardized by the 3rd Generation Partnership Project (3GPP). This new standard is a natural evolution to the existing second (2G) and third (3G) generation wireless networks in order to respond to the growing demand in terms of data rates and speed and marketed as 4G Long Term Evolution (LTE). In LTE, data throughput and the speed of wireless data are increased by using a combination of new methods and technologies like Orthogonal Frequency Division Multiplexing (OFDM) and Multiple-Input Multiple-Output (MIMO) techniques.

In the downlink, the LTE transmission is based on Orthogonal Frequency Division Multiple Access (OFDMA), known as a technique of encoding digital data on multiple carrier frequencies. It was shown that OFDMA is an efficient technique to improve the spectral efficiency of wireless systems. By converting the wide-band frequency selective channel into a set of several flat fading subchannels, OFDM technique becomes more resistant to frequency selective fading than single carrier systems. As OFDM signals are in time and frequency domain, they allow adding frequency domain scheduling to time domain scheduling. In LTE, for a given transmission power, the system data throughput and the coverage area can be optimized by employing Adaptive Modulation and Coding (AMC) techniques. The role of the user scheduler at the transmitter side is to assign the data rate for each user according to the channel conditions from the serving cell, the interference level from other cells, and the noise level at the receiver side.

In LTE standard, the use of MIMO has been considered as an essential technique in order to achieve the target in terms of data throughput and reliability. MIMO is known to be a very powerful technique to improve the system performance of wireless communication systems. The diversity and multiplexing modes are the two main modes of operation of multiple antennas systems. The principle of diversity mode is based on transmitting the same signal over multiple antennas and hence to improve the reliability of the system by the diversity gain. In this mode, the mapping function of transmit symbols used at the transmit antennas is called Space Time Block Code (STBC). On the other hand, multiplexing mode uses two or more different spatial streams and send them through two different antennas, consequently, the data rate can be improved.

To study the performance of LTE systems a MATLAB based downlink physical layer simulator for Link Level Simulation (LLS) has been developed in [1] [2]. A System Level Simulation of the Simulator is also available [3]. The goal of developing the LTE simulator was to facilitate comparison with the work of different research groups and it is publicly available for free under academic non-commercial use license [2]. The main features of the simulator are adaptive coding and modulation, MIMO transmission and scheduling. As the simulator includes many physical layer features, it can be used for different applications in research [3]. In [4], the simulator was used to study the channel estimation of OFDM systems and the performance evaluation of a fast fading channel estimator was presented. In [5] and [6], a method for calculating the Precoding Matrix Indicator (PMI), the Rank Indicator (RI), and the Channel Quality Indicator (CQI) were studied and analyzed with the simulator.

In this paper, the Bit Error Rate (BER) analysis of two transmit diversity schemes known as Space Frequency Block Codes (SFBC) and Frequency Switched Transmit Diversity (FSTD) MIMO schemes in LTE system for M-QAM modulation scheme are presented in terms of SNR using the moment generating function of the SNR. The results obtained from the analysis are then compared to the results of Monte-Carlo simulation using the Link Level LTE simulator [1] [2].

The remainder of this paper is organized as follows. In Section II, we present the system and channel model used in the simulation. In Section III, we present a performance analysis for the average BER of SFBC and FSTD MIMO schemes. The numerical and simulation results and discussions are presented in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL

In this section, the structure of the OFDM LTE signal is described. The OFDM signal has a time and a frequency domains. In the time domain, the LTE signal is composed of successive frames. Each frame has a duration of $T_{\text{frame}} = 10$ *msec*. Each frame is divided into 10 subframes with equal length of 1 *msec*. Each subframe consists of two equal length time-slots with a time duration of $T_{\text{slot}} = 0.5$ *msec*. For a normal cyclic prefix length, each time-slot consists of $N_s = 7$ OFDM symbols. In the frequency domain, the OFDM technique converts the LTE wideband signal into several narrowband signals. Each narrowband signal is transmitted on one subcarrier frequency. In LTE, the spacing between subcarriers is fixed to 15 KHz. Twelves adjacent subcarriers, occupying a total of 180 KHz, of one slot forms the so-called Resource Block (RB). The number of Resource Blocks in an LTE slot depends on the allowed system bandwidth. The minimum number of RB is equal to 6 corresponding to 1.4 MHz system bandwidth. For 20 MHz system bandwidth (Maximum Allowed bandwith in LTE) the number of RB is equal to 100. In a MIMO system with M_R receive antennas and M_T transmit antennas, the relation between the received and the transmitted signals on subcarrier frequency k ($k \in 1, ...K$), at sampling instant time n ($n \in 1, ...N$) is given by

$$\mathbf{y}_{k,n} = \mathbf{H}_{k,n}\mathbf{x}_{k,n} + \mathbf{n}_{k,n} \tag{1}$$

where $\mathbf{y}_{k,n} \in C_{M_R \times 1}$ is the received vector, $\mathbf{H}_{k,n} \in C_{M_R \times M_T}$ represents the channel matrix on subcarrier k at instant time n, $\mathbf{x}_{k,n} \in C_{M_R \times 1}$ is the transmit symbol vector and $\mathbf{n}_{k,n} \sim \mathcal{CN}(0, \sigma_n^2.\mathbf{I})$ is a white, complex valued Gaussian noise vector with variance σ_n^2. Assuming perfect channel estimation, the channel matrix and noise variance are considered to be known at the receiver. A linear equalizer filter given by a matrix $\mathbf{F}_{k,n} \in C_{M_R \times M_R}$ is applied on the received symbol vector $\mathbf{y}_{k,n}$ to determine the post-equalization symbol vector $\mathbf{r}_{k,n}$ as follows [6]

$$\mathbf{r}_{k,n} = \mathbf{F}_{k,n}\mathbf{y}_{k,n} = \mathbf{F}_{k,n}\mathbf{H}_{k,n}\mathbf{x}_{k,n} + \mathbf{F}_{k,n}\mathbf{n}_{k,n}. \tag{2}$$

The Zero Forcing (ZF) or Minimum Mean Square Error (MMSE) design criterion [7] are typically used for the linear receiver and the input signal vector is normalized to unit power. In MIMO-OFDM systems, the key factor of link error prediction and performances is the signal to noise ratio (SNR) which represents the measurement for the channel quality information. In this study, the SNR is defined as follows [1]:

$$\gamma_{k,n} = \frac{\|\mathbf{H}_{k,n}\mathbf{x}_{k,n}\|_{\text{F}}^2}{N_T \sigma_n^2} \tag{3}$$

where $\mathbf{x}_{k,n}$ is the transmitted symbol vector, $\|.\|_F^2$ is the squared Frobenius norm of a matrix.

III. AVERAGE BER PERFORMANCE ANALYSIS

In the following, we present a BER performance analysis for the 2×1 MIMO SFBC and 2×2 MIMO SFBC systems, over slow fading channels. Using the Moment Generating Function (MGF)-based approach we obtain closed-form expressions for the average BER performance of the system. We then present numerical evaluation results obtained from the closed-form expressions. Finally, we present Monte-carlo simulations to verify the accuracy of our analysis.

A. BER Analysis of SFBC

In LTE, the transmit diversity techniques are defined only for 2 and 4 transmit antennas and one data stream. When two eNodeB antennas are available for transmit diversity operation, the Space Frequency Block Code (SFBC) is used [8]. SFBC is based on the well known Space Time Block Codes (STBC), also known as Alamouti codes for two transmit antennas [9]. STBC is employed with the UMTS and it operates on pairs of adjacent symbols in the time domain. Since the signal in LTE is two dimensional (time and frequency domains) and the number of available OFDM symbols in a subframe is not always an even number, the direct application of STBC is not straightforward. In LTE, for SFBC transmission, the symbols are transmitted from two eNodeB antenna ports on each pair of adjacent subcarriers as follows [8]:

$$\begin{bmatrix} y^{(0)}(1) & y^{(0)}(2) \\ y^{(1)}(1) & y^{(1)}(2) \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \tag{4}$$

where $y^{(p)}(k)$ denotes the symbols transmitted on the k^{th} subcarrier from antenna port p. An important characteristic of such codes is that the transmitted signal streams are orthogonal and a simple linear receiver is required for optimal performances.

Since OFDM converts the multipath channel into N frequency flat fading channel, we first derive the BER expressions over flat Rayleigh fading channels, given by $P_b\left(E\right)$. Then, the overall average BER over N subcarriers, in each case can be calculated from

$$BER = \frac{1}{N}\sum_{k=1}^{N} P_{b,k}\left(E\right) \tag{5}$$

where the index k (subcarrier index) is ignored for the sake of brevity. In addition, the impact of cyclic prefix in OFDM is assumed to be negligible.

For the 2×1 SFBC MIMO scheme, the probability density function of the SNR for each subcarrier is given by a chi-square distribution function as follows [10]:

$$f(\gamma)=\frac{2}{\bar{\gamma}^2}\gamma e^{-\frac{2}{\bar{\gamma}}\gamma} \tag{6}$$

where $\bar{\gamma}$ is the average SNR per symbol given by $\bar{\gamma}=E_s/N_0$.

The moment generating function (MGF) can be determined using the following equation:

$$M_{\bar{\gamma}}(s)=\int_0^{\infty} e^{-s\gamma} f(\gamma)d\gamma. \tag{7}$$

Inserting (6) into (7) and solving the integral yields

$$M_{\bar{\gamma}}(s)=\frac{4}{\bar{\gamma}^2(s+\frac{2}{\bar{\gamma}})^2}. \tag{8}$$

The average BER expression for M-QAM modulation scheme can be obtained from [11] (equation (8.111; Page 255))

$$P_b\left(E\right)\cong B\sum_{i=1}^{\sqrt{M}/2}\frac{1}{\pi}\int_0^{\pi/2} M_{\bar{\gamma}}\left(A_{i,\theta}\right)d\theta \tag{9}$$

where $A_{i,\theta}=\frac{(2i-1)^2}{2\sin^2\theta}\frac{3}{(M-1)}$ and B is defined by

$$B=4\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)\left(\frac{1}{\log_2 M}\right). \tag{10}$$

Then, using the MGF expression in (8), we obtain

$$M_{\bar{\gamma}}(A_{i,\theta})=\frac{4}{\bar{\gamma}^2\left(\left[\frac{(2i-1)^2}{2\sin^2\theta}\frac{3}{(M-1)}\right]+\frac{2}{\bar{\gamma}}\right)^2}. \tag{11}$$

Substituting (11) into (9) and after some manipulations, we obtain

$$P_b\left(E\right)\cong B\sum_{i=1}^{\sqrt{M}/2}\frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+c_i}\right)^2 d\theta \tag{12}$$

where $c_i=\frac{3(2i-1)^2}{2(M-1)}\frac{\bar{\gamma}}{2}$.

The average BER performance as a function of $\bar{\gamma}=E_s/N_0$ can be evaluated by numerical evaluation of the integral in (12) for M-QAM modulation schemes. Alternatively, by solving the integral, we obtain a closed-form expression for the average BER of M-QAM modulation as follows

$$P_b\left(E\right)\cong B\sum_{i=1}^{\sqrt{M}/2}\mathcal{I}_2\left(\pi/2,c_i\right) \tag{13}$$

where the closed-form expression for $\mathcal{I}_2(.,.)$ can be obtained from [11](eq.5A.24) as follows

$$\begin{aligned}\mathcal{I}_n\left(\phi,D\right)&=\frac{1}{\pi}\int_0^{\phi}\left(\frac{\sin^2\theta}{\sin^2\theta+D}\right)^n d\theta, \qquad -\pi\le\phi\le\pi\\ &=\frac{\phi}{\pi}-\frac{\beta}{\pi}\left\{\left(\frac{\pi}{2}+\tan^{-1}\alpha\right)\sum_{q=0}^{n-1}\begin{pmatrix}2q\\q\end{pmatrix}\frac{1}{\left(4\left(1+D\right)\right)^q}\right.\\ &\left.+\sin\left(\tan^{-1}\alpha\right)\sum_{q=1}^{n-1}\sum_{p=1}^{q}\frac{T_{p\,q}}{(1+D)^q}\left[\cos\left(\tan^{-1}\alpha\right)\right]^{2(q-p)+1}\right\}\end{aligned} \tag{14}$$

where $T_{p\,q}=\begin{pmatrix}2q\\q\end{pmatrix}\left[\begin{pmatrix}2(q-p)\\q-p\end{pmatrix}4^p\left[2(q-p)+1\right]\right]^{-1}$, $\beta=\sqrt{\frac{D}{1+D}}\,\mathrm{sgn}\phi$, and $\alpha=-\beta\cot\phi$.

B. BER Analysis of FSTD

In LTE, the frequency space code, designed for 4 transmit antennas is defined as follows:

$$\begin{bmatrix} y^{(0)}(1) & y^{(0)}(2) & y^{(0)}(3) & y^{(0)}(4)\\ y^{(1)}(1) & y^{(1)}(2) & y^{(1)}(3) & y^{(1)}(4)\\ y^{(2)}(1) & y^{(2)}(2) & y^{(2)}(3) & y^{(2)}(4)\\ y^{(3)}(1) & y^{(3)}(2) & y^{(3)}(3) & y^{(3)}(4)\end{bmatrix}=$$

$$\begin{bmatrix} x_1 & x_2 & 0 & 0\\ 0 & 0 & x_3 & x_4\\ -x_2^* & x_1^* & 0 & 0\\ 0 & 0 & -x_4^* & x_3^*\end{bmatrix}. \tag{15}$$

For the 4×2 FSTD MIMO scheme, we can show that the instantaneous SNR of the system, for k-th subcarrier, is equivalent to that for a 2×2 STBC MIMO system. Therefore, the probability density function of the SNR is given by a chi-square distribution function as follows [10]:

$$f(\gamma)=\frac{8}{3\bar{\gamma}^4}\gamma^3 e^{-\frac{2}{\bar{\gamma}}\gamma}. \tag{16}$$

In this case, the MGF expression can be obtained by substituting (16) into (7), which yields

$$M_{\bar{\gamma}}(s)=\frac{16}{\bar{\gamma}^4(s+\frac{2}{\bar{\gamma}})^4}. \tag{17}$$

Similarly to the SFBC case discussed in previous Section, inserting (17) into (9), the average BER expression with M-QAM modulation for FTSD can be written as

$$P_b\left(E\right)\cong B\sum_{i=1}^{\sqrt{M}/2}\frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+c_i}\right)^4 d\theta \tag{18}$$

where $c_i=\frac{3(2i-1)^2}{2(M-1)}\frac{\bar{\gamma}}{2}$, and the integral can be calculated numerically.

Alternatively, by solving the integral, we obtain a closed-form expression for the average BER of M-QAM modulation as follows

$$P_b\left(E\right) \cong B \sum_{i=1}^{\sqrt{M}/2} \mathcal{I}_4\left(\pi/2, c_i\right) \tag{19}$$

where the closed-form expression for $\mathcal{I}_4(.,.)$ can be obtained from (14).

Finally, for the sake of comparisons, we express the average BER of the SISO case, that has been derived for Rayleigh fading channels for M-QAM signals [11] (eq. 8.112; Page 256), as follows:

$$P_b(E) \cong B/2 \sum_{i=1}^{\sqrt{M}/2} \left(1 - \sqrt{\frac{1.5(2i-1)^2\overline{\gamma}\log_2 M}{M-1+1.5(2i-1)^2\overline{\gamma}\log_2 M}}\right) \tag{20}$$

where B is defined earlier.

IV. SIMULATION AND ANALYTICAL RESULTS

In this section, we provide the results obtained from the mathematical expressions derived in this paper. Monte-Carlo simulation results are also provided to show the accuracy of the analysis. The common simulation settings for Monte-Carlo simulations are summarized in Table I.

The average BER performance as a function of $\overline{\gamma} = Es/N_0$ for SISO and MIMO schemes are shown in Fig. 1 and Fig.2. Fig. 1 shows the results for 16-QAM modulation and Fig.2 presents the results for 64-QAM modulation. It can be seen that the average BER performances of 16-QAM and 64-QAM schemes at high SNRs decrease by factors $\overline{\gamma}^1$, $\overline{\gamma}^2$, and $\overline{\gamma}^4$, for SISO, 2×1, and 4×2 cases, respectively. Thus, the diversity order (slope of the curves) are equal to 1, 2 and 4, respectively, for the considered cases. As stated earlier, since in 4×2 FSTD, at each time-slot/frequency-slot 2 out of 4 transmit antennas are in use, therefore the diversity order will be $2\times2=4$. In fact, the corresponding average BER curve for 4×2 FSTD is somehow like the classical 2×2 STBC system, when the channel is not a time-varying channel.

From the figures it is clear that the BER performance improves as the number of transmit or receive antennas increases, as expected. It can be observed that the negative slope of the BER curve for the SISO case is equal to 1, meaning that the diversity order for the SISO case is equal to 1, as expected. The second curves in Fig.1 and Fig.2 represent the BER results of the 2×1 diversity scheme. Asymptotically, the slope of these curves can be observed to be equal to 2, which corresponds to the diversity order of 2×1 system. An SNR (Es/N_0) gain improvement can also be observed compared to the SISO scheme. From Fig.1, it can be observed that to achieve the BER value of 10^{-3}, the 2×1 diversity scheme needs about 10 dB less in Es/N_0, compared to the SISO case.

TABLE I
SIMULATION SETTINGS

Parameter	Setting
Transmission Schemes	SISO; 2×1 SFBC; 4×2 FSTD
Bandwidth	5 MHz
Simulation length	5000 subframes
Channel Type	Flat Rayleigh
Channel knowledge	Perfect
CQI	9 (16-QAM) and 16 (64-QAM)

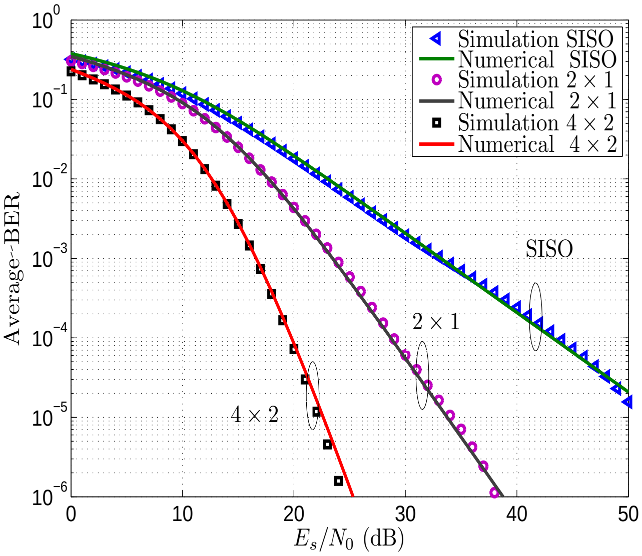

Fig. 1. Numerical Evaluation and Monte-Carlo Simulations of the average BER for 16-QAM modulation.

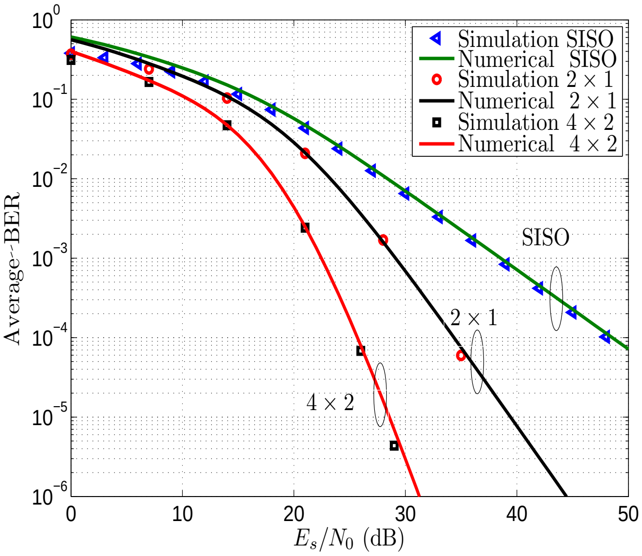

Fig. 2. Numerical Evaluation and Monte-Carlo Simulations of the average BER for 64-QAM modulation.

In Fig.2 for 64-QAM modulation, the BER of 10^{-3} is achieved at $Es/N_0 = 39$ dB in SISO configuration, however the same value of BER is achieved with only at $Es/N_0 = 29$ dB in the 2×1 diversity scheme. Thus a SNR gain of 10 dB is clearly observed for the 2×1 diversity scheme. The BER results of the 4×2 diversity scheme for both modulation schemes 16-QAM and 64-QAM are also shown. As described earlier, in high SNR region the slope of that curve tends to be equal to 4. This value corresponds to the diversity order of a 2×2 system.

Finally, it can be observed from Fig.1 and Fig.2 that numerical evaluation results obtained from BER formulas match closely to the BER results obtained from Monte-Carlo simulations. This verifies the accuracy of the mathematical analysis.

V. CONCLUSION

In this paper, we have presented an average BER performance analysis for MIMO schemes in the 3GPP Long Term Evolution (LTE) system over a flat Rayleigh fading channel. The theoretical analysis for two different MIMO schemes in a 5 MHz bandwidth LTE system were presented. To verify the accuracy of the analysis the results of Monte-Carlo simulation for the studied schemes were provided and compared with the theoretical analysis. To show the BER performance improvement in the MIMO schemes, the performance of a SISO configuration was also presented. The results show a good agreement between numerical results and Monte-Carlo simulation results.

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