# Second Best Codeword for MIMO Broadcast Channels with Limited Feedback

Mouncef Benmimoune and Daniel Massicotte
Electrical and Computer Engineering Department
Laboratory of Signal and System Integrations
Université du Québec à Trois-Rivières
3351, Boul. des Forges, Trois-Rivières, QC, CANADA
Email:{mouncef.benmimoune, daniel.massicotte}@uqtr.ca

Sébastien Roy
Electrical and Computer Engineering Department
Sherbrooke University
2500, BLD de l'Université, Sherbrooke, QC, CANADA
Email: s.roy@ieee.org

***Abstract*—In this paper, we propose a simple design for a multi-user MIMO precoder using a Grassmannian codebook. The precoder design is applied to both sides of the wireless link under the assumption of limited feedback and the same codebook. The proposed scheme aims to correct the precoding vectors choice in order to avoid more than one user having the same vector, a situation which degrades system performance. Simulation results show that our approach provides greater system performance enhancement in both error probability and sum rate.**

***Keywords-MIMO broadcast channels; precoding; codebook; codeword correction; limited feedback.***

## I. Introduction

Recently, Multi-Input Multi-Output (MIMO) technology has been introduced in several applications, such as wireless LANs and cellular telephony because of its high gain in both channel capacity and reliability. The MIMO concept consists in deploying multiple antennas at both the transmitter and the receiver in order to exploit the spatial dimension. This new dimension can be used in a single-user scenario (SU-MIMO), providing a system capacity that increases linearly with the multiplexing gain, regardless of the availability or otherwise of the channel state information at the transmitter (CSIT) [1]. Or it can be leveraged in a multiuser configuration (MU-MIMO), where several mobile stations (MS) communicate with a base station (BS) while sharing the same time-frequency resource.

In multi-user case, the situation vis-a-vis the sum capacity is substantially different, since the interference must be taken into account and balanced in a trad-off against the data rate [2]. As a result, the CSIT is required since it critically affects the multiplexing gain [3].

In information theory, it is well known that the optimal strategy for achieving sum capacity in a MIMO broadcast channel is dirty paper coding (DPC) [4]. However, deploying DPC in real-time systems is impractical due to the complexity of the encoder and the decoder. Moreover, it is sensitive to CSIT inaccuracy. For low complexity and suboptimal performance, several non-linear and linear precoders have been proposed [5], [6], [7], [8]. However, all these schemes need perfect CSIT, which is not practical.

In the literature, a considerable number of publications have assumed partial CSIT, where unitary precoding is the main, denoted also as limited feedback (see, e.g., [9] and the references therein). Recently, this scheme was introduced in IEEE 802.16 e/m and LTE Advanced for its performance. In fact, it was shown in [10] that for SU-MIMO systems, even a few bits of feedback, of the order of the number of transmit antennas, is sufficient to achieve near-optimal performance. For MU-MIMO systems, however, the amount of feedback per user must grow linearly with the number of transmit antennas and the SNR in decibel [1].

The use of the same codebook in all MS causes a rank loss in the channel matrix since two or more users can choose the same codeword. Few solutions to avoid this situation are found in the literature. Indeed, most papers suggest using different codebooks (see, e.g., [1]). But this is not quite feasible. In Ding *et al.* [11], the authors propose to modify the codebook of each user by a rotation of the general codebook by a random unitary matrix that is known at the BS. However, this solution is impractical because the BS must know all unitary matrices. All the more, the latter requires a storage and more feedback overhead for matrix's identification. To address this problem, in this paper, we propose to make a correction of the precoding vectors choices at the BS while keeping the same codebook.

Throughout the paper, lower-case bold letters are used for vectors and upper-case bold letters for matrices; $\|\cdot\|_2$ denotes the Euclidean norm of the vector; $|\cdot|$ denotes the cardinality of a set or the absolute value of the scalar; $Tr(\cdot)$ denotes the matric trace; and $(\cdot)^H$ and $(\cdot)^{-1}$ denote Hermitian and matrix inversion, respectively. The identity matrix is denoted by $\mathbf{I}$.

The rest of this paper is organized as follows. The MU-MIMO system model and the Grassmannian codebook construction are depicted in the next section. The problem formulation and the proposed precoding design under limited feedback is discussed in Section III. Simulation results are given in Section IV, and we conclude with Section V.

## II. System Model

### A. Multi-User MIMO System with Limited Feedback

Let us consider a closed loop MU-MIMO broadcast channel with $N_t$ antennas at the BS and $N_r$ antennas at each of $K$ MS. The vector data $\mathbf{s} \in \mathbb{C}^{K\times 1}$ is preprocessed by the

 ISBN: 978-1-61208-284-4 

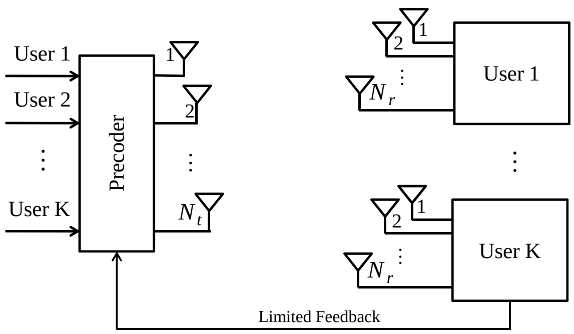


Fig. 1. Downlink Multi-User MIMO system

precoding matrix $\mathbf{W} \in \mathbb{C}^{N_t \times K}$ and then transmitted via flat-fading channel $\mathbf{H}_k \in \mathbb{C}^{N_r \times N_t}$ between the BS and the $k^{th}$ MS. At the $k^{th}$ MS, the equivalent baseband input-output relationship can be written as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W} \mathbf{s} + \mathbf{n}_k, \tag{1}$$

where $\mathbf{n}_k \in \mathbb{C}^{N_r \times 1}$ is the noise term which follows an independent complex Gaussian distribution with zero mean and $N_0$ variance, i.e. $\mathcal{CN}(0, N_0)$.

At the receiver, a linear MMSE detector is used to estimate the symbols transmitted, the decoding matrix of the $k^{th}$ user being given by [12]

$$\tilde{\mathbf{g}}_k = \tilde{\mathbf{h}}_k^H \left( \tilde{\mathbf{H}}_k \tilde{\mathbf{H}}_k^H + \frac{K N_0}{P_t} \mathbf{I}_{N_r} \right)^{-1}, \tag{2}$$

where

$$\tilde{\mathbf{H}}_k = \mathbf{H}_k \mathbf{W}. \tag{3}$$

$$\tilde{\mathbf{h}}_k = \mathbf{H}_k \mathbf{w}_k. \tag{4}$$

In this case, the received signal-to-interference and noise ratio (SINR) for the $k^{th}$ user is [12]

$$sinr_k\left(\mathbf{H}_k, \mathbf{w}_k\right) = \frac{\frac{P_t}{N_t} |\tilde{\mathbf{g}}_k \mathbf{H}_k \mathbf{w}_k|^2}{\sum_{i=1, i\neq k}^{K} \frac{P_t}{N_t} |\tilde{\mathbf{g}}_k \mathbf{H}_k \mathbf{w}_i|^2 + N_0 \|\tilde{\mathbf{g}}_k\|_2^2}, \tag{5}$$

where $P_t$ represents the total transmit power which must satisfy the following constraint

$$E[\|\mathbf{W}\mathbf{s}\|_2^2] = Tr(\mathbf{W}\mathbf{W}^H) \leq P_t, \tag{6}$$

where $\mathbf{W}$ is given by $\mathbf{W} = [\mathbf{w}_1 \mathbf{w}_2 \ldots \mathbf{w}_K]$.

The downlink sum rate can be expressed as follows

$$r\left(\mathbf{H}_k, \mathbf{w}_k\right) = \sum_{k=1}^{K} \log_2\left(1 + sinr_k\left(\mathbf{H}_k, \mathbf{w}_k\right)\right). \tag{7}$$

### *B. Grassmannian Codebook*

The problem of Grassmannian line packing is to find the packing of $P$ lines in $\mathbb{C}$ such that the distance between any pair of lines is maximal. This packing of $P$ lines is captured by the matrix $\mathbf{V} = [\mathbf{v}_1 \mathbf{v}_2 \ldots \mathbf{v}_P]$, where each column $\mathbf{v}_i$ represents a unit vector with $\mathbf{v}_i^H \mathbf{v}_i = 1$ and $\mathbf{v}_i^H \mathbf{v}_j \neq 1 (i \neq j)$. The distance between two lines produced from unit vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ can be defined by the function $d(\mathbf{v}_1, \mathbf{v}_2)$, which represents the sine of the angle formed by these two lines [13], [12]

$$d(\mathbf{v}_1, \mathbf{v}_2) = \sin(\theta_{1,2}) = \sqrt{1 - |\mathbf{v}_1^H \mathbf{v}_2|^2}. \tag{8}$$

To maximize the precoding gain with codebook vectors is to maximize the minimum distance between any pair of lines spanned by the codebook vectors [12].

The minimum distance of a packing is the sine of the smallest angle between any pair of lines, i. e.

$$\delta(\mathbf{V}) = \min_{1 \leq k \leq l \leq P} \{ \sqrt{1 - |\mathbf{v}_1^H \mathbf{v}_2|^2} \}. \tag{9}$$

The Grassmannian codebook is designed so that

$$\{\mathbf{V}\} = \arg \max_{\mathbf{X} \in \mathcal{U}_{N_t}^P} \{\delta(\mathbf{X})\}, \tag{10}$$

where $\mathcal{U}_{N_t}^P$ is defined as the set containing all unitary matrices.

## III. MULTI-USER MIMO PRECODER DESIGN

In this paper, we address a problem that is common to all common codebook in limited feedback broadcast systems. As stated in the introduction, prior work has mainly focused on rotational schemes. Moreover, the standardization processes have so far ignored this problem. Thus, this paper comes up with a very simple and interesting solution.

In our proposal, we share the same codebook between the BS and all MS. To this end, we consider a Grassmannian codebook of size $P$. We consider a practical scenario where the partial CSI is provided at the BS through limited feedback. We assume that each user has a knowledge of its own CSI and no information about the channels of other users. In our study, we assume also that the feedback channel is error free and delay free and the Channel Quality Indicator (CQI) is sent directly (without quantization), since we are interested only in the effects of limited feedback on the precoding. However, the number of bits needed for the quantization of CQI is relatively low [14].

The detailed procedure of the proposed scheme is as follow :

1) First, each MS calculates the CQI, such as :

$$\mathbf{cqi}_k = \|\mathbf{H}_k \mathbf{v}_p\|_2^2; p = 1, \ldots, P. \tag{11}$$

2) Each MS must find the index of the best precoding vector in the Grassmannian codebook that can maximize the CQI :

$$\mathbf{w}_k = \arg \max_{\mathbf{v}_p \in \mathbf{V}} \{\mathbf{cqi}_k \mid p = 1, \ldots, P\}. \tag{12}$$

3) After obtaining the best precoding vector, each MS transmits its own index and the CQI to the BS through finite rate feedback.
4) The BS recovers the position of the precoding vector.
5) If two or more precoding vectors are identical, the BS asks the users with low CQI to transmit their higher CQI and respective index, excluding the first choice.
6) Else, the precoder combines the signal of each user with the selected $\mathbf{w}_k$.

 ISBN: 978-1-61208-284-4 

## IV. Simulation Results

In this section, we compare the performance in terms of error probability and sum rate of the proposed scheme regarding the proposals in [11] and [12]. In the simulations, the approach of Ding *et al.* [11] implies strictly the codebook rotation procedure. Also, for the sake of simplicity we consider the scheme of Fang *et al.* [12] without user selection.

In all simulations, a MU-MIMO broadcast channel is considered where the BS is equipped with two antennas transmitting to two MS with two receiver antennas each. We also consider that the BS transmits one data stream to each user with the same power allocation, QPSK modulation is taken into account and linear MMSE detector is adopted.

Results in Fig. 2 shows the comparison of BER performance versus the SNR. According to Fig. 2, we can see clearly that our proposal has the best performance. A gain of 2 dB and 4 dB is measured at $10^{-1}$ and $10^{-2}$ of BER, respectively.

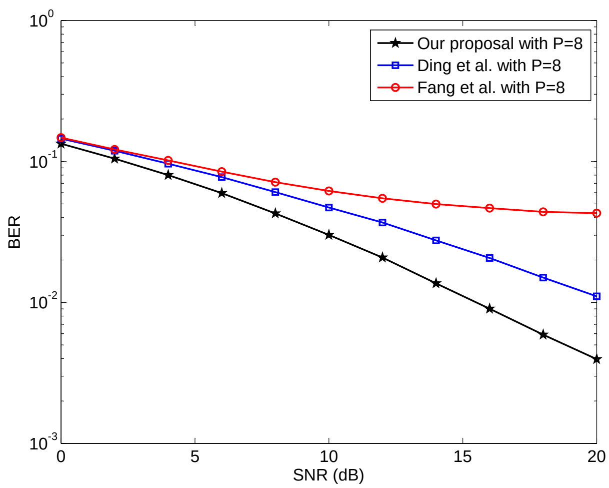


Fig. 2. BER comparison versus the SNR for 2x2 MU-MIMO system with $K = 2$ and $P = 8$.

Fig. 3 presents the sum rate CDF for the three schemes. The figure shows that the proposed scheme outperforms methods [11] and [12]. The figure shows also almost the same performance for methods [11] and [12]. It should be noted that the curves are plotted for SNR= 5dB.

The comparison of ergodic sum rate versus the SNR is depicted in Fig. 4. The figure shows that our proposal is superior regardless of the SNR level. As expected, a gain of 1 dB and 2 dB is observed for a low and high SNR, respectively.

## V. Conclusion

In this paper, we proposed to use a simple method to correct the choice of the precoding vectors in MU-MIMO broadcast channels. The proposed precoder is designed for the assumption of imperfect channel knowledge at the transmitter through limited feedback. The simulation results show a significant performance gain with respect to existing methods. Compared to [11], our scheme uses the commun codebook in all users.

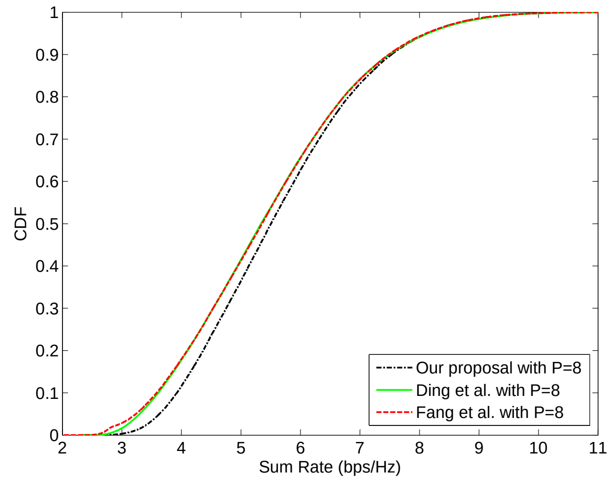


Fig. 3. Comparison of CDF of sum rate for 2x2 MU-MIMO system with $K = 2$ and $P = 8$.

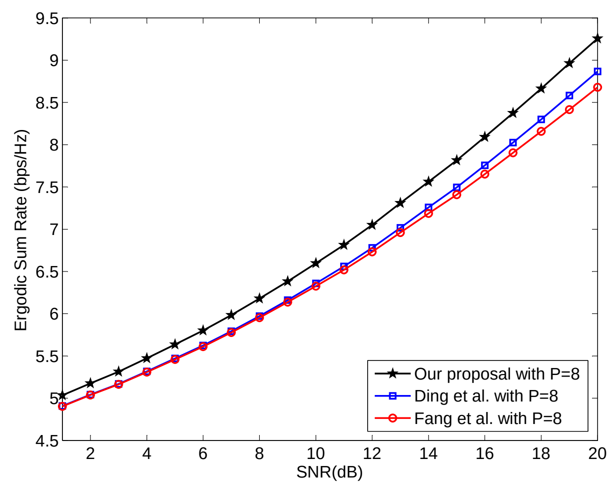


Fig. 4. Ergodic sum rate comparison for 2x2 MU-MIMO system with $K = 2$ and $P = 8$.

Moreover, the choice of precoding vectors is performed at both sides of the wireless link.

## Acknowledgment

The authors wish to thank the Microsystems Strategic Alliance of Quebec and the Natural Sciences and Engineering Research Council of Canada for its financial support.

## References

[1] N. Jindal, "MIMO broadcast channels with finite rate feedback," *IEEE Trans. Inform. Theory*, vol. 52, no. 11, pp. 5045-5059, Nov. 2006.
[2] A. B. Gershman and N. D. Sidiropoulos, *Space-Time Processing for MIMO Communications*, Chichester, U.K.: Wiley, 2005.
[3] G. Caire and S. Shamai (Shitz), "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. Inf. Theory*, vol. 49, no. 7, pp. 1691-1706, Jul. 2003.
[4] M. Costa, "Writing on dirty paper," *IEEE Trans. on Inf. Theory*, vol. 29, no.3, pp. 439-441, May 1983.
[5] J. Jing, R. Buehrer, and W. Tranter, "Spatial T-H precoding for packet data systems with scheduling," *in IEEE Fall Veh. Tech. Conf.*, Vol. 1, pp. 537-541, 2003.

[6] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, “Zero forcing methods for downlink spatial multiplexing in multiuser MIMO channels,” *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp.461-471, 2004.

[7] C. Peel, B. Hochwald, A. Swindlehurst, “Vector-perturbation technique for near-capacity multiantenna multiuser communication-part I: channel inversion and regularization,” *IEEE Trans. on Communications*, vol. 53, no.1, pp. 195-202, 2005.

[8] L. U. Choi and R. D. Murch, “A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach,” *IEEE Trans. on Wireless Commun.*, vol. 3, no. 1, pp. 20-24, 2004.

[9] D. J. Love, R. W. Heath, V. K. N. Lau, D. Gesbert, B. D. Rao, and M. Andrews, “An overview of limited feedback in wireless communication systems,” *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1341-1365, Oct. 2008.

[10] D. J. Love, R. Heath, W. Santipach, and M. Honig, “What is the value of limited feedback for MIMO channels?,” *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 54-59, Oct. 2004.

[11] P. Ding, D. J. Love, and M. D. Zoltowski, “Multiple antenna broadcast channels with shape feedback and limited feedback,” *IEEE Trans. Signal Processing*, vol. 55, pp. 3417-3428, July 2007.

[12] S. Fang, G. Wu, Y. Xiao, and S. Q. Li, “Multi-User MIMO Linear Precoding with Grassmannian Codebook,” *International Conference on Communications and Mobile Computing*, vol. 1, pp.250-255, 2009.

[13] D. J. Love, R. W. Heath, Jr., and T. Strohmer, “Grassmannian beamforming for multiple-input multiple-output wireless systems,” *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2735-2747, 2003.

[14] T. Yoo, N. Jindal, and A. Goldsmith, “Multi-antenna downlink channels with limited feedback and user selection,” *IEEE J. Select. Areas Commun.*, vol. 25, no. 7, pp. 1478-1491, Sept. 2007.

 ISBN: 978-1-61208-284-4 