The Performance Analysis of Complex SSC/MRC Combiner in Rice Fading Channel

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Abstract— **In this paper, the complex Switch and Stay Combining/Maximal Ratio Combining (SSC/MRC) combiner will be considered. The output signal is observed at two time moments in the presence of Rice fading. Both combiners, SSC such as MRC, are dual branches. The probability density function (PDF) at the output of the complex combiner will be obtained and the bit error rate (BER) for binary phase swift keying (BPSK) modulation will be determined. The results will be shown graphically in some figures. The improvement of using this kind of complex SSC/MRC combiner, with regard to classical MRC and SSC combiners at one time moment, is emphasized.**

Keywords-Bit error rate, Probability density function; Complex SSC/MRC combiner; Rice fading; two time moments

I. INTRODUCTION

The fading is one of the most important reason for system performance degradation in mobile communication. Many different communication systems are subjected to fading caused by multipath propagation due to reflection, refraction and scattering by buildings, trees and other large structures [1]. Because of that received signal is a sum of different signals that arrive via different propagation paths.

Some statistical models are used in the literature to describe the fading envelope of the received signal such as Rayleigh and Rician distributions [2]. They are used to characterize the envelope of faded signals over small geographical areas (short term fading). The log-normal and gamma distribution are used when much wider geographical areas is involved.

There are several ways to reduce fading influence on system performances without increasing the signal power and channel capacity. The diversity reception techniques are used extensively in fading radio channels to reduce the effects of fading on system performances [3].

The use of diversity is significant if there is statistical independence in the fading of the received signal in each of the diversity branches. The assumption of statistical independence between the diversity channels is valid only if they are sufficiently separated [2]. In mobile radio systems the signals at the mobile station become decorrelated as the antenna separation increases. In space diversity systems an antenna separation of 30 to 50 wavelengths is required to have correlation coefficients strictly between zero and one-third, in which case, for a two-channel maximal-ratio system in a Rayleigh-fading environment, the effect of correlation may be ignored [4].

However, there are other cases of practical interest where the assumption of statistical independence is not valid [5]. A long time ago Al-Hussani and Al-Bassion studied the effect of correlation on the performance of a dual-branch MRC combiner for correlated Nakagami fading channel. They found that for the Nakagami-fading environment and for a worst case fading condition and identical signal-to-noise ratio (SNR) in each of two branches, the performance difference between a single channel and dual channels system increases from 3 to 24 dB as the correlation coefficient decreases from unity to zero.

There are several kinds of diversity combining schemes such as Maximal-Ratio Combining (MRC), which is the optimal combining scheme [3]. In this combiner signals from all inputs are summed. Because MRC requires cognition of the channel fading parameters, it is the most complicated combining model [3].

The next by performances is Equal Gain Combining (EGC) [6], and then Selection Combining (SC) and Switch and Stay Combining (SSC), with lower performances. These combining models are simpler and cheaper and they are very often implemented in practice whereas SC and SSC combining models do not require signal cophasing and fading envelope evaluation [7].

SSC is simplification of the system complexity, but with losing in quality. In this model, the receiver selects an antenna until its value drops below predetermined threshold. Then, the receiver switches to other antenna and stay with it for the next time slot, regardless the channel quality of that antenna is above or below the threshold. In the literature, mainly dual SSC schemes have been analyzed [8].

By the authors' knowledge in the new open literature these problems are not treated by sampling in two time moments, except by this group of authors. We derived the expressions for joint probability density function of the SSC combiner output signal in the presence of different fading distributions in two time moments (for example in [9] for log-normal) and based on them determined the performance analysis of SSC/SC combiner at two time moments in the presence of log-normal and Rayleigh fading in [10], [11]. The bit error rates for SSC/MRC combiner at two time moments in the presence of log-normal, Rayleigh and Hoyt fading we determined in [12]-[14], respectively.

In this paper, the probability density function and the bit error rate, based on it, for SSC/MRC combiner output signal in the channel with Rice fading and sampling at two time moments during one time interval, will be considered. The system is more complicated compared to classical MRC and SSC systems at one time moment, but with better performances. That imply that bit error rate can be increased and transmit power can be reduced comparing to classical systems.

This paper is organized as follows: Section II presents related works; Section III gives the complex SSC/MRC system model and the probability density functions derivations, such as the bit error rate of the SSC/MRC combiner output signal at two time moments. Sections IV shows the numerical results obtained for performances introduced in previous sections. Finally, the main gains obtained in the paper are pointed out in Section V, Conclusion.

II. SYSTEM MODEL

The model of the SSC/MRC combiner with two inputs, considering in this paper, is presented in Fig. 1.

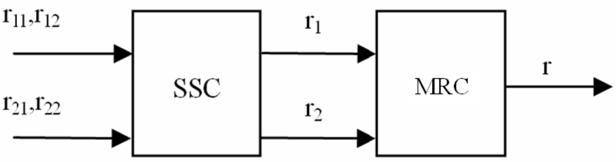

Figure 1. Complex dual SSC/MRC combiner.

The complex SSC/MRC combiner with two branches at two time moments is considered.

At the inputs of the first part of complex combiner the signals are r_{11} and r_{21} at first time moment and they are r_{12} and r_{22} at second time moment. The output signals from SSC part of complex combiner are r_1 and r_2. The first index represents the branch ordinal number and the other one signs the time moment observed. The indices at the output signal correspond to the time moments considered. The SSC combiner output signals r_1 and r_2, are the inputs for the MRC combiner. Finally, the overall output signal is r.

The joint probability density function of correlated signals r_1 and r_2 at the SSC combiner output, at two time moments, Rice distributed and with same parameters σ_i and A_i [15], is obtained in closed form from expressions in [9, eq. (3) - (6)] as

$$C_1(r,\sigma,A)=\frac{1}{\sigma^2}e^{-\frac{A^2}{\sigma^2(1+\rho)}}\sum_{i,l_1,l_2,l_3=0}^{\infty}\varepsilon_i\cdot$$
$$\cdot\frac{1}{l_1!l_2!l_3!(i+l_1)!(i+l_2)!(i+l_3)!}\frac{\rho^{i+2l_1}}{\left(2\sigma^2(1-\rho^2)\right)^{2i+l_1+l_2+2l_3}}\cdot$$
$$\cdot A^{2i+2l_2+2l_3}(1-\rho)^{2i+2l_2+2l_3}r^{2i+2l_1+2l_3+1}\cdot$$
$$\cdot e^{-\frac{r^2}{2\sigma^2(1-\rho^2)}}\gamma\left(i+l_1+l_2+1,\frac{r_t^2}{2\sigma^2(1-\rho^2)}\right) \quad (1)$$

$$C_2(r_1,r_2,\sigma,A)=\frac{r_1r_2}{\sigma^4(1-\rho^2)}e^{-\frac{r_1^2+r_2^2+2A^2(1-\rho)}{2\sigma_2^2(1-\rho^2)}}\cdot$$
$$\cdot\sum_{i=0}^{\infty}\varepsilon_i I_i\left(\frac{\rho r_1 r_2}{\sigma^2(1-\rho^2)}\right)I_i\left(\frac{A r_1}{\sigma^2(1+\rho)}\right)I_i\left(\frac{A r_2}{\sigma^2(1+\rho)}\right) \quad (2)$$

where $\varepsilon_i=\begin{cases}1, & i=0\\ 2, & i>0\end{cases}$, parameters σ_i and A_i are noise variance and amplitude, respectively.

For $r_1<r_T$, $r_2<r_T$ it is:

$$p_{r_1r_2}(r_1,r_2)=P_1C_1(r_1,\sigma_2,A_2)C_1(r_2,\sigma_1,A_1)+$$
$$+P_2C_1(r_1,\sigma_1,A_1)C_1(r_2,\sigma_2,A_2) \quad (3)$$

For $r_1\geq r_T$, $r_2<r_T$:

$$p_{r_1r_2}(r_1,r_2)=P_1C_1(r_1,\sigma_1,A_1)\frac{r_2}{\sigma_2^2}e^{-\frac{r_2^2+A_2^2}{2\sigma_2^2}}I_0\left(\frac{r_2A_2}{\sigma_2^2}\right)+$$
$$+P_1C_1(r_1,\sigma_2,A_2)C_1(r_2,\sigma_1,A_1)+$$
$$+P_2C_1(r_1,\sigma_2,A_2)\frac{r_2}{\sigma_1^2}e^{-\frac{r_2^2+A_1^2}{2\sigma_1^2}}I_0\left(\frac{r_2A_1}{\sigma_1^2}\right)+$$
$$+P_2C_1(r_1,\sigma_1,A_1)C_1(r_2,\sigma_2,A_2) \quad (4)$$

For $r_1<r_T$, $r_2\geq r_T$:

$$p_{r_1r_2}(r_1,r_2)=P_1\left(1-Q_1\left(A/\sigma_1,r_t/\sigma_1\right)\right)C_2(r_1,r_2,\sigma_2,A_2)+$$
$$+P_1C_1(r_1,\sigma_2,A_2)C_1(r_2,\sigma_1,A_1)+$$
$$+P_2\left(1-Q_1\left(A/\sigma_2,r_t/\sigma_2\right)\right)C_2(r_1,r_2,\sigma_1,A_1)+$$

$$+P_2C_1(r_1,\sigma_1,A_1)C_1(r_2,\sigma_2,A_2) \tag{5}$$

For $r_1 \geq r_T$, $r_2 \geq r_T$:

$$p_{r_1r_2}(r_1,r_2)=P_1C_2(r_1,r_2,\sigma_1,A_1)+$$

$$+P_1C_1(r_1,\sigma_1,A_1)\frac{r_2}{\sigma_2^2}e^{-\frac{r_2^2+A_2^2}{2\sigma_2^2}}I_0\left(\frac{r_2A_2}{\sigma_2^2}\right)+$$

$$+P_1C_1(r_1,\sigma_2,A_2)C_1(r_2,\sigma_1,A_1)+$$

$$+P_1\left(1-Q_1\left(A/\sigma_1,r_t/\sigma_1\right)\right)C_2(r_1,r_2,\sigma_2,A_2)+$$

$$+P_2C_2(r_1,r_2,\sigma_2,A_2)+$$

$$+P_2C_1(r_1,\sigma_2,A_2)\frac{r_2}{\sigma_1^2}e^{-\frac{r_2^2+A_1^2}{2\sigma_1^2}}I_0\left(\frac{r_2A_1}{\sigma_1^2}\right)+$$

$$+P_2C_1(r_1,\sigma_1,A_1)C_1(r_2,\sigma_2,A_2)+$$

$$+P_2\left(1-Q_1\left(A/\sigma_2,r_t/\sigma_2\right)\right)C_2(r_1,r_2,\sigma_1,A_1) \tag{6}$$

The outputs of SSC combiner are used as inputs for MRC combiner.

Total conditional signal value at output of the MRC combiner, for equally transmitted symbols of L branch MRC receiver, is given by [1]

$$r=\sum_{l=1}^{L}r_l \tag{7}$$

For coherent binary signals the conditional BER $P_b(e|\{r_l\}_{l=1}^L)$ is given by [3]

$$P_b(e|\{r_l\}_{l=1}^L)=Q\left(\sqrt{2gr}\right) \tag{8}$$

where Q is one-dimensional Gaussian Q-function [3]

$$Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt \tag{9}$$

Gaussian Q-function can be defined using alternative form as [3, 16]

$$Q(x)=\frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{x^2}{2\sin^2\phi}\right)d\phi \tag{10}$$

Using the alternative representation of Gaussian-Q function (10), the conditional BER can be expressed as

$$P_b(e|\{r_l\}_{l=1}^L)=\frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{gr}{\sin^2\phi}\right)d\phi=\frac{1}{\pi}\int_0^{\pi/2}\prod_{l=1}^{L}\left(-\frac{gr_l}{\sin^2\phi}\right)d\phi \tag{11}$$

The unconditional BER is obtained by

$$P_b(e)=\underbrace{\int_0^\infty\int_0^\infty\cdots\int_0^\infty}_{L}P_b\left(\{r_l\}_{l=1}^L\right)\prod_{l=1}^{L}p_{r_1,r_2,\ldots r_L}(r_1,r_2,\ldots r_L)dr_1dr_2\ldots dr_L \tag{12}$$

Substituting (11) in (12), $P_b(e)$ is obtained as

$$P_b(e)=\underbrace{\int_0^\infty\int_0^\infty\cdots\int_0^\infty\frac{1}{\pi}\int_0^{\pi/2}\prod_{l=1}^{L}\left(-\frac{gr_l}{\sin^2\phi}\right)d\phi}_{L}\, p_{r_1,r_2,\ldots r_L}(r_1,r_2,\ldots r_L)dr_1dr_2\ldots dr_L \tag{13}$$

For dual branch MRC combiner, $P_b(e)$ is

$$P_b(e)=\underbrace{\int_0^\infty\int_0^\infty\cdots\int_0^\infty\frac{1}{\pi}\int_0^{\pi/2}\prod_{l=1}^{L}\left(-\frac{gr_l}{\sin^2\phi}\right)d\phi}_{L}\, p_{r_1,r_2,\ldots r_L}(r_1,r_2,\ldots r_L)dr_1dr_2\ldots dr_L \tag{14}$$

Substituting (3 - 6) in (14), $P_b(e)$ of SSC/MRC combiner can be obtained as:

$$P_b(e)=\frac{1}{\pi}\int_0^{r_t}\int_0^{r_t}\int_0^{\pi/2}dr_1dr_2d\phi\left(-\frac{gr_1}{\sin^2\phi}\right)\left(-\frac{gr_2}{\sin^2\phi}\right)\cdot$$

$$\cdot\left[P_1C_1(r_1,\sigma_2,A_2)C_1(r_2,\sigma_1,A_1)+\right.$$

$$\left.+P_2C_1(r_1,\sigma_1,A_1)C_1(r_2,\sigma_2,A_2)\right]+$$

$$+\frac{1}{\pi}\int_{r_t}^{\infty}\int_0^{r_t}\int_0^{\pi/2}dr_1dr_2d\phi\left(-\frac{gr_1}{\sin^2\phi}\right)\left(-\frac{gr_2}{\sin^2\phi}\right)\cdot$$

$$\cdot\left[P_1C_1(r_1,\sigma_1,A_1)\frac{r_2}{\sigma_2^2}e^{-\frac{r_2^2+A_2^2}{2\sigma_2^2}}I_0\left(\frac{r_2A_2}{\sigma_2^2}\right)+\right.$$

$$+P_1C_1(r_1,\sigma_2,A_2)C_1(r_2,\sigma_1,A_1)+$$

$$+P_2C_1(r_1,\sigma_2,A_2)\frac{r_2}{\sigma_1^2}e^{-\frac{r_2^2+A_1^2}{2\sigma_1^2}}I_0\left(\frac{r_2A_1}{\sigma_1^2}\right)+$$

$$\left.+P_2C_1(r_1,\sigma_1,A_1)C_1(r_2,\sigma_2,A_2)\right]+$$

$$+\frac{1}{\pi}\int_0^{r_t}\int_{r_t}^{\infty}\int_0^{\pi/2}dr_1dr_2d\phi\left(-\frac{gr_1}{\sin^2\phi}\right)\left(-\frac{gr_2}{\sin^2\phi}\right)\cdot$$

$$\cdot\left[P_1\left(1-Q_1\left(A/\sigma_1,r_t/\sigma_1\right)\right)C_2(r_1,r_2,\sigma_2,A_2)+\right.$$

$$+P_1C_1(r_1,\sigma_2,A_2)C_1(r_2,\sigma_1,A_1)+$$

$$+P_2\left(1-Q_1\left(A/\sigma_2,r_t/\sigma_2\right)\right)C_2(r_1,r_2,\sigma_1,A_1)+$$

$$\left.+P_2C_1(r_1,\sigma_1,A_1)C_1(r_2,\sigma_2,A_2)\right]+$$

$$+\frac{1}{\pi}\int_{r_t}^{\infty}\int_{r_t}^{\infty}\int_{0}^{\pi/2} dr_1 dr_2 d\phi\left(-\frac{gr_1}{\sin^2\phi}\right)\left(-\frac{gr_2}{\sin^2\phi}\right)\cdot$$
$$\cdot\left[P_1C_2(r_1,r_2,\sigma_1,A_1)+\right.$$
$$+P_1C_1(r_1,\sigma_1,A_1)\frac{r_2}{\sigma_2^2}e^{-\frac{r_2^2+A_2^2}{2\sigma_2^2}}I_0\left(\frac{r_2A_2}{\sigma_2^2}\right)+$$
$$+P_1C_1(r_1,\sigma_2,A_2)C_1(r_2,\sigma_1,A_1)+$$
$$+P_1(1-Q_1(A/\sigma_1,r_t/\sigma_1))C_2(r_1,r_2,\sigma_2,A_2)+$$
$$+P_2C_2(r_1,r_2,\sigma_2,A_2)+$$
$$+P_2C_1(r_1,\sigma_2,A_2)\frac{r_2}{\sigma_1^2}e^{-\frac{r_2^2+A_1^2}{2\sigma_1^2}}I_0\left(\frac{r_2A_1}{\sigma_1^2}\right)+$$
$$P_2C_1(r_1,\sigma_1,A_1)C_1(r_2,\sigma_2,A_2)+$$
$$\left.+P_2(1-Q_1(A/\sigma_2,r_t/\sigma_2))C_2(r_1,r_2,\sigma_1,A_1)\right] \tag{15}$$

III. NUMERICAL RESULTS

Some values of the bit error rate for different types of combiners and correlation parameters, are presented in Fig. 2 and 3, where it is assumed that both inputs have the same channel parameters. It is adopted that r_t is the optimal threshold for the SSC decision [3]:

$$r_t=\sigma\sqrt{\frac{\pi}{2}}e^{-\frac{A^2}{4\sigma^2}}\left[(1+\frac{A^2}{2\sigma^2})I_0\left(\frac{A^2}{4\sigma^2}\right)+\frac{A^2}{2\sigma^2}I_1\left(\frac{A^2}{4\sigma^2}\right)\right] \tag{16}$$

The family of curves for the BER is shown in Fig. 2 versus different distribution parameter *A*.

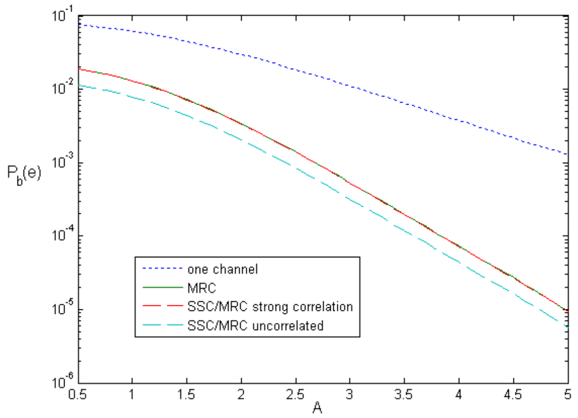

Figure 2. Bit error rate for different types of combiners versus parameter *A* for σ=1

It consists of four curves: one for one channel receiver, the other for MRC combiner at one time moment, third for SSC/MRC combiner at two time moments for uncorrelated case, and fourth for very strong correlation.

We can see that SSC/MRC combiner has remarkably better performances for uncorrelated case then MRC combiner considered at one time moment. For ρ =1 the BER curves for complex SSC/MRC combiner and for MRC combiner coincide with each other. Thus, it is visible that utilization of this complex SSC/MRC combiner provides better performance of the system for uncorrelated signals, but for strong correlation it is not economic to use complex combiner.

The influence of correlation to the bit error rate of complex SSC/MRC combiner is presented in Fig. 3. The benefits of using this type of combiner increases with decreasing of correlation between input signals.

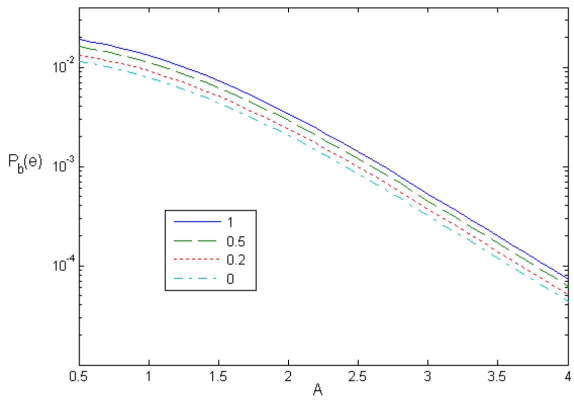

Figure 3. Bit error rate for SSC/MRC combiner versus parameter *A* for σ=1 and for different values of ρ

It is obvious that using of complex SSC/MRC combiner results in better performance of the system, because the BER for uncorrelated SSC/MRC combiner decreases regarding MRC combiner. Also, it is apparent that there is no economic reason for the use of complex SSC/MRC combiner in the case of strong correlations between input signals since then characteristics are not better than for simple MRC combiner.

IV. CONCLUSIONS

The probability density function of output signal at two time moments, for complex dual SSC/MRC combiner, is determined in this paper in closed form expressions. The bit error probability is calculated on that basis.

This is important for the system performance determination for system deciding by two samples. First, the joint probability density function of the SSC combiner output signal at two time moments is derived and used as input of MRC combiner. The obtained results for bit error rate are calculated and shown graphically. The performance improvement of SSC/MRC combiner at two time moments, comparing with classical SSC and MRC combiners, is described and pointed out for different values of correlation coefficient.

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