On the Performance of Selective Decode and Forward Relaying over Imperfectly Known Fading Relay Channels

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***Abstract*—In this paper, we investigate the effects of imperfect channel estimation on the performance of a Selective-Decode-and-Forward (SDF) relay-assisted communication system. In the system studied, a pilot symbol-assisted modulation (PSAM) scheme is used along with a channel estimation scheme based on minimum mean square error (MMSE). In particular, we derive an approximate expression for the bit error probability (BEP) in the presence of channel estimation error for the SDF cooperative protocol. Numerical simulations are presented to show that the derived approximate BEP expression is very close to the actual BEP. We also provide power-allocation that optimally assigns power constraint to training and data transmission phases to minimize the BEP.**

Keywords-cooperative communication; fading; channel estimation

I. INTRODUCTION

Recent advances in information and wireless technologies have led to growing interests in the development of multiple-input and multiple-output (MIMO) systems, which improves spectral and power efficiency of wireless networks. However, installing multiple antennas at transmitter and/or receiver results in size expansion of the devices, which is not practical in many wireless applications. A cooperative communication system can be considered as an alternative to MIMO systems. Cooperative techniques can exploit cooperative diversity by means of providing several copies of a signal that have experienced channel gains with low correlations [1][2][3]. Furthermore, early research in cooperative communication has shown that energy efficient transmitters operating in relay networks help extend the battery life [4].

According to the way in which the information is transmitted from the source terminal to the relay terminals and the way it is processed at the relay terminals, the existing cooperative protocols can usually be divided into three types – such as decode-and-forward (DF) protocols, selective decode-and-forward (SDF) protocols and amplify-and-forward (AF) protocols [1][2][3][5].

In the DF protocol, the relay terminals decode the received signal, then the sequence will be re-encoded and it will be forwarded to the destination terminal. In the SDF protocol, the relay terminals detect the signal and check if the detected signal contains any error. If the information sequence is considered error free, then the sequence will be re-encoded by either the same or a different code before it will be finally transmitted to the destination terminal. In AF protocols, the relay terminals simply re-transmit a scaled version of the signal that they receive from the source terminal to the destination terminal.

Most of the existing work on cooperative communications has assumed perfect knowledge of the channel fading coefficients at the receiver side, which is an overly optimistic assumption, and does not match with reality. For instance, W. Su et al. [6] and A. S. Ibrahim et al. [7] assume that the receiver has access to perfect information about the channel and the transmitter knows the instantaneous channel gain without the phase component. Various expressions for the symbol error probability of a cooperative communication system have been derived in [8][9][10][11] under the assumption of perfect channel state information (CSI) at both the relay and destination. Although research results based on these assumptions provide valuable insights, in practical systems these coefficients must be estimated and then used in the detection process. Especially, in mobile applications, the assumption of perfect channel knowledge is unwarranted as randomly varying channel conditions are learned by the receivers imperfectly.

In recent work (e.g., [12][13]), the effects of the channel estimation error on the bit error rate (BER) performance of cooperative communication systems have been studied by using a simple model for the channel estimation error, where the variance of the channel estimation error is assumed to be fixed for all values of the signal-to-noise ratio (SNR). Y. Wu et al. [14] investigate effects of channel estimation errors on the symbol-error-rate (SER) performance of a cooperative communication system operating in AF mode. J. Zhang et al. [15] assume that the receiver estimates the channel imperfectly, based on the pilot signal sent by the transmitter. Moreover, both the source and the relay perform an optimization on the power to be allocated to the pilot and data, assuming that achievable rate is the only factor to be optimized.

To the best of our knowledge, no previous work exists

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on the BEP performance of selective decode-and-forward relay communication system that uses pilot symbol-assisted modulation for channel estimation.

In the present paper, we derive a lower bound for BEP performance of binary phase shift keying (BPSK) modulation for the selective decode-and-forward relay communication system. We then apply the lower bound expression of BEP as the performance metric of the system, and solve a power allocation problem to optimally allocate power to training and data sequences.

The rest of the paper is organized as follows. We first introduce our system model and channel estimation in Section II. In Section III, the BEP expression is derived for the system under consideration. Simulation results are presented in Section IV, followed by conclusion and future work in Section V.

Notation: $\mathrm{E}\{\cdot\}$, $\{\cdot\}^*$, $\hat{x}$ denote the expected value, the complex conjugate and the estimate of the variable x, respectively.

II. SYSTEM MODEL AND CHANNEL ESTIMATION

We consider a three-node relay network, which consists of a source, relay, and destination node. This relay network model is depicted in Figure 1. We assume BPSK transmission over flat fading channels. Note that this assumption is imposed only for the convenience of notation. The analysis in this paper can also be applied, after some simple modifications, to the case wherein the source and the relay transmit using M-ary phase shift keying (MPSK) modulation. Let h_{sd}, h_{sr} and h_{rd} represent the source-destination, source-relay, and relay-destination channel gains, respectively. Each node-to-node channel gain is modelled by a zero-mean complex Gaussian random variable with variance σ_h^2. Furthermore, each channel is assumed to be constant during the frame transmission where each frame consists of a fixed number of symbols. A practical relay node with low cost usually cannot transmit and receive signal at the same time in the same frequency band. We assume that nodes transmit under half-duplex constraint in the same frequency band.

As in [1], the transmission protocol can be described as follows: First, the source transmits data to both relay and destination with power P_s. The received signals at the relay and the destination can be written as:

$$y_{sr} = \sqrt{P_s} h_{sr} x + n_{sr} \tag{1}$$

$$y_{sd} = \sqrt{P_s} h_{sd} x + n_{sd} \tag{2}$$

where n_{sr} and n_{sd} represent the additive noise terms, x represents the transmitted symbol with unit average energy, i.e., $\mathrm{E}\{|x|^2\} = 1$ and P_s is the source power (energy per symbol time) for data transmission. In the second time slot, if the relay correctly detects the received message in the first time slot, then it will forward it to the destination; otherwise, the relay will remain silent to avoid the propagation of errors [7]. Thus, the received signal at the destination in the second time slot is in the following form

$$y_{rd} = \theta \sqrt{P_r} h_{rd} x + n_{rd} \tag{3}$$

where θ can be either 0 or 1 indicating whether the relay was silent or not, n_{rd} is additive noise term, and P_r is the relay power (energy per symbol time) for data transmission. At the end of the second time slot, the destination will combine the desired signals from the source and the relay, if any, and attempt to detect the symbol.

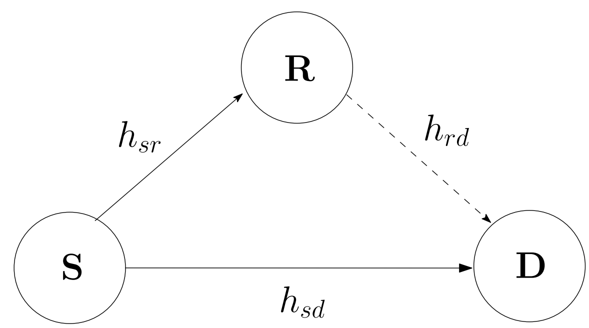

Fig. 1. Three-node relay network model

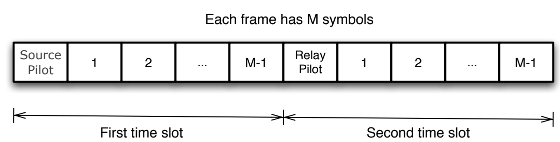

Fig. 2. Transmission structure in a block of M symbols

The transmission block consists of two phases−training phase and data transmission phase. We assume that the communication between nodes is through single-input-single-output (SISO) channel. For channel gain estimation of a SISO system, only one pilot symbol is required [15][16]. To reduce the variance of channel estimation errors, one can either increase the number of pilot symbols or use a higher pilot transmit power. As in [15], we assume that only one pilot symbol is used to estimate the channel coefficient. We point out that since block fading channels are considered, the accuracy of MMSE channel estimation depends only on the total training signal power regardless of the training duration, and increasing the number of training symbols in each block results in decreasing the throughput. We assume that, in the considered scheme, the source and relay can allocate power to pilot phase and data phase in different proportions. At the beginning of the frame, the source sends a pilot symbol, which we denote as s_p, to the relay and destination. The received signals, r_{sr}^p and r_{sd}^p can be expressed as follows

$$r_{sr}^p = h_{sr} s_p + n_{sr}$$

$$r_{sd}^p = h_{sd}s_p + n_{sd} \tag{4}$$

where r_{sr}^p and r_{sd}^p are the received pilot signals at the relay and at the destination, respectively. Then, the relay sends pilot symbol s_p' to the destination. The received pilot signal at the destination is as follows

$$r_{rd}^p = h_{rd}s_p' + n_{rd} \tag{5}$$

The noise terms n_{sd}, n_{sr}, and n_{rd} are modelled as zero mean complex Gaussian random variables with equal variance N_0 ($N_0/2$ per real dimension).

The system conveys source information as a sequence of frames, and each frame in the first time slot accommodates M symbols sent from the source–one symbol is dedicated for pilot and the rest are for data transmission (Figure 2). The second time slot of the frame accommodates M symbols sent from the relay–one symbol is dedicated for pilot and the rest are for data transmission (Figure 2). The energy allocated to a frame for the source's transmission is denoted by P_1, and the energy allocated to a frame for the relay's transmission is denoted by P_2 (that is, if the relay ends up transmitting in that frame). Parameter α denotes the fraction of source's energy allocated for transmission during the training phase, and β denotes the fraction of relay's energy allocated for transmission during the training phase. Thus, $|s_p|^2$ and $|s_p'|^2$, the transmit energies for the training phase are equal to αP_1 (if transmitted by the source) and βP_2 (if transmitted by the relay), respectively, where $0 < \alpha, \beta < 1$. Source power (energy per symbol time) for data transmission (P_s) and the relay power for data transmission (P_r) can be obtained as:

$$P_s = \frac{(1-\alpha)P_1}{M-1}, \quad P_r = \frac{(1-\beta)P_2}{M-1} \tag{6}$$

The relayed communication system considered in the present paper assumes that the relay and the destination both estimate the wireless channel gain from the received pilot signal using MMSE channel estimation method.

MMSE estimate of the channel is obtained at the relay and destination by using $\hat{h}_{sr} = \mathrm{E}\{h_{sr}r_{sr}^{p\,*}\} \cdot (\mathrm{E}\{r_{sr}^p r_{sr}^{p\,*}\})^{-1} \cdot r_{sr}^p$, $\hat{h}_{sd} = \mathrm{E}\{h_{sd}r_{sd}^{p\,*}\} \cdot \left(\mathrm{E}\{r_{sd}^p r_{sd}^{p\,*}\}\right)^{-1} \cdot r_{sd}^p$ and $\hat{h}_{rd} = \mathrm{E}\{h_{rd}r_{rd}^{p\,*}\} \cdot \left(\mathrm{E}\{r_{rd}^p r_{rd}^{p\,*}\}\right)^{-1} \cdot r_{rd}^p$ [15]. Based on [17], we can write

$$h_{ij} = \hat{h}_{ij} + e_{ij} \tag{7}$$

where e_{ij} is the channel estimation error modelled as a zero mean complex Gaussian random variable with variance $\sigma_{e_{ij}}^2$ and we have [17]:

$$\begin{aligned} \hat{h}_{ij} &\sim \mathcal{CN}\left(0, \frac{\sigma_h^4|s_p|^2}{\sigma_h^2|s_p|^2 + N_0}\right) \\ e_{ij} &\sim \mathcal{CN}\left(0, \frac{\sigma_h^2 N_0}{\sigma_h^2|s_p|^2 + N_0}\right) \end{aligned} \tag{8}$$

where $|s_p|^2$ is the power allocated to pilot symbol, which is equal to αP_1 or βP_2, depending on whether the source or relay is transmitting and $\mathcal{CN}(\cdot,\cdot)$ denotes complex Gaussian distribution. It can be easily shown that e_{ij} and $\hat{h}_{ij}$ are uncorrelated random variables and since they have jointly Gaussian distribution, they are also independent.

In the next section, we will use the analysis presented so far to derive the BEP expression.

III. BEP Analysis and optimal power allocation

In SDF protocols, the relay forwards a symbol only if the relay has high confidence that the symbol has been decoded correctly, and remains silent otherwise. It is reasonable to add a module for such decision making. From the destination node's vantage point, that decision made by the relay can only be guessed. Traditionally, three methods can be used to detect the presence of a signal [18]: energy detector, matched filter, and cyclostationary feature detection. Energy detection, which is the most popular method, is suboptimal and non-coherent and can be simply implemented [19]. Matched filter is a coherent detection that maximizes the signal to noise ratio. Cyclostationary feature detection exploits the inherent periodicity of the received signal. Besides all the methods mentioned above for detecting the presence of the signal, flag-based methods are also of interest [20]. However, sending a flag signal has the drawback of consuming additional energy and bandwidth. In this section, we assume that the destination knows θ in (3) perfectly. In other words, at each time slot, the destination knows whether the relay is in transmission mode or not. Then, we derive the optimal signal detection rule and a closed-form BEP expression for the system that uses MMSE channel estimator. Using (7), the received signals at the destination during two time slots can be written as

$$\begin{aligned} y_{sd}^i &= \sqrt{P_s}\hat{h}_{sd}x_i + \sqrt{P_s}e_{sd}x_i + n_{sd}^i \\ y_{rd}^i &= \theta_i\sqrt{P_r}\hat{h}_{rd}x_i + \theta_i\sqrt{P_r}e_{rd}x_i + n_{rd}^i \end{aligned} \tag{9}$$

where $i = 1, 2, ..., M-1$ and $\theta_i \in \{0, 1\}$. We consider a method in which symbols are detected individually (i.e., in a symbol-by-symbol fashion), which can be employed by a system that does not have sufficient memory space to store and consider the preceding symbols. With θ known by the destination, optimal detection rule to be used by the destination can be written as

$$\hat{x} = \arg \max_{x\in\{-1,+1\}} p(y_{sd}, y_{rd}|x, \hat{h}_{sd}, \hat{h}_{rd}, \theta) \tag{10}$$

By using (9), conditioned on the estimated channel gains ($\hat{h}_{sd}$ and $\hat{h}_{rd}$), transmitted symbol (x) and θ, it can be easily seen that y_{sd} and y_{rd} are two independent complex Gaussian random variables with means $\sqrt{P_s}\hat{h}_{sd}x$, $\theta\sqrt{P_r}\hat{h}_{rd}x$ and variances $P_s\sigma_{e_{sd}}^2 + N_0$, $\theta P_r\sigma_{e_{rd}}^2 + N_0$, respectively. The decision rule becomes

$$\hat{x} = \arg \max_{x\in\{-1,+1\}} \left(\frac{1}{\pi(P_s\sigma_{e_{sd}}^2 + N_0)} e^{\frac{-|y_{sd} - \sqrt{P_s}\hat{h}_{sd}x|^2}{P_s\sigma_{e_{sd}}^2 + N_0}} \times \frac{1}{\pi(\theta P_r\sigma_{e_{rd}}^2 + N_0)} e^{\frac{-|y_{rd} - \theta\sqrt{P_r}\hat{h}_{rd}x|^2}{\theta P_r\sigma_{e_{rd}}^2 + N_0}} \right) \tag{11}$$

The decision rule is simply the minimum-distance decision rule, where

$$\hat{x} = \arg \min_{x\in\{-1,+1\}} \left(\frac{|y_{sd} - \sqrt{P_s}\hat{h}_{sd}x|^2}{P_s\sigma^2_{e_{sd}} + N_0} + \frac{|y_{rd} - \theta\sqrt{P_r}\hat{h}_{rd}x|^2}{\theta P_r\sigma^2_{e_{rd}} + N_0} \right)$$
$$= \arg \max_{x\in\{-1,+1\}} \mathrm{Re}\{ \left(\frac{\sqrt{P_s}}{P_s\sigma^2_{e_{sd}} + N_0}\hat{h}^*_{sd}y_{sd} + \frac{\theta\sqrt{P_r}}{P_r\sigma^2_{e_{rd}} + N_0}\hat{h}^*_{rd}y_{rd} \right) x\} \quad (12)$$

In order to implement this decision rule, we can use maximal ratio combining (MRC) that treats the estimated channels as true channels [21]

$$y^{\mathrm{MRC}} = \frac{\sqrt{P_s}}{P_s\sigma^2_{e_{sd}} + N_0}\hat{h}^*_{sd}y_{sd} + \frac{\theta\sqrt{P_r}}{P_r\sigma^2_{e_{rd}} + N_0}\hat{h}^*_{rd}y_{rd} \quad (13)$$

Conditioned on the channel gains, by substituting (9) in (13) and after some simple manipulations, the SNR at the receiver can be written as follows [24]

$$\gamma^{\mathrm{MRC}} = \frac{P_s}{P_s\sigma^2_{e_{sd}} + N_0}|\hat{h}_{sd}|^2 + \frac{\theta P_r}{P_r\sigma^2_{e_{rd}} + N_0}|\hat{h}_{rd}|^2 \quad (14)$$

We now examine a closed form expression of BEP. Conditioned on $\hat{h}_{sd}$, $\hat{h}_{rd}$, and θ, the BEP can be written as [24]

$$P\left(e|\hat{h}_{sd}, \hat{h}_{rd}, \theta\right) = Q\left(\sqrt{2\left(A|\hat{h}_{sd}|^2 + \theta B|\hat{h}_{rd}|^2\right)}\right) \quad (15)$$

where

$$A = \frac{P_s}{P_s\sigma^2_{e_{sd}} + N_0}, \; B = \frac{P_r}{P_r\sigma^2_{e_{rd}} + N_0} \quad (16)$$

We denote $|\hat{h}_{sd}|^2$ and $|\hat{h}_{rd}|^2$ by X and Y respectively. In accordance with the complex Gaussian channel model, X and Y are exponentially distributed; we can therefore write

$$f_X(x) = \lambda_{sd}e^{-\lambda_{sd}x}, \; x \geq 0, \; \lambda_{sd} = \frac{1}{\sigma^2_{\hat{h}_{sd}}}$$
$$f_Y(y) = \lambda_{rd}e^{-\lambda_{rd}y}, \; y \geq 0, \; \lambda_{rd} = \frac{1}{\sigma^2_{\hat{h}_{rd}}} \quad (17)$$

where

$$\lambda_{sd} = \frac{\alpha P_1\sigma^2_h + N_0}{\alpha P_1\sigma^4_h}, \; \lambda_{rd} = \frac{\beta P_2\sigma^2_h + N_0}{\beta P_2\sigma^4_h} \quad (18)$$

Conditioned on θ and using (15), (17), and $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2\varphi}}d\varphi$ [24], BEP can be written as

$$P(e|\theta) = \int_0^\infty P\left(e|x, y, \theta\right) f_X(x)f_Y(y)dxdy$$
$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\int_0^\infty\int_0^\infty \exp\left(-\frac{Ax + \theta By}{\sin^2\varphi}\right) f_X(x)f_Y(y)dxdyd\varphi \quad (19)$$

Conditional probability of error can be rewritten as

$$P(e|\theta) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\int_0^\infty \lambda_{sd}e^{-(\frac{A}{\sin^2\varphi}+\lambda_{sd})x}dx \times \int_0^\infty \lambda_{rd}e^{-(\frac{B\theta}{\sin^2\varphi}+\lambda_{rd})y}dy\, d\varphi$$
$$= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\lambda_{sd}\sin^2\varphi}{\lambda_{sd}\sin^2\varphi + A} \cdot \frac{\lambda_{rd}\sin^2\varphi}{\lambda_{rd}\sin^2\varphi + B\theta}d\varphi \quad (20)$$

After some simple manipulation, we can write

$$P(e|\theta) = \frac{1}{2} + \frac{1}{\pi}\frac{1}{\frac{B\theta}{\lambda_{rd}} - \frac{A}{\lambda_{sd}}}\left[\int_0^{\frac{\pi}{2}} \frac{(\frac{A}{\lambda_{sd}})^2}{\sin^2\varphi + \frac{A}{\lambda_{sd}}}d\varphi - \int_0^{\frac{\pi}{2}} \frac{(\frac{B\theta}{\lambda_{rd}})^2}{\sin^2\varphi + \frac{B\theta}{\lambda_{rd}}}d\varphi\right] \quad (21)$$

By using the result from [25, p. 177, 2.562.1], which is

$$\int \frac{dx}{a + b\sin^2 x} = \frac{\mathrm{sign}\, a}{\sqrt{a(a+b)}}\tan^{-1}\left(\sqrt{\frac{a+b}{a}}\tan x\right),$$

and

$$P(e) = P(e|\theta = 0)P(\theta = 0) + P(e|\theta = 1)P(\theta = 1) \quad (22)$$

and after some manipulations, the probability of error can be expressed as in (23) at the top of the next page. We point out that the probability that the relay detects the symbol with error (i.e., $P(\theta = 0)$) can be written as $P(\theta = 0) = \frac{1}{2}\left(1 - \sqrt{\frac{A}{A+\lambda_{sd}}}\right)$.

Remark: In the derivation of (23), it is assumed that the relay is perfectly capable of diagnosing whether there has been an error in the detection of the symbol transmitted by the source. Based on this diagnosis, if the detection was erroneous the relay would not forward that symbol to the destination in the second time slot. In a practical system, error detection scheme (e.g., cyclic redundancy check) could be implemented. In the following, we prove (23) is a lower bound of the BEP for a more practical system in which the relay uses a block error detection scheme:

I) The case in which the relay mistakenly concludes that there is a symbol that is detected wrong in the frame (while all the symbols were detected correctly at the relay): In this case, at the second time slot, the relay remains silent and the whole frame is kept from being forwarded. In the practical system, the final symbol detection at the destination is made only based on the received signal at the first time slot, which results in a higher BEP than the ideal system of Section II.

II) The case in which the relay's block error detection mistakenly concludes that all the detected symbols in the frame are error-free (while there is a symbol that is detected with error at the relay): In the practical system, the relay forwards the frame which includes some symbol different from the source's and the frame may contain some corrected decoded symbols. In the ideal system of Section II, which leads to (23), only correctly decoded symbols will be forwarded. For the

$$P(e)=\frac{1}{4}\left[1+\sqrt{\frac{A}{A+\lambda_{sd}}}\right]\left[1+\frac{\lambda_{sd}\lambda_{rd}}{B\lambda_{sd}-A\lambda_{rd}}\left(\frac{A}{\lambda_{sd}}\sqrt{\frac{A}{A+\lambda_{sd}}}-\frac{B}{\lambda_{rd}}\sqrt{\frac{B}{B+\lambda_{rd}}}\right)\right]+\frac{1}{4}\left[1-\sqrt{\frac{A}{A+\lambda_{sd}}}\right]^2 \tag{23}$$

correctly decoded symbols, the destination's symbol detection, which combines the signal from the source and the signal from the relay, will result in the same probability of error in both the practical system and the ideal system. For a symbol that is detected by the relay with error, in the practical system the destination will end up combining the signal from the source and a signal carrying a different symbol from the relay, while in the ideal system of section II the destination will detect it based only on the signal form the source. Therefore, the ideal system will have less likelihood of making a symbol (bit) error.

III) In the case in which the relay correctly concludes that the frame contains a symbol that is decoded with error at the relay: In this case, in the system the whole frame is kept from being forwarded. In the ideal system, only the corrected decoded symbols will be forwarded, and for these symbols the ideal system will combine both source's and relay's signals. Therefore, the ideal system has a lower chance of making error.

IV) The case in which the relay correctly concludes that the frame contains no symbol error: In both the practical system and the ideal system of Section II, the whole frame is forwarded to the destination. In this case, both systems will have the same detection mechanism of individual symbols in the destination. Finally, from I, II, III, and IV, it can be easily concluded that (23) is a lower bound for a system that uses block error detection at the relay.

In order to optimally allocate power to training and data transmission phases so that (23) is minimized, we formulate an optimization problem as

$$(\alpha_{opt},\beta_{opt},r_{opt})=\arg\min_{\alpha,\beta,r} P_e(\alpha,\beta,r)$$
$$\text{subject to}: 0\le \text{r}\le 1,\ \ 0\le\alpha\le 1,\ \ 0\le\beta\le 1 \tag{24}$$

where $r=P_1/P$ is the ratio of source power constraint to total power constraint (i.e., $P_1=r\cdot P$ and $P_2=(1-r)\cdot P$).

IV. SIMULATION RESULTS

In this section, Monte-Carlo simulation results are presented and compared with (23). Matlab was used for Monte-Carlo simulation, and 10^8 transmitted symbols were drawn from the BPSK constellation in order to estimate the BEP. The node-to-node channels are modelled by zero mean independent Gaussian random variables with unit variance.

Figure 3 shows the BEP analysis proposed in this paper in comparison with the simulation results for M=4, 8, 16 and 64 where M is the frame length. Parameters α, β and $r=P_1/P$ are set to 0.30, 0.30 and 0.61 respectively. As the figure shows, all the simulation curves and analytical expression are very close. Note that the energy allocated to each data symbol can be obtained from (6). In order to numerically compare the BEP curve obtained by simulation and the BEP curve obtained by analytical expression, for each frame length (M) we define the ratio d_M as $d_M=(\text{BEP}_{\text{simulation}}\text{-BEP}_{\text{analytical expression}})/\text{BEP}_{\text{simulation}}$. For P/N_0=15 dB, α=β=0.30 and $r=P_1/P$=0.61, we have d_{64}=0.0005, d_{16}=0.0011, d_8=0.0011, and d_4=0.0010. Also

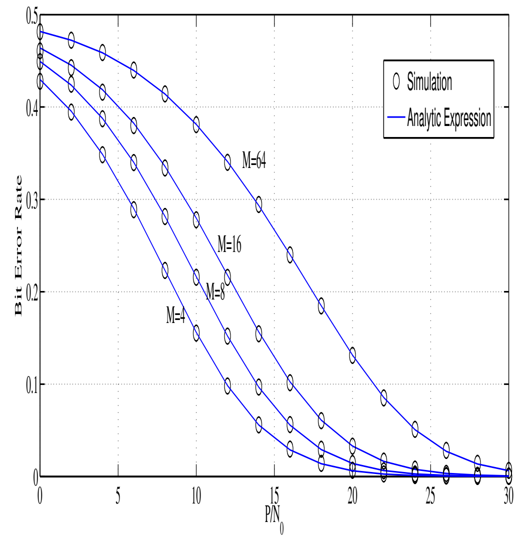

Fig. 3. BEP per P/N_0 for α=β=0.3, $r=P_1/P=0.61$ and different frame lengths

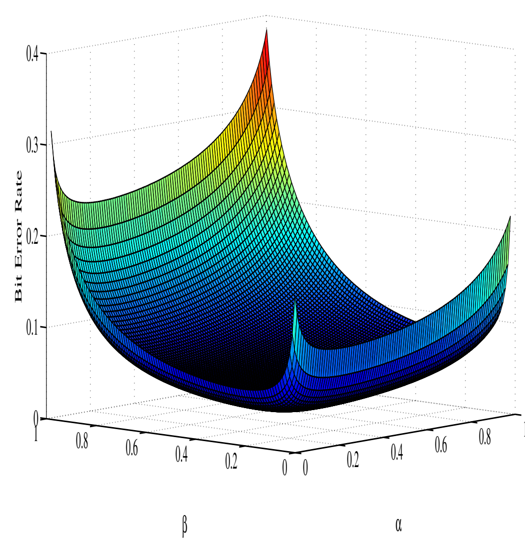

Fig. 4. BEP per α and β, P/N_0=15 dB, $P_1/P=0.61$ and M=4

TABLE I
RESULTS OF OPTIMIZATION FOR M= 4, 8, 16 AND 64 UNDER P/N_0 =15 dB and 25 dB

		M= 4	M=8	M=16	M= 64
$\mathrm{P/N_0} = 15\mathrm{dB}$	r	0.61	0.61	0.71	1
	α	0.35	0.29	0.24	0.18
	β	0.35	0.29	0.26	-
$\mathrm{P/N_0} = 25\mathrm{dB}$	r	0.61	0.61	0.61	0.61
	α	0.34	0.24	0.19	0.13
	β	0.34	0.20	0.16	0.13

for P/N_0=30 dB we have: d_{64}=0.0010, d_{16}=0.0024, d_8=-0.0075, and d_4=-0.0053. These results indicate that there is little difference between the values of BEP obtained from simulation and analytical expression.

In Figure 4, the BEP is plotted against both α and β for P/N_0=15 dB, $r = 0.61$, and M=4. From the derived BEP expression (23), it is apparent that for a fixed total power, the power allocation among pilot and data affects the BEP performance. If we allocate too much power for pilots, channel estimation error will be reduced but because of low data SNR, detection of the data in noise is more difficult. On the other hand, lower power for pilots results in poor channel estimation and thus in poor detection [22], [23]. The minimum value of BEP under P/N_0=15 dB, $r = 0.61$ and M=4, turns out to be at $\alpha = \beta = 0.35$.

In Table I, the optimum values for α, β and r for M= 4, 8, 16 and 64 under P/N_0 =15 dB and 25 dB are tabulated. To obtain the optimal values of α, β and r, an optimization problem was formulated as in (24) and solved.

V. CONCLUSION AND FUTURE WORK

We have investigated the impact of channel estimation error on the performance of a selective decode-and-forward relay communication network that uses a practical model for channel estimation. We have derived a lower bound expression for the BEP performance of considered scheme. We presented numerical simulation to show the proposed approximate analytical formulation is very close to the actual BEP. It is observed that channel estimation error causes loss of BEP performance. For a given power budget, this loss is decreased, by optimal power allocation between pilot symbol and data symbols based on the lower bound expression for BEP.

This paper assumed that the destination node perfectly knows whether the relay at each time slot is in silent mode or not. We are currently designing the destination's schemes for detecting whether the relay has forwarded the symbols in the frame or not. The study of these schemes and analyzing the BEP performance are left for future research.

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