A Semi-Analytical Performance Prediction of Turbo Coded SC-FDMA

Fatima Ezzahra Naamane[1], Mostafa Belkasmi[2]
Departement of Communication Networks
ENSIAS
Rabat, Morocco
Email: [1]naamane20@gmail.com,[2]m.belkasmi@um5s.net.ma

Mohamed Et-tolba
Departement of Communications Systems
INPT
Rabat, Morocco
Email: mohamed.ettolba@gmail.com

***Abstract*—Single carrier frequency division multiple access (SC-FDMA) is the access technique which is used in the long term evolution (LTE) uplink in order to reduce the peak-to-average power ratio (PAPR). The LTE uplink uses this technique in joint application with turbo coding and high order modulations to achieve peak data rates up to 86 Mbps within 20 MHz bandwidth. This paper introduces a semi analytical method for predicting the turbo coded SC-FDMA performance in terms of bit error rate (BER). Simulation results have shown that the BER obtained with the proposed method is close to that measured by Monte-Carlo simulation method.**

Keywords-SC-FDMA; LTE turbo code; Monte Carlo Method; BER Simulation; PDFs

I. Introduction

LTE [1] is designed to support peak data rates up to 300 Mbps and 86 Mbps for downlink and uplink, respectively, within 20 MHz bandwidth. In terms of system performance, it targets to improve user throughput, coverage and spectrum efficiency. In order to reach this performance, the LTE radio interface uses orthogonal frequency division multiple access (OFDMA) [2] as access technique for its robustness against both inter-symbol interference and channel impairments.

The LTE uplink transmission is based on SC-FDMA [3] to its reduced PAPR compared to OFDM. These features are used in joint application with multiple input multiple output (MIMO), adaptive modulation and coding (AMC) [4] and hybrid automatic repeat request (Hybrid ARQ) [5]. For reliable data transmission, the LTE uplink transmitter consists of a combination of error detection code, channel coding, interleaving, and SC-FDMA modulator. To meet the LTE requirement, the channel coding is performed, using a 1/3 rate turbo encoder which is often decoded with an iterative soft input soft output (SISO) decoder. The turbo encoder internal interleaver is based on quadratic permutation polynomials (QPP) to support high data rates. The performance of LTE uplink physical layer can be measured in terms of BER, block error rate (BLER) or throughput. In this work, our focus is BER evaluation which has been made by simulating the 3GPP LTE uplink transmitter, the transmission channel and the receiver. This simulation is done using Monte-Carlo (MC) method. However, it has been proven that it is prohibitive and time consuming. To makeupfor this limitation of MC simulation method, semi-analytical performance prediction has been proposed in several studies.

Bohdanowicz [6] proposed the importance sampling (IS) method for BER prediction. It has been found that for simple memoryless systems (e.g. a BPSK modem [7]), the efficiency of the IS technique is high and its implementation is relatively easier. However, its accuracy can be severely degraded, especially that of the coded systems.

For such systems, several solutions of performance prediction problem for complex system are carried out in several studies. In [8], a low complexity prediction technique for Turbo-Like code has been proposed. It is based on estimating the probability density function (pdf) of the log likelihood ratio (LLR) at the output of the decoder. The BER prediction is made assuming that the probability density functions of the decoder output LLRs are normal densities.

A semi-analytical approach of the BER prediction has been presented by Saoudi et al. [9]. The authors have proposed to predict the BER by using the known kernel estimator of the pdf of the soft decision made at the output of the detector. This method assumes that any prior information on the pdf of received samples is available at the receiver side. Moreover, its accuracy depends on the estimation of the smoothing parameter which is very important in the prediction process. Saoudi et al. [10] proposed an unsupervised soft BER prediction method for any digital communications systems. This technique considers that the pdf at the detector output is estimated with a Gaussian Kernel. The accuracy of this prediction method is very sensitive to smoothing parameter especially for high signal to noise ratios (SNRs).

In this paper, we propose a semi-analytical BER prediction method which is based on the pdf estimation using Gaussian kernel. We assume that no knowledge on the distribution of the received soft samples is currently available. In the proposed method, we have derived a new expression of the smoothing parameter which takes into account the histogram of the soft samples at the detector output. It has been shown that our method exhibits a significant accuracy in term of BER compared to Monte Carlo simulation method.

The remainder of the paper is organized as follows. In Section II, we present the coded SC-FDMA system. We introduce the problem of error probability derivation in Section III. Section IV details the probability density function estimation using kernel method and how one should select the bandwidth h to optimize the properties of the probability density function. Simulations and numerical results are given in Section V. Finally, the paper is concluded in Section VI.

II. Coded SC-FDMA System

The coded SC-FDMA system is presented in Figure 1; it consists of a source, a turbo code LTE, modulation mapper, modulation SC-FDMA, transmission channel and a receiver.

The coded SC-FDMA system will be the case of study the performance of the proposed method for BER prediction.

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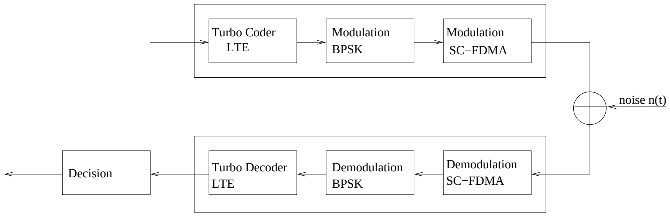

Fig. 1: Coded SC-FDMA System

A. Analytical form of SC-FDMA signal

This technique consists of distributing a large number of carriers, not directly as the source symbols in OFDM, but their frequency representation after having spread over the band of the system [11].

The signal of user k to the output of the system will be given by the expression:

$$S^k(t) = \sum_{n \in \Omega_n^k} U_n^k p(t - nT_s) e^{j2\pi f_n t} \tag{1}$$

- Let $\{f_n\}_{0<n<N-1}$ all orthogonal carrier frequencies modulated system, and f_c the center frequency (RF frequency) of the transmitted signal in the channel. It has the following relationship:

$$f_n = f_c + n\Delta f \tag{2}$$

Or $\Delta f = \frac{1}{T_s}$ is the spacing between sub-channels, with T_s the duration of a symbol.

- $\{U_n^k\}_{n \in \Omega_n^k}$, frequency representation of the symbols of the modulation block of user $k \in [0, 1, \cdots, L-1]$ with DFT obtained after the modulation.

We recall that the spectral spreading factor of the system is denoted L and the maximum number of users that can communicate simultaneously in the system.

- Ω_n^k represents the set of Q sub-carriers modulated by user k.

- $p(t)$ shows the shaping filter.

B. SC-FDMA demodulation

The principle of the demodulation of the SC-FDMA system is to demodulate the signal on each sub-carrier system. As a result, the received signal is first reduced to baseband, before being sampled for the digital signal processing. After removing the guard interval, a demodulator DFT provides the symbols modulating each carrier. An equalizer is then implemented, as in the technical SC / FDE, in order to eliminate the contribution of the channel on each subcarrier signal, and there by recover the symbol frequency. A demodulator IDFT can then retrieve the source symbols of the system. The signal received at user k receiver on symbol duration is written as follows:

$$y^k(t) = \sum_{n \in \Omega_n^k} U_n^k \int [h_n^k(t-\tau) p(t - nT_s) e^{j2\pi f_n t}\, d\tau \tag{3}$$

C. Turbo Coder LTE

The LTE system [12] has adopted a new structure for the turbo encoder. This is an improvement to turbo encoder interleaver by a new permutation polynomial based deterministic interleaver, called QPP interleaver and anti-interleaver with advantages beyond other interleavers. The encoder is characterized by a new structure simple, flexible operation and the most important is that parallel turbo decoding and register contention problem is solved successfully, which effectively increases the efficiency of the high-speed block parallel Turbo decoding [13].

In addition, the receiver contains an LTE turbo decoder based on the theory of iterative decoding. This is an important feature in the turbo- decoding, so decoding complexity lineally increases, with the size of the sequence information. In order to achieve a better decoding performance, component decoding must adopt soft input soft output (SISO) algorithm. MaxMAP decoding algorithm and SOVA decoding algorithm are two kinds of common soft-input soft-output Turbo decoding methods [14][15].

III. BIT ERROR PROBABILITY DERIVATION

To derive the bit error probability, we consider the general digital communication system presented in Figure 2. It consists of a source, a transmitter, transmission channel and a receiver. The source is considered to be digital and delivers the information bits $b \in \{0, 1\}$. These bits are processed by a transmitter which can include channel coding, interleaving, and modulation. After that, the information bits at the output of the transmitter are transmitted over a channel.

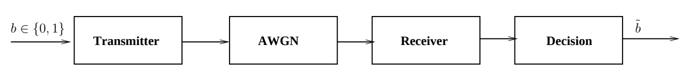

Fig. 2: General system model

For simplicity, the channel is assumed to be Gaussian [7]. The channel output is delivered to the receiver, which tries to detect the information bits from a noisy signal by using a detector, a sampling process and a decision. Due to the channel effect, the receiver can make a wrong decision on information bits at its output $\widetilde{b}$. So, it is important to measure the communication system efficiency. The most popular mean to do this is the BER evaluation. According to figure above, the bit error probability is written as :

$$p_e = P_1.Pr(\widetilde{b} = 0 \backslash b = 1) + P_0.Pr(\widetilde{b} = 1 \backslash b = 0) \tag{4}$$

where $P_k, k = 0, 1$, is the probability that $b = k$. $Pr(.)$ is the conditional probability. In terms of the decision threshold, the probability in (4) can be rewritten as follows:

$$p_e = P_1.Pr(y(t_0) < \mu\backslash b = 1) + P_0.Pr(y(t_0) \geq \mu\backslash b = 0) \quad (5)$$

where t_0 is the sampling time, μ is the decision threshold and:

$$\begin{cases} y(t_0) \geq 0; & \text{if } \tilde{b}_i = 1 \\ y(t_0) < 0; & \text{if } \tilde{b}_i = 0 \end{cases}$$

The conditional probabilities in (5) can be evaluated by integrating the probability density functions of the random variable y. If $f(y)$ denotes the pdf of y, the error probability is expressed as:

$$p_e = P_1. \int_{-\infty}^{\mu} f_1(y)\,dy + P_0. \int_{-\mu}^{+\infty} f_0(y)\,dy \quad (6)$$

So, to predict the error probability P_e, one has to estimate the probability density $f(y)$.

IV. PROBABILITY DENSITY FUNCTION AND ERROR PROBABILITY ESTIMATION :

A. Kernel estimator

Several types of non parametric estimation approaches are suggested to estimate a probability density function. This is due to the recent development in statistics theory. The most known of these methods is the kernel estimator [16] which we have adopted in this work. For a given set S of N received samples $y_1, y_2, \ldots, y_N$, the kernel estimator of the probability density function $\widetilde{f}(y)$ is [17][18]:

$$\widetilde{f}(y) = \frac{1}{Nh} \sum_{i=1}^{N} K(\frac{y - y_i}{h}) \quad (7)$$

where h is the smoothing parameter and $K(.)$ is the kernel function. To guaranty that $f(y)$ is a density; the kernel is a function that satisfies $\int_{-\infty}^{+\infty} K(u)\,du = 1$. Moreover, it is assumed to be symmetric about 0. In this work, we used the Gaussian kernel, which satisfies the above properties. The estimated density $\widetilde{f}(y)$ can be rewritten as:

$$\widetilde{f}(y) = \frac{(Nh)^{-1}}{\sqrt{2\pi}} \sum_{i=1}^{N} e^{-(\frac{y-y_i}{h})^2/2} \quad (8)$$

To get the expression of the estimated bit error probability p_e, we divide the set of observed samples into two subsets S_0 and S_1. The first subset contains N_0 observed samples, which correspond to the transmission of $b = 0$. The second subset consist of N_1 observed samples when $b = 1$ is transmitted. In this manner, and by substituting the probability density $f(y)$ by its estimate $\widetilde{f}(y)$, the estimated bit error probability is expressed as:

$$\widetilde{p}_e = \frac{P_0}{N_0} \sum_{i=1}^{N_0} Q(\frac{-(y_i)_0}{h_0}) + \frac{P_1}{N_1} \sum_{i=1}^{N_1} Q(\frac{(y_i)_1}{h_1}) \quad (9)$$

where $h_k, k = 0, 1$ is the smoothing parameter and $Q(.)$ denotes the complementary unit cumulative Gaussian distribution, that is,

$$Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt. \quad (10)$$

From (9), it is very clear that the accuracy of bit error probability estimation depends on the choice of the optimal smoothing parameter.

B. Choice of optimum smoothing parameter h

The mean integrated squared error (MISE) criterion is one of several methods used for selecting the optimum bandwidth. In MISE-based method, this optimum bandwidth is obtained by minimizing the MISE, which is expressed as:

$$MISE = E[\int_{-\infty}^{+\infty} [\widetilde{f}(y) - f(y)]^2\,dy] \quad (11)$$

The bandwidth that mimimizes MISE is given by:

$$h_{opt} = arg_h\ min\ (MISE(h)) \quad (12)$$

Under additional assumptions:

$$\lim_{N\to\infty} h = 0; \lim_{N\to\infty} Nh = 0; \quad (13)$$

Several types of MISE criterion have been suggested in litterature. Hereafter, we details the most populer ones.

1) Kernel based MISE Criterion:

For Kernel based MISE, we can use the normal reference method in selecting the bandwidth h for kernel estimator. As in [10], the optimum smoothing parameter is expressed as:

$$h_{opt,Kernel} = C1(f)C2(K)N^{-1/5} \quad (14)$$

$$C1(f) = [\int_{-\infty}^{+\infty} [f''(x)]^2\,dx]^{-1/5} \quad (15)$$

$$C2(K) = \frac{[\int_{-\infty}^{+\infty} [K(x)]^2\,dx]^{1/5}}{[\int_{-\infty}^{+\infty} [x^2 K(x)]^2\,dx]^{2/5}} \quad (16)$$

From (14), it is clear that the optimum parameter depends on the unknown pdf and also on the kernel $K(.)$. For a Gaussian kernel, we get $C2(K) = (2\sqrt{\pi})^{-1/5}$ and for the normal reference method $C1(f) = (8\sqrt{\pi}/3)^{1/5}\sigma$, yielding:

$$h_{opt,Kernel} = (4/3)^{1/5}\sigma N^{-1/5} = 1.06\sigma N^{-1/5} \quad (17)$$

Other way are used to select the bin size h. We focus on the normal reference method that uses the histogram of the PDF. Our approach is based on the latter method of calculating h. This method will be detailled in the following paragraph.

2) Histogram based MISE Criterion:

The histogram is used to measure the probability of observing a particular interval length [19]. It is a way to estimate the pdf by taking origin x_0 and a bin width h and define the bins of the histogram as the intervals $[x_0 + mh, x_0 + (m + 1)h]$ for positive and negative integers m. The histogram estimate of the pdf is then defined by:

$$H_N(x) = \frac{1}{Nh}(\text{number of } x_i \text{ in the same bin as } x) \quad (18)$$

Based on the same principle in the previous section, the MISE between $H_N(x)$ and the true pdf is written as:

$$MISE = \varepsilon[\int_{-\infty}^{+\infty} [H_N(x) - f(x)]^2\,dx] \quad (19)$$

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For a very large N ($N \to \infty$), the value of h needed in (18) can be shown [20] as:

$$h_{opt,Hist} = C1(f)N^{-1/3} \tag{20}$$

$$C1(f) = 6^{1/3}[\int_{-\infty}^{+\infty} [f''(x)]^2 \, dx]^{-1/3} \tag{21}$$

As can be seen, the optimum smoothing parameter depends only on the unknown pdf f. We need to find $C1(f)$ and under the assumption that $f \sim N(\mu, \sigma^2)$. This gives a simple data-based strategy for choosing the bin width h.

$$C1(f) = (24\sqrt{\pi})^{-1/3}\sigma \tag{22}$$

The optimum smoothing parameter is finaly written as:

$$h_{opt,Hist} = (24\sqrt{\pi})^{-1/3}\sigma N^{-1/3} \tag{23}$$

In practice, the choice of an efficient method for the calculation of h; for an observed data sample is a more complex problem, because of the effect of the bandwidth on the shape of the corresponding estimator. If the bandwidth is small, we will obtain an under-smoothed estimator, with high variability. On the contrary, if the value of h is important, the resulting estimator will be very smooth and farther from the function that we are attempting to estimate.

An example is drawn in Figure 3, where we show kernel estimators using the kernel function (the standard Gaussian density) and two different values for the bandwidth. The data sample consists of 1000 random numbers of an exponential distribution.

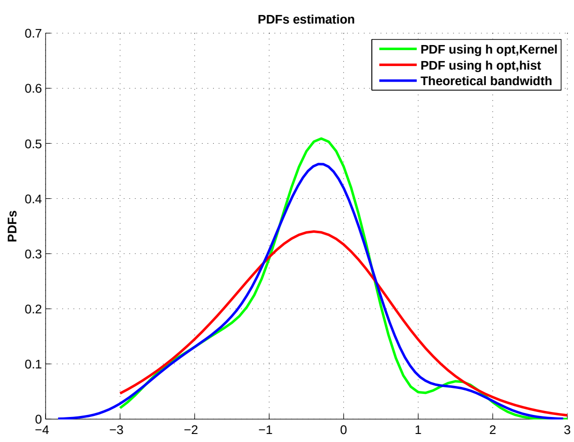

Fig. 3: Estimated densities for bandwidths chosen using different methods

For the data from a SC-FDMA system, the kernel estimators using the kernel function (the standard Gaussian density) and two different values for the bandwidth provides the result in Figure 4.

Figure 4 shows Gaussian kernel density estimates based on two different bandwidths for a sample of 500 data points from the SC-FDMA system. The second method MISE of bandwidth calculation h is of a good performance and can obtain an under smoothed estimator as the Kernel based MISE criterion.

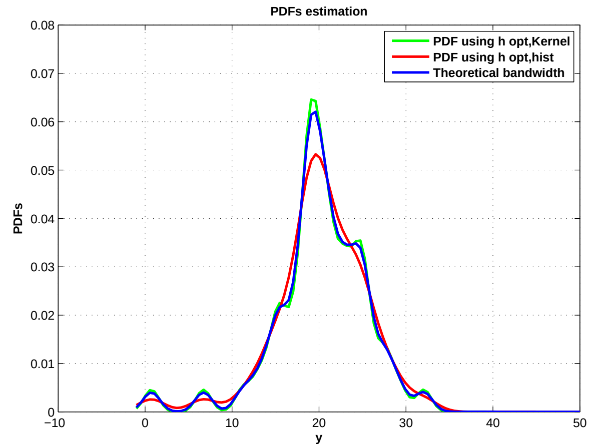

Fig. 4: Estimated densities for bandwidths chosen using different methods

Our practice shows that the histogram based MISE Criterion appears to be a suitable method for the choice of the bandwidth. As this method is based on estimating the histogram of samples, we have compared it to another method that is also based on the estimation of MISE using Kernel estimator. It is seen that the histogram based MISE Criterion ouperforms the Kernel based MISE Criterion in terms of squared errors.

Thus, we choose the histogram based MISE Criterion to study the performance of the new BER estimation method proposed in this paper.

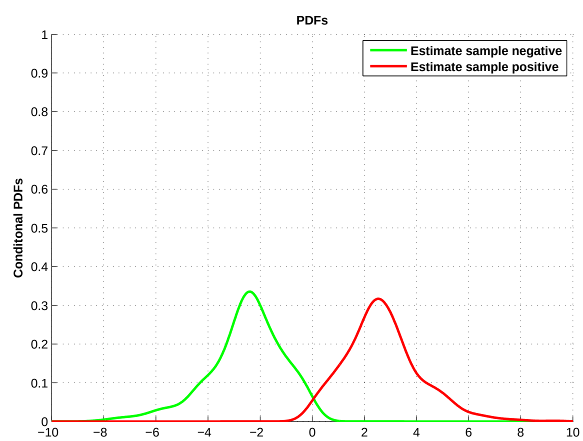

Fig. 5: Estimated Conditional pdfs for $SNR = 4dB$

Once the optimal smoothing parameter is calculated, we can present both the estimated conditional pdfs for $(y_i)_0$ and $(y_i)_1$ by using (9). We consider that the number of soft outputs

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whose simulator result is shown in Figure 5 is $N = 1024$ observations whose simulation result is shown in Figure 5.

V. Simulations and Numerical Results

To evaluate the performance of the proposed new method of estimating the BER, we consider two systems to simulate, namely BPSK system and a coded SC-FDMA system.

A. Validation of BER Prediction

In this section, we have shown that the new proposed BER estimator (Figure 6) provides the same performance as the Monte Carlo method whose data sample consists of 1000 random numbers of an exponential distribution.

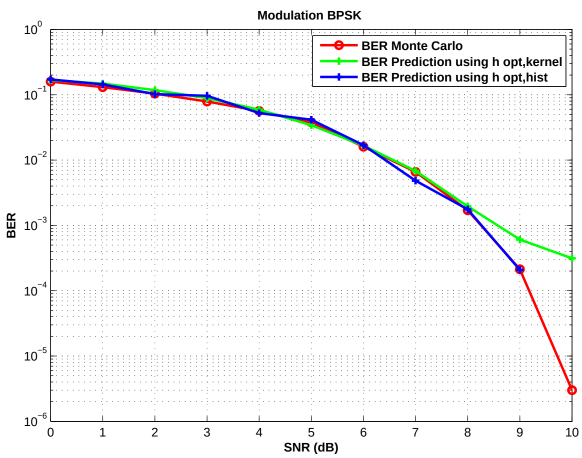

Fig. 6: BER perfomance for BPSK

This validation encourages us to study this performance for a coded SC-FDMA system.

B. BER prediction results for SC-FDMA

The objective of this section is to study and evaluate the implementation of the SC-FDMA technology encoded by a turbo encoder LTE on a Gaussian channel.

Our focus is on the behavior of the estimator proposed on the transmission bits by a coded SC-FDMA system. In such a system, it is difficult to have a reliable estimate of BER using MC-aided techniques with a limited number of soft observations and in the regions where the SNR is very high. In other words, the BER estimation performances of coded SC-FDMA system are studied in LTE uplink simulation system under Gaussien Channel.

In Figure 7, we can see that the new proposed BER estimator (9) provides the same performance as the Monte Carlo method, based on the perfect knowledge of the transmission bits. The proposed method is characterized, however, by the lack of knowledge of the received samples distribution of a SC-FDMA demodulation. We also present in Figure 7, the BER estimation performance using turbo encoded with SC-FDMA technique. The proposed technique provides reliable estimates, comparable to the Monte Carlo technique.

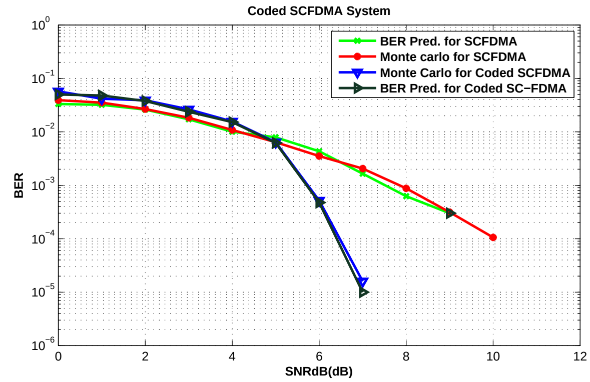

Fig. 7: BER performance comparison for Turbo Coded SC-FDMA

Note that the new technique allows a reliable estimate of BER for SNR up to $8dB$ values, while the MC-technique is unable to do so. It stops at $SNR = 8dB$ due to the very limited number of bits of information transmitted. This last conclusion presents a major advantage of the new estimator proposed for digital communications.

VI. Conclusion

The purpose of this article is to address the problem of estimation of BER for digital communication systems. In this context, we have proposed a semi-analytical method for predicting the turbo coded SC-FDMA performance in terms of bit error rate (BER), which is based on the pdf estimation using Gaussian kernel. We have assumed that no knowledge on the distribution of the received soft samples is available.

In the proposed method, we have derived a new expression of the smoothing parameter, which takes into account the histogram of the soft samples at the detector output. After, application to turbo coded BPSK modulation and coded SC-FDMA; we have concluded that have the same performance with either Monte Carlo technique (MC) or the new proposed BER estimator.

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