

Uplink Performance Evaluation of Broadband Systems which Adopt a Massive MU-MIMO Approach

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Abstract—This paper deals with an Orthogonal Frequency Division Multiplexing (OFDM)-based uplink within a Multi User (MU)-Multi-Input Multi-Output (MIMO) system where a "massive MIMO" approach" is adopted. In this context, either an optimum Minimum Mean-Squared Error (MMSE) linear detection or a reduced-complexity Matched Filter (MF) linear detection are considered. Regarding performance evaluation by simulation, several semi-analytical methods are proposed: one performance evaluation method in the optimum (MMSE) case; two performance evaluation methods in the MF case. This paper includes performance results for uncoded 4-Quadrature Amplitude Modulation (QAM)/OFDM transmission and a MU-MIMO channel with uncorrelated Rayleigh fading, under the assumptions of perfect power control and perfect channel estimation. The accuracy of performance results obtained through the semi-analytical simulation methods is assessed by means of parallel conventional Monte Carlo simulations [10]. The performance results are discussed in detail and we also emphasize the achievable "massive MIMO" effects, even for the reduced-complexity detection techniques, provided that the number of BS antennas is much higher than the number of antennas which are jointly employed in the terminals of the multiple autonomous users.

Keywords- *Broadband Wireless Communications; MU-MIMO Systems; Massive MIMO; Performance Evaluation; OFDM.*

I. INTRODUCTION

The development of MIMO technologies has been crucial for the "success story" of broadband wireless communications in the last two decades [1]. Through spatial multiplexing schemes, following and extending ideas early presented in [2], MIMO systems are currently able to provide very high bandwidth efficiencies and a reliable radiotransmission at data rates beyond 1 Gigabit/s. Appropriate MIMO detection schemes, offering a range of performance/complexity tradeoffs, have been essential for the technological improvements in this area [1][3][4]. In the last decade, MU-MIMO systems have been successfully implemented and introduced in several broadband communication standards [5]; in such "space division multiple access" systems, the more antennas the Base Station (BS) is equipped with, the more users can simultaneously communicate in the same time-frequency resource.

In recent years, the adoption of MU-MIMO systems with a very large number of antennas in the BS, much larger than the number of Mobile Terminal (MT) antennas in its cell, was

proposed by Marzetta [6]. This "massive MIMO" approach has been shown to be recommendable for several reasons [6][7][8][9]: simple linear processing for MIMO detection becomes nearly optimal; both MultiUser Interference (MUI) effects and fast fading effects of multipath propagation tend to disappear; both power efficiency and bandwidth efficiency become substantially increased.

This paper deals with an OFDM-based uplink within a MU-MIMO system where the BS is constrained to adopt simple, linear detection techniques but can be equipped with a large number of receiver antennas. In this context, either an optimum (MMSE) linear detection or a reduced-complexity MF linear detection are considered in Section II. Regarding performance evaluation by simulation, several semi-analytical methods are proposed in Section III, all of them combining simulated channel realizations and analytical computations of BER performance which are conditional on those channel realizations: one performance evaluation method in the optimum (MMSE) case; two performance evaluation methods in the MF case.

In Section IV, this paper includes performance results for uncoded 4-QAM/OFDM transmission and a MU-MIMO channel with uncorrelated Rayleigh fading effects regarding the several transmitter/receiver (TX/RX) antenna pairs, under the assumptions of perfect power control and perfect channel estimation. The accuracy of performance results obtained through the semi-analytical simulation methods is assessed by means of parallel conventional Monte Carlo simulations (involving an error counting procedure). The performance results are discussed in detail and we also emphasize the achievable "massive MIMO" effects, even for the reduced-complexity detection techniques, provided that the number of BS antennas is much higher than the number of antennas which are jointly employed in the terminals of the multiple autonomous users. Section V includes the main conclusions of the paper.

II. SYSTEM MODEL

A. OFDM-based Radiotransmission

We consider here a Cyclic Prefix (CP)-assisted, OFDM-based, block transmission, within a MU-MIMO system with N_T TX antennas and N_R RX antennas - for example (but not necessarily) one antenna per MT, as depicted in Figure 1(a).

We assume, in the j th TX antenna ($j = 1, 2, \dots, N_T$), a length- N block $\mathbf{S}^{(j)} = [S_0^{(j)}, S_1^{(j)}, \dots, S_{N-1}^{(j)}]^T$ of frequency-domain data symbols in accordance with the corresponding binary data block. The insertion of a length- L_s CP, long enough to cope with the time-dispersive effects of multipath propagation, is also assumed after the IDFT that is required to bring the data block information to the time domain.

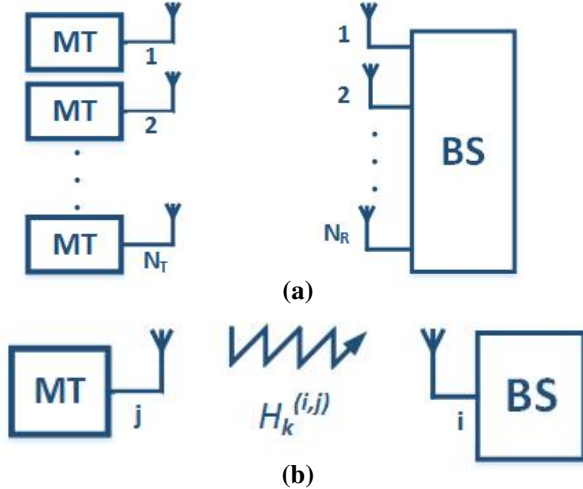


Figure 1. Uplink of an $N_T \times N_R$ MU-MIMO system (a) and channel characterization, at subcarrier k , concerning the antenna pair (i, j) (b).

The frequency-domain data symbols $S_k^{(j)}$ ($k = 0, 1, \dots, N-1; j = 1, 2, \dots, N_T$) are randomly and independently selected from a QAM alphabet ($E[S_k^{(j)}] = 0$ and $E[|S_k^{(j)}|^2] = \sigma_S^2$ for any (j, k)).

For any subcarrier, the frequency domain transmission rule can be described as follows:

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{S}_k + \mathbf{N}_k, \quad (1)$$

where $\mathbf{S}_k = [S_k^{(1)}, S_k^{(2)}, \dots, S_k^{(N_T)}]^T$ is the "input" vector, $\mathbf{N}_k = [N_k^{(1)}, N_k^{(2)}, \dots, N_k^{(N_R)}]^T$ is the Gaussian noise vector ($E[N_k^{(i)}] = 0$ and $E[|N_k^{(i)}|^2] = \sigma_N^2 = N_0 N$), \mathbf{H}_k

denotes the $N_R \times N_T$ channel matrix with entries $H_k^{(i,j)}$, concerning a given channel realization (RX antenna i and TX antenna j , for each subcarrier, as shown in Figure 1(b)) and $\mathbf{Y}_k = [Y_k^{(1)}, Y_k^{(2)}, \dots, Y_k^{(N_R)}]^T$ is the resulting "output" vector.

By assuming $E[H_k^{(i,j)}] = 0$ and a constant

$$E[|H_k^{(i,j)}|^2] = P_\Sigma, \quad (2)$$

for any (i, j, k) , and a 4-QAM/OFDM block transmission, the average bit energy at each BS antenna is given by

$$E_b = \frac{\sigma_S^2}{2\eta N} P_\Sigma, \quad (3)$$

where $\eta = \frac{N}{N+L_s}$, with L_s denoting the CP length.

B. Optimum (MMSE) and MF Linear Detection Techniques

With regard to subcarrier k , two linear detection techniques are considered, as shown in Figure 2, both directly providing frequency-domain decisions $\hat{\mathbf{S}}_k$ based on the frequency-domain detector output $\tilde{\mathbf{Y}}_k$ ($k = 0, 1, \dots, N-1$).

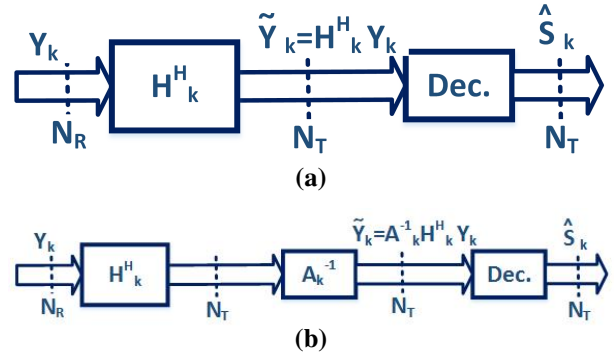


Figure 2. Reduced-complexity (MF) (a) and optimum (MMSE) (b) linear detection techniques, regarding subcarrier k .

As Figure 2(a) indicates,

$$\tilde{\mathbf{Y}}_k = \mathbf{H}_k^H \mathbf{Y}_k \quad (4)$$

for the reduced-complexity linear detection technique. The $\tilde{Y}_k^{(j)}$ components of $\tilde{\mathbf{Y}}_k$ ($k = 0, 1, 2, \dots, N-1; j = 1, 2, \dots, N_T$) are given by

$$\tilde{Y}_k^{(j)} = \sum_{i=1}^{N_R} H_k^{(i,j)*} Y_k^{(i)}, \quad (5)$$

which clearly means, for any (j, k) pair, a Maximal Ratio Combining (MRC) procedure, involving MF for each component of the length- N_R received vector \mathbf{Y}_k .

For the optimum (MMSE) linear detection technique, it can be shown that [4]

$$\tilde{\mathbf{Y}}_k = \mathbf{A}_k^{-1} \mathbf{H}_k^H \mathbf{Y}_k, \quad (6)$$

where

$$\mathbf{A}_k = \mathbf{H}_k^H \mathbf{H}_k + \alpha \mathbf{I}_{N_T}, \quad (7)$$

with $\alpha = \frac{\sigma_N^2}{\sigma_S^2} = N_0 \frac{N}{\sigma_S^2}$.

III. SEMY-ANALYTICAL METHODS FOR PERFORMANCE EVALUATION

A. Performance Evaluation Method in the Optimum (MMSE) Case

The frequency-domain output $\tilde{\mathbf{Y}}_k$ of the MMSE detector in Figure 2 (b) includes Gaussian noise and residual MUI terms in its N_T components. Regarding the j th component of $\tilde{\mathbf{Y}}_k$, the resulting Signal-to-Interference-plus-Noise-Ratio (SINR) can be derived by resorting to well-known "MMSE estimation" principles. It can be written as

$$\text{SINR}_{j,k} = \frac{\Gamma_k^{(j,j)}}{1 - \Gamma_k^{(j,j)}}, \quad (8)$$

where $\Gamma_k^{(j,j)}$ is the (j, j) entry of

$$\mathbf{\Gamma}_k = [\mathbf{H}_k^H \mathbf{H}_k + \alpha \mathbf{I}_{N_T}]^{-1} \mathbf{H}_k^H \mathbf{H}_k, \quad (9)$$

$$\left(\Gamma_k^{(j,j)} = \frac{E[S_k^{(j)*} \tilde{Y}_k^{(j)}]}{\sigma_S^2}, \right. \quad \text{since}$$

$$\tilde{Y}_k^{(j)} = \Gamma_k^{(j,j)} S_k^{(j)} + \text{'uncorrelated noise - like term'} \left. \right).$$

For 4-QAM/OFDM, the resulting $BER_{j,k}$ - conditional on the channel realization \mathbf{H}_k ($k = 0, 1, \dots, N-1$) - is given by

$$BER_{j,k} \approx Q\left(\sqrt{SINR_{j,k}}\right), \quad (10)$$

with $SINR_{j,k}$ according to (8). The average BER for the overall channel realization \mathbf{H}_k ($k = 0, 1, \dots, N-1$) can be computed as follows:

$$BER = \frac{1}{N_T} \sum_{j=1}^{N_T} BER_j, \quad (11)$$

where

$$BER_j = \frac{1}{N} \sum_{k=0}^{N-1} BER_{j,k}, \quad (12)$$

B. Performance Evaluation Methods in the MF Case

The N_T components of the frequency domain output $\tilde{\mathbf{Y}}_k$, in the MF detector of Figure 2 (a), can be decomposed - into "useful signal", MUI and "Gaussian noise" - as follows:

$$\tilde{Y}_k^{(j)} = \sum_{i=1}^{N_R} |H_k^{(i,j)}|^2 S_k^{(j)} + \sum_{l \neq j} \sum_{i=1}^{N_R} H_k^{(i,l)} H_k^{(i,j)*} S_k^{(l)} + \sum_{i=1}^{N_R} H_k^{(i,j)*} N_k^{(i)}, \quad (13)$$

($j = 1, 2, \dots, N_T$). Regarding the j th component of $\tilde{\mathbf{Y}}_k$, the resulting SINR can be given by

$$SINR_{j,k} = \frac{\sigma_S^2 \left(\sum_{i=1}^{N_R} |H_k^{(i,j)}|^2 \right)^2}{\sigma_S^2 \sum_{l \neq j} \left| \sum_{i=1}^{N_R} H_k^{(i,l)} H_k^{(i,j)*} \right|^2 + NN_0 \sum_{i=1}^{N_R} |H_k^{(i,j)}|^2}$$

$$= \frac{1}{\alpha_k^{(j)}} \sum_{i=1}^{N_R} |H_k^{(i,j)}|^2, \quad (14)$$

where

$$\alpha_k^{(j)} = \alpha + \sum_{l \neq j} \alpha_k^{(l,j)}, \quad (15)$$

with $\alpha = N_0 \frac{N}{\sigma_S^2}$ and

$$\alpha_k^{(l,j)} = \frac{\left| \sum_{i=1}^{N_R} H_k^{(i,j)*} H_k^{(i,l)} \right|^2}{\sum_{i=1}^{N_R} |H_k^{(i,j)}|^2}, \quad (16)$$

For 4-QAM/OFDM, the BER computation for the overall channel realization \mathbf{H}_k ($k = 0, 1, \dots, N-1$) can be performed in accordance with eqns. (10), (11) and (12), through the use of $SINR_{j,k}$ given by eqns. (14), (15) and (16).

The second semi-analytical method for performance evaluation considers the MUI as it is, and does not necessarily regard it as a "quasi-Gaussian interference". By assuming $\sigma_S = \sqrt{2}$ (i.e., $S_k^{(j)} = \pm 1 \pm j$), it is easy to derive

$$BER_{j,k} = \frac{1}{2^{2N_T-1}} \sum_{\{\tilde{S}_k^{(l)}; l \neq j\}} \left[Q \left(\frac{H_\Sigma + \Re e \left\{ \sum_{l \neq j} \left(\sum_{i=1}^{N_R} H_k^{(i,l)} H_k^{(i,j)*} \right) \tilde{S}_k^{(l)} \right\}}{\sqrt{N \frac{N_0}{2} \sum_{i=1}^{N_R} |H_k^{(i,j)}|^2}} \right) + \right. \\ \left. + Q \left(\frac{H_\Sigma + \Im m \left\{ \sum_{l \neq j} \left(\sum_{i=1}^{N_R} H_k^{(i,l)} H_k^{(i,j)*} \right) \tilde{S}_k^{(l)} \right\}}{\sqrt{N \frac{N_0}{2} \sum_{i=1}^{N_R} |H_k^{(i,j)}|^2}} \right) \right], \quad (17)$$

with $H_\Sigma = \sum_{i=1}^{N_R} |H_k^{(i,j)}|^2$; then, by resorting to eqns. (11) and (12), we can get the average BER for the "overall channel" realization \mathbf{H}_k ($k = 0, 1, \dots, N-1$).

IV. NUMERICAL PERFORMANCE RESULTS AND DISCUSSION OF MASSIVE MIMO EFFECTS

In the following, we present a set of performance results for uncoded 4-QAM/OFDM uplink block transmission, with $N = 256$ and $L_s = 64$, within a MU-MIMO $N_T \times N_R$ Rayleigh fading channel. The fading effects regarding the several TX/RX antenna pairs are assumed to be uncorrelated, with all zero-mean complex Gaussian $H_k^{(i,j)}$ channel coefficients having the same variance P_Σ (see Section II-A).

Figure 5 and Figure 6 involve subsets of BER performance curves taken from Figure 3 and Figure 4, respectively: in both cases, we selected the $N_T \times N_R$ MU-MIMO MF linear detection and the SIMO $1 \times N_R$ (single-path) performance, for several values of N_R . Figure 7 shows BER performance results, under MF linear detection, for $N_R = 100$ and several values of N_T [SIMO $1 \times N_R$ (single-path) case also included.].

With regard to both linear detection techniques of Section II-B, the several performance results concerning the MU-MIMO system have been obtained by random generation of a large number of channel realizations, analytical BER computation - according to the methods of Section III - for each channel realization, and an averaging operation over the set of channel realizations. The accuracy of performance results obtained through these semi-analytical simulation methods was assessed by means of parallel conventional Monte Carlo simulations (involving an error counting procedure). As expected, having in mind the subcarrier-by-subcarrier detection procedure in the uncoded QAM/OFDM block transmission context, the achieved performances turned out to be the same for frequency-flat and frequency-selective fading conditions (under the assumption of a CP long enough to cope with the channel time dispersion).

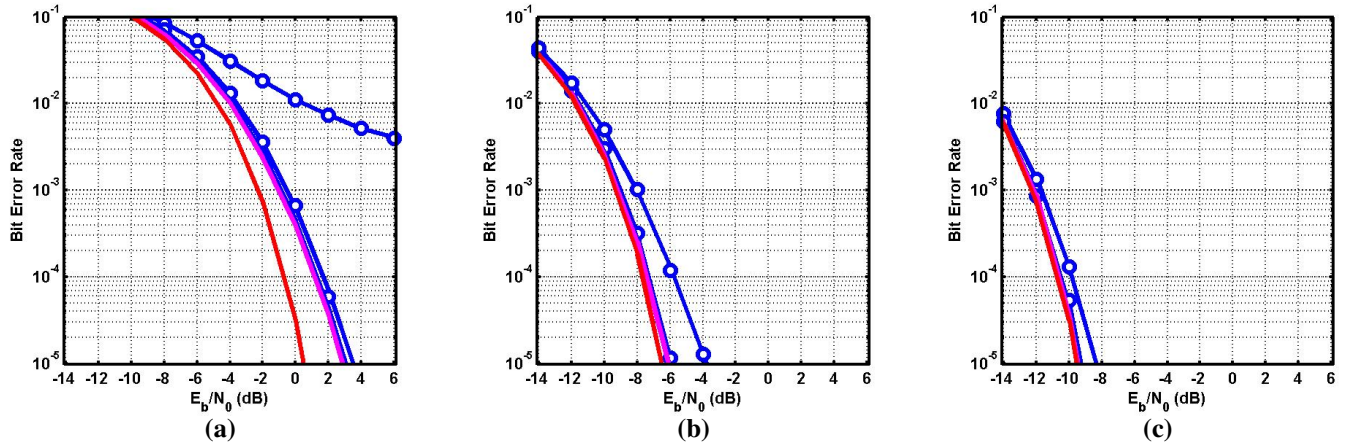


Figure 3. BER performances for OFDM-based MU-MIMO with $N_T = 2$, and $N_R = 10$ (a), 50 (b) or 100 (c), under linear (MF, MMSE) and ML detection [SIMO $1 \times N_R$ (single-path, multipath) reference BER performances are also included, and the five BER performances are ordered, from the worst to the best, as explained in section IV].

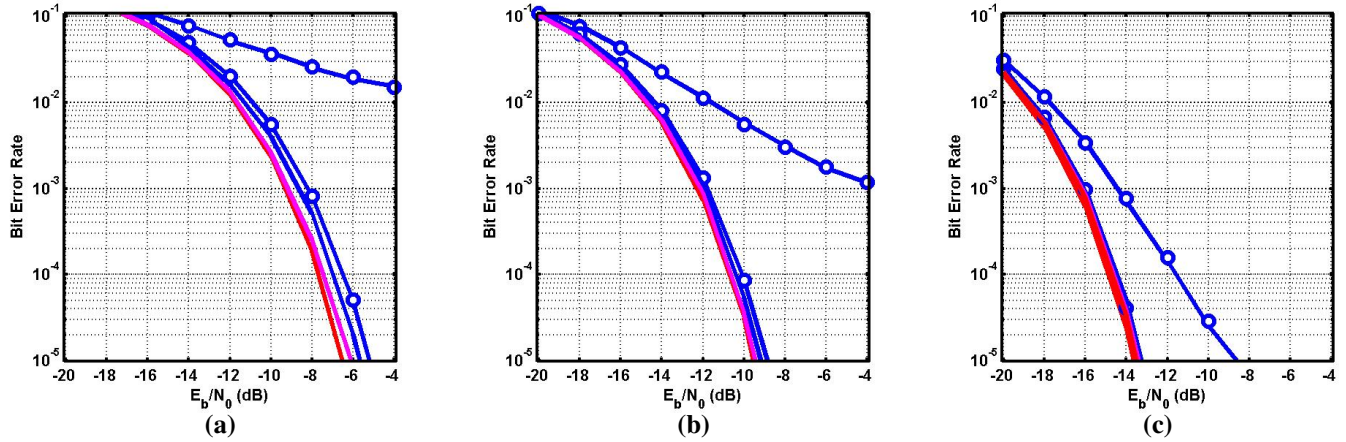


Figure 4. BER performances for OFDM-based MU-MIMO with $N_T = 10$, and $N_R = 50$ (a), 100 (b) or 250 (c), under linear (MF, MMSE) and ML detection [SIMO $1 \times N_R$ (single-path, multipath) reference BER performances are also included, and the five BER performances are ordered, from the worst to the best, as explained in section IV].

When $N_R \gg N_T$, both the MUI effects and the fading effects of multipath propagation tend to disappear: consequently, the BER performances for the MU-MIMO $N_T \times N_R$ Rayleigh fading channel become very close to those concerning a SIMO $1 \times N_R$ channel with single-path propagation for all N_R TX/RX antenna pairs. The achievable performances under a "truly massive" MU-MIMO implementation can be analytically derived as explained in the following.

Entries of \mathbf{H}_k are i.i.d. Gaussian-distributed random variables with zero mean and variance P_Σ . According to the law of large numbers [10],

$$\lim_{N_R \rightarrow \infty} \left[\frac{1}{N_R} \sum_{i=1}^{N_R} |H_k^{(i,j)}|^2 \right] = E \left[|H_k^{(i,j)}|^2 \right] = P_\Sigma \quad (18)$$

and

$$\begin{aligned} \lim_{N_R \rightarrow \infty} \left[\frac{1}{N_R} \sum_{\substack{i=1 \\ (l \neq j)}}^{N_R} H_k^{(i,j)*} H_k^{(i,l)} \right] &= \quad (19) \\ = \frac{E}{(l \neq j)} \left[H_k^{(i,j)*} H_k^{(i,l)} \right] &= 0. \end{aligned}$$

Therefore, having in mind (14), (15), (16) and (3),

$$\begin{aligned} \lim_{N_R \rightarrow \infty} \left(\frac{SINR_{j,k}}{N_R} \right) &= \frac{\sigma_S^2}{N_0 N} \lim_{N_R \rightarrow \infty} \left(\frac{\sum_{i=1}^{N_R} |H_k^{(i,j)}|^2}{N_R} \right) \\ &= \frac{\sigma_S^2}{N_0 N} P_\Sigma = 2\eta \frac{E_b}{N_0} \quad (20) \end{aligned}$$

When $N_R \gg N_T$ (which also implies $N_R \gg 1$)

$$SINR_{j,k} \approx N_R \lim_{N_R \rightarrow \infty} \left[\frac{SINR_{j,k}}{N_R} \right] = 2\eta N_R \frac{E_b}{N_0}, \quad (21)$$

leading to

$$BER \approx Q \left(\sqrt{2\eta N_R \frac{E_b}{N_0}} \right). \quad (22)$$

Figures 3 and 4 show the simulated BER performances for an OFDM-based MU-MIMO uplink and several possibilities regarding N_T and N_R , when using the linear (MF and

MMSE) detection techniques of Section II. In both figures, for the sake of comparisons, we include Maximum Likelihood (ML) detection results concerning the $N_T \times N_R$ system; we also include SIMO $1 \times N_R$ reference performances, for both the multipath propagation channel - which implies a Rayleigh fading concerning each TX/RX antenna pair - and an ideal single-path propagation channel. For the linear detection techniques, the semi-analytical methods of Section III have been adopted; the complementary conventional Monte Carlo simulation (involving error counting) results correspond to the superposed circles in the solid lines.

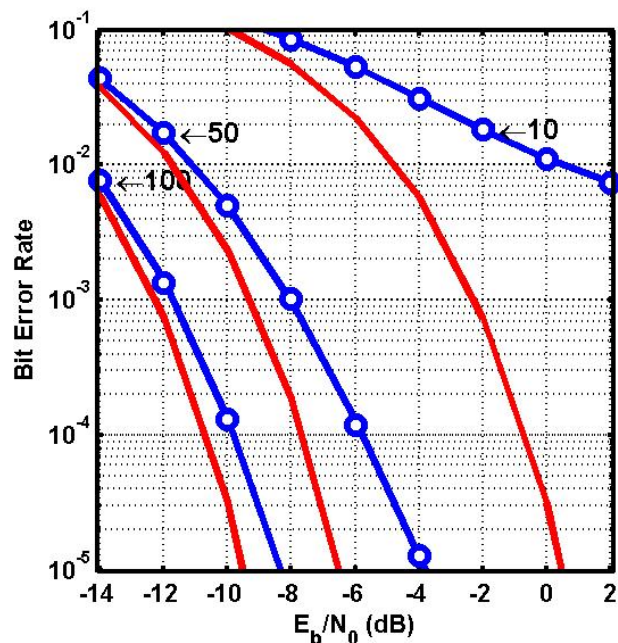


Figure 5. BER performances for OFDM-based MU-MIMO with $N_T = 2$, and $N_R = 10, 50$ or 100 , under MF linear detection [SIMO $1 \times N_R$ (single-path) reference performances are also included].

In all figures, where the $1 \times N_R$ (single-path case) SIMO detection performance was analytically computed according to (22) - an excellent agreement of the semi-analytical simulation results with conventional Monte Carlo simulation results can be observed.

In the simulation results concerning all subfigures of both Figure 3 and Figure 4, the five BER performance curves have been shown to be ordered, from the worst to the best, as follows: $N_T \times N_R$ MU-MIMO with MF linear detection; $N_T \times N_R$ MU-MIMO with MMSE linear detection; $N_T \times N_R$ MU-MIMO with ML detection; $1 \times N_R$ (multipath case) SIMO detection; $1 \times N_R$ (single-path case) SIMO detection. These figures clearly show that the performance penalty which is inherent to the reduced-complexity (MF) linear detection - as compared with the optimum (MMSE) linear detection and even the optimum (ML) detection - can be made quite small, by increasing N_R significantly; they also show that, under highly increased N_R values, the "MUI-free" SIMO (multipath) performance and the ultimate bound - the "MUI-free and fading-free" SIMO (single-path) performance - can be closely approximated, even when adopting the reduced-complexity (MF) linear detection. Figure 5 and Figure 6 emphasize the performance benefits of an increased N_R , for a given N_T , and

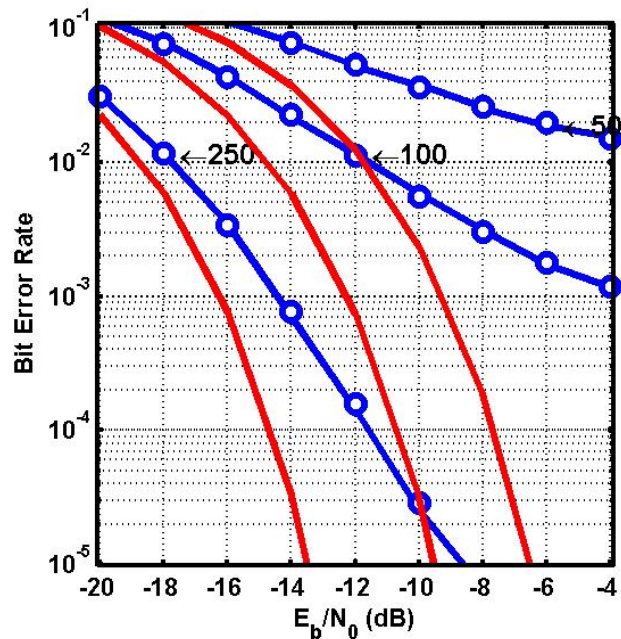


Figure 6. BER performances for OFDM-based MU-MIMO with $N_T = 10$, and $N_R = 50, 100$ or 250 , under MF linear detection [SIMO $1 \times N_R$ (single-path) reference performances are also included].

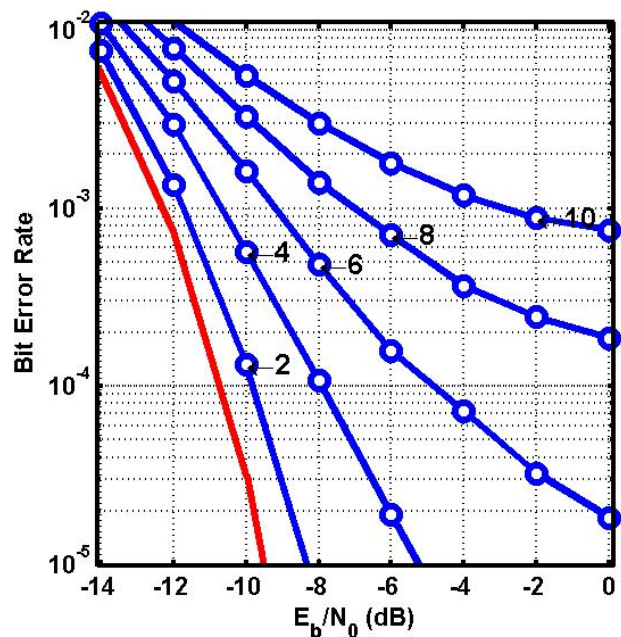


Figure 7. BER performances for OFDM-based MU-MIMO with $N_R = 100$, and $N_T = 2, 4, 6, 8$ or 10 , under MF linear detection [SIMO $1 \times N_R$ (single-path) reference performance is also included].

Figure 7 emphasizes the more or less acceptable performance degradation levels which are unavoidable when N_R is kept fixed and N_T increases (under the reduced-complexity detection in all cases). This set of figures emphasizes a "massive MIMO" effect when $N_R \gg 1$, especially when $N_R \gg N_T$ too, which leads to BER performances very close to the ultimate "MUI-free and fading-free" SIMO (single-path) performance

bound.

V. CONCLUSIONS

This paper was dedicated to the uplink performance evaluation of a MU-MIMO system with OFDM transmission, when adopting a large number of antennas and linear detection techniques at the BS. The accuracy of performance results obtained by semi-analytical means, as proposed in Section III, was demonstrated.

The numerical performance results, discussed in detail in Section IV, show the "massive MIMO" effects provided by a number of BS antennas much higher than the number of antennas which are jointly employed in the terminals of the multiple autonomous users, even when a reduced-complexity (MF) linear detection technique is adopted.

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