On the SC/FDE Uplink Alternative to OFDM in a Massive MU-MIMO Context

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Abstract—This paper deals with Single Carrier (SC)/Frequency Domain Equalization (FDE) as an uplink alternative to Orthogonal Frequency Division Multiplexing (OFDM) for a Multi User (MU)-Multi-Input Multi-Output (MIMO) system where a "massive MIMO" approach is adopted. In this context, either an optimum Minimum Mean-Squared Error (MMSE) linear detector or appropriate reduced-complexity linear detection techniques are considered. Regarding performance evaluation by simulation, two semi-analytical methods are proposed - one method in the optimum (MMSE) case and the other one in the reduced-complexity cases. This paper includes performance results for uncoded 4-Quadrature Amplitude Modulation (QAM) SC/FDE transmission and a MU-MIMO channel with uncorrelated Rayleigh fading, under the assumptions of perfect power control and perfect channel estimation. The accuracy of performance results obtained through the semi-analytical simulation methods is assessed by means of parallel conventional Monte Carlo simulations. The performance results are discussed in detail and we also emphasize the achievable "massive MIMO" effects, even for the reducedcomplexity detection techniques, provided that the number of BS antennas is much higher than the number of antennas which are jointly employed in the terminals of the multiple autonomous users. Appropriate "SC/FDE vs OFDM" comparisons are also included in this discussion of performance results.

Keywords-Broadband Wireless Communications; MU-MIMO Systems; Massive MIMO; Performance Evaluation; SC/FDE.

I. INTRODUCTION

Cyclic Prefix (CP)-assisted block transmission schemes were proposed and developed, in the last two decades, for broadband wireless systems, which have to deal with strongly frequency-selective fading channel conditions. These schemes take advantage of current low-cost, flexible, Fast Fourier Transform (FFT)-based signal processing technology, with both OFDM and SC/FDE alternative choices [1][2][3]. Mixed air interface solutions, with OFDM for the downlink and SC/FDE for the uplink, as proposed in [2], are now widely accepted; the main reason for replacing OFDM by SC/FDE, with regard to uplink transmission, is the lower envelope fluctuation of the transmitted signals when data symbols are directly defined in the time domain, leading to reduced power amplification problems at the mobile terminals.

Also in the last two decades, the development of MIMO technologies has been crucial for the "success story" of broadband wireless communications. Through spatial multiplexing schemes, early introduced by Foschini [4], and appropriate MIMO detection schemes [5][6] - offering a range of performance/complexity tradeoffs -, MIMO systems are currently able to provide very high bandwidth efficiencies and a reliable radiotransmission at very high data rates. In the last decade, MU-MIMO systems [7] - able to serve multiple autonomous users in the same time-frequency resource, thereby providing a true "space division multiple access" - also have been successfully implemented and introduced in several broadband communication standards.

In recent years, the adoption of a very large number of antennas in the BS, much larger than the number of Mobile Terminal (MT) antennas in its cell, was proposed in [8][9]. This "massive MIMO" approach has been shown to be recommendable for several reasons [8][9]: simple linear processing for MIMO detection becomes nearly optimal; both MultiUser Interference (MUI) effects and fast fading effects of multipath propagation tend to disappear; both power efficiency and bandwidth efficiency become substantially increased.

This paper deals with SC/FDE as an uplink alternative to OFDM [10] for a MU-MIMO system where the BS is constrained to adopt simple, linear detection techniques, but can be equipped with a large number of receiver antennas. In this context, either an optimum (MMSE) linear detector or appropriate reduced-complexity, Matched Filter (MF)based, linear detection techniques are considered in Section II. Regarding performance evaluation by simulation, two semianalytical methods are proposed in Section III - one method in the optimum (MMSE) case and the other one in the reduced-complexity cases -, both combining simulated channel realizations and analytical computations of BER performance which are conditional on those channel realizations.

In Section IV, this paper includes performance results for uncoded 4-QAM SC/FDE transmission and a MU-MIMO channel with uncorrelated Rayleigh fading effects regarding the several transmitter/receiver (TX/RX) antenna pairs, under the assumptions of perfect power control and perfect channel estimation. The accuracy of performance results obtained through the semi-analytical simulation methods is assessed by means of parallel conventional Monte Carlo simulations (involving an error counting procedure). The performance results are discussed in detail and we also emphasize the achievable "massive MIMO" effects, even for the reducedcomplexity detection techniques, provided that the number of BS antennas is much higher than the number of antennas which are jointly employed in the terminals of the multiple autonomous users. Appropriate "SC/FDE vs OFDM" comparisons are also included in this discussion of performance results. Section V includes the main conclusions of the paper.

II. SYSTEM MODEL

A. SC/FDE-based Radiotransmission

We consider here a CP-assisted, SC/FDE-based, block transmission, within a MU-MIMO system with N_T TX antennas and N_R RX antennas - for example (but not necessarily) one antenna per MT. We assume, in the *j*th TX antenna $(j = 1, 2, ..., N_T)$, a length-N block $\mathbf{s}^{(j)} = [s_0^{(j)}, s_1^{(j)}, ..., s_{N-1}^{(j)}]^T$ of time-domain data symbols in accordance with the corresponding binary data block. The insertion of a length-Ls CP, long enough to cope with the time-dispersive effects of multipath propagation, is also assumed.

The time-domain data symbols $s_n^{(j)}$ $(n = 0, 1, \dots, N - 1; j = 1, 2, \dots, N_T)$ are randomly and independently selected from a QAM alphabet $\left(E\left[s_n^{(j)}\right] = 0 \text{ and } E\left[\left|s_n^{(j)}\right|^2\right] = \sigma_s^2\right)$ for any (j, n).

By using the frequency-domain version of the time-domain data blocks $\mathbf{s}^{(j)} = \left[s_0^{(j)}, s_1^{(j)}, \cdots, s_{N-1}^{(j)}\right]^T$, given by $\mathbf{S}^{(j)} = \left[S_0^{(j)}, S_1^{(j)}, \cdots, S_{N-1}^{(j)}\right]^T = DFT(\mathbf{s}^{(j)}) \ (j = 1, 2, \cdots, N_T)$, we can describe the frequency-domain transmission rule as follows, for any subchannel k:

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{S}_k + \mathbf{N}_k,\tag{1}$$

where $\mathbf{S}_{k} = \begin{bmatrix} S_{k}^{(1)}, S_{k}^{(2)}, \dots, S_{k}^{(N_{T})} \end{bmatrix}^{T}$ is the "input vector", $\mathbf{N}_{k} = \begin{bmatrix} N_{k}^{(1)}, N_{k}^{(2)}, \dots, N_{k}^{(N_{R})} \end{bmatrix}^{T}$ is the Gaussian noise vector $\left(E \begin{bmatrix} N_{k}^{(i)} \end{bmatrix} = 0$ and $E \begin{bmatrix} \left| N_{k}^{(i)} \right|^{2} \end{bmatrix} = \sigma_{N}^{2} = N_{0}N \right)$, \mathbf{H}_{k} denotes the $N_{R} \times N_{T}$ channel matrix with entries $H_{k}^{(i,j)}$, concerning a given channel realization, and $\mathbf{Y}_{k} = \begin{bmatrix} Y_{k}^{(1)}, Y_{k}^{(2)}, \dots, Y_{k}^{(N_{R})} \end{bmatrix}^{T}$ is the resulting, frequency-domain, "output" vector.

As to a given MIMO channel realization, it should be noted that the Channel Frequency Response (CFR) $\mathbf{H}^{(i,j)} = \begin{bmatrix} H_0^{(i,j)}, H_1^{(i,j)}, \dots, H_{N-1}^{(i,j)} \end{bmatrix}^T$, concerning the antenna pair (i,j), is the DFT of the Channel Impulse Response (CIR) $\mathbf{h}^{(i,j)} = \begin{bmatrix} h_0^{(i,j)}, h_1^{(i,j)}, \dots, h_{N-1}^{(i,j)} \end{bmatrix}^T$, where $h_n^{(i,j)} = 0$ for n > Ls $(n = 0, 1, \dots, N-1)$. Regarding a statistical channel model - which encompasses all possible channel realizations -, let us assume that $E \begin{bmatrix} h_n^{(i,j)*}h_{n'}^{(i,j)} \end{bmatrix} = 0$ for $n' \neq n$.

By assuming, for any (i, j, k), a constant

$$E\left[\left|H_{k}^{(i,j)}\right|^{2}\right] = \sum_{n=0}^{N-1} E\left[\left|h_{n}^{(i,j)}\right|^{2}\right] = P_{\Sigma},$$
(2)

(of course, with $h_n^{(i,j)} = 0$ for $L_s < n \le N-1$) and a 4-QAM SC/FDE block transmission, the average bit energy at

each Base Station (BS) antenna is given by

$$E_b = \frac{\sigma_s^2}{2\eta} P_{\Sigma},\tag{3}$$

where $\eta = \frac{N}{N+L_s}$.

B. Optimum (MMSE) Linear Detection Techniques

Linear detection techniques are considered in this paper for dealing with both MUI and Inter Symbol Interference (ISI). An appropriate linear detector can be implemented by resorting to frequency-domain processing, so as to jointly perform frequency-domain MultiUser Detection (MUD) and FDE procedures. After CP removal, a DFT operation leads to the required set { \mathbf{Y}_k ; $k = 0, 1, \dots, N-1$ } of length- N_R inputs to the frequency-domain detector (\mathbf{Y}_k given by (1)); it works, for each k, as shown in Fig. 1(a), leading to a set { $\tilde{\mathbf{Y}}_k$; $k = 0, 1, \dots, N-1$ } of length- N_T outputs $\tilde{\mathbf{Y}}_k = [\tilde{Y}_k^{(1)}, \tilde{Y}_k^{(2)}, \dots, \tilde{Y}_k^{(Nt)}]^T$ ($k = 0, 1, \dots, N-1$).

For the optimum (MMSE) linear detection technique, in the frequency-domain, it can be shown that

$$\widetilde{\mathbf{Y}}_k = \mathbf{A}_k^{-1} \mathbf{H}_k^H \mathbf{Y}_k, \tag{4}$$

(see Fig. 1(a)), where

$$\mathbf{A}_{k} = \mathbf{H}_{k}^{H} \mathbf{H}_{k} + \alpha \mathbf{I}_{N_{T}}, \tag{5}$$

with
$$\alpha = \frac{\sigma_N^2}{\sigma_S^2} = \frac{N_0}{\sigma_s^2} \left(\sigma_S^2 = E\left[\left| S_k^{(j)} \right|^2 \right] = N \sigma_s^2 \right).$$

For each user j, the required time-domain decisions are then based on the IDFT of the length-N block $\tilde{\mathbf{Y}}^{(j)} = \left[\tilde{Y}_0^{(j)}, \tilde{Y}_1^{(j)}, \cdots, \tilde{Y}_N - 1^{(j)}\right]^T$ $(j = 1, 2, \cdots, N_T)$, as shown in Fig. 1(c).



Figure 1. Linear, frequency-domain detection procedure ($\mathbf{k} = 0, 1, ..., N-1$) (a), reduced-complexity implementation regarding $\mathbf{A_k}^{-1}$ (b) and time-domain decision procedure for user j ($j = 1, 2, ..., N_T$) (c).

C. Reduced-complexity Linear Detection Techniques

Instead of the optimum (MMSE) linear detector, a reducedcomplexity linear detection technique can be implemented by replacing the A_k matrix in (4) by a diagonal A'_k matrix sharing the same entries

$$A_{k}^{'(j,j)} = A_{k}^{(j,j)} = \alpha + \sum_{i=1}^{N_{R}} \left| H_{k}^{(i,j)} \right|^{2}, \tag{6}$$

in the main diagonal. Therefore, the required matrix inversion in (4) becomes a very easy task, and the corresponding reduced-complexity implementation of the second block in Fig. 1 (a) can be done according to Fig. 1 (b), with

$$C_{k}^{(j)} = \frac{1}{\alpha + \sum_{i=1}^{N_{R}} \left| H_{k}^{(i,j)} \right|^{2}},$$
(7)

Consequently, the N_T components of $\mathbf{\tilde{Y}}_k$ can be decomposed - into "useful signal", MUI and "Gaussian noise" - as follows:

$$\begin{split} \widetilde{Y}_{k}^{(j)} &= C_{k}^{(j)} \sum_{i=1}^{N_{R}} \left| H_{k}^{(i,j)} \right|^{2} S_{k}^{(j)} + \\ &+ C_{k}^{(j)} \sum_{l \neq j} \sum_{i=1}^{N_{R}} H_{k}^{(i,l)} H_{k}^{(i,j)*} S_{k}^{(l)} + \\ &+ C_{k}^{(j)} \sum_{i=1}^{N_{R}} H_{k}^{(i,j)*} N_{k}^{(i)}, \end{split}$$
(8)

 $(j=1,2,\cdots,N_T).$

As an alternative to the $C_k^{(j)}$ coefficients given by (7) in the context of a simplified signal processing structure as jointly depicted in Fig. 1(a) and Fig. 1(b) - one can adopt $C_k^{(j)}$ coefficients so as to meet the MMSE criterion.

It can be shown that, under the reduced-complexity constraint, the resulting MMSE coefficients can be written as

$$C_k^{(j)} = \frac{1}{\alpha_k^{\prime(j)} + \sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2},\tag{9}$$

where

$$\alpha_k^{'(j)} = \alpha + \sum_{l \neq j} \alpha_k^{(l,j)}, \tag{10}$$

with $\alpha = \frac{N_0}{\sigma_s^2}$ and

$$\alpha_k^{(l,j)} = \frac{\left|\sum_{i=1}^{N_R} H_k^{(i,j)*} H_k^{(i,l)}\right|^2}{\sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2}$$
(11)

III. SEMI-ANALYTICAL METHODS FOR PERFORMANCE EVALUATION

A. Performance Evaluation Method in the Optimum (MMSE) Case

The frequency-domain output \mathbf{Y}_k of the MMSE detector in Fig. 1(a) can be written as

$$\mathbf{Y}_k = \mathbf{\Gamma}_k \mathbf{S}_k + 'Noise - like \ term' \tag{12}$$

where

$$\boldsymbol{\Gamma}_{k} = \left[\mathbf{H}_{k}^{H}\mathbf{H}_{k} + \alpha\mathbf{I}_{N_{T}}\right]^{-1}\mathbf{H}_{k}^{H}\mathbf{H}_{k}$$
(13)

For 4-QAM SC/FDE transmission and optimum (MMSE) detection, the resulting BER_j $(j = 1, 2, \dots, N_T)$ - conditional on the channel realization $\{\mathbf{H}_k; k = 0, 1, \dots, N-1\}$ - is given by

$$BER_j \approx Q\left(\sqrt{SINR_j}\right)$$
 (14)

where $SINR_{j}$ denotes the signal-to-"Interference plus Noise" Ratio regarding the components of the *j*th, timedomain, output block $\tilde{\mathbf{y}}^{(j)} = \begin{bmatrix} \tilde{y}_{0}^{(j)}, \tilde{y}_{1}^{(j)}, \cdots, \tilde{y}_{N-1}^{(j)} \end{bmatrix}^{T} = IDFT\left(\begin{bmatrix} \tilde{Y}_{0}^{(j)}, \tilde{Y}_{1}^{(j)}, \cdots, \tilde{Y}_{N-1}^{(j)} \end{bmatrix}^{T} \right)$. It can be shown that $SINR_{j} = \frac{\gamma^{(j)}}{1 - \gamma^{(j)}}$ (15)

in eqn. (14), with

$$\gamma^{(j)} = \frac{1}{N} \sum_{k=0}^{N-1} \Gamma_k^{(j,j)}, \qquad (16)$$

where $\Gamma_k^{(j,j)}$ denotes the (j,j) entry of Γ_k defined in (13) $\left(\gamma^{(j)} = \frac{E[s_n^{(j)*}\tilde{y}_n^{(j)}]}{\sigma_s^2}$, since $\tilde{y}_n^{(j)} = \gamma^{(j)}s_n^{(j)} +$ 'uncorrelated noise - like term').

Of course, the average BER for the channel realization $\{\mathbf{H}_k; k = 0, 1, \dots, N-1\}$ can be easily derived from (14):

$$BER = \frac{1}{N_T} \sum_{j=1}^{N_T} BER_j,$$
 (17)

B. Performance Evaluation Method in the Reduced-complexity Cases

When using the MMSE criterion under the constraint of a simplified detection structure, based on Fig. 1(a) and Fig. 1(b), the $C_k^{(j)}$ coefficients are given by (9). In this case, we can still write (12), but now with

$$\Gamma_k = \mathbf{A}_k^{'-1} \mathbf{H}_k^H \mathbf{H}_k, \tag{18}$$

where $\mathbf{A'}_k$ is a diagonal matrix with entries

$$A_{k}^{'(j,j)} = \frac{1}{C_{k}^{(j)}} = \alpha_{k}^{'(j)} + \sum_{i=1}^{N_{R}} \left| H_{k}^{(i,j)} \right|^{2},$$
(19)

which replaces \mathbf{A}_k ($\alpha_k^{\prime(j)}$ given by (10) and (11)).

Therefore, eqns. (15) and (16), for the $SINR_j$ are still valid; however, due to the different Γ_k matrix,

$$\gamma^{(j)} = \frac{1}{N} \sum_{k=0}^{N-1} \frac{\sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2}{\alpha_k^{\prime(j)} + \sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2}$$
(20)

in (15), in this case. Of course, the resulting BER (conditional on the channel realization $\{\mathbf{H}_k; k = 0, 1, \dots, N-1\}$) can then be computed according to eqns. (14) and (17).

With regard to the reduced-complexity technique which uses $C_k^{(j)}$ coefficients according to (7) (not according to (9)), it should be noted that it cannot be regarded, in general, as a true "MMSE technique"- The only exception is the case where $N_T = 1$, leading to $\alpha_k^{'(j)} = \alpha$: in this special, Single User (SU) case - with ISI but not MUI- we still could adopt (15), with

$$\gamma^{(j)} = \frac{1}{N} \sum_{k=0}^{N-1} \frac{\sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2}{\alpha + \sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2};$$
(21)

consequently, for $N_T = 1$, we should get

$$SINR_{j,SU} = \frac{1}{\alpha} \frac{\sum_{k=0}^{N-1} \frac{\sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2}{\alpha + \sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2}}{\sum_{k=0}^{N-1} \frac{1}{\alpha + \sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2}}$$
(22)

For $N_T > 1$, an appropriate MUI term should be added to the " $ISI + noise \ term$ ". The resulting $SINR_j$ is given by

$$SINR_{j} = \frac{SINR_{j,SU}}{1 + SINR_{j,SU} \frac{\sigma_{MUI}^{2}(j)}{\sigma_{2}^{2}}},$$
(23)

where

$$\sigma_{MUI}^{2} = \frac{\sigma_{s}^{2}}{N\left(\gamma^{(j)}\right)^{2}} \sum_{l \neq j} \sum_{k=0}^{N-1} \left| \frac{\sum_{i=1}^{N_{R}} H_{k}^{(i,l)} H_{k}^{(i,j)*}}{\alpha + \sum_{i=1}^{N_{R}} \left| H_{k}^{(i,j)} \right|^{2}} \right|^{2}, \quad (24)$$

with $\gamma^{(j)}$ obtained from (21), concerns the MUI when $N_T > 1$.

IV. NUMERICAL PERFORMANCE RESULTS, DISCUSSION OF MASSIVE MIMO EFFECTS AND SC/FDE VS OFDM COMPARISONS

The set of performance results which are presented here are concerned to 4-QAM SC/FDE uplink block transmission, with N = 256 and Ls = 64, in a MU-MIMO $N_T \times N_R$ Rayleigh fading channel. The fading effects regarding the several TX/RX antenna pairs are assumed to be uncorrelated, and two possibilities are considered for the CIRs of the channel realizations (the first possibility only for some performances of Fig. 4):

- A zero-mean, complex Gaussian h₀^(i,j) with variance P_Σ, and h_n^(i,j) = 0 for n = 1, ..., 255 (i.e. a frequency-flat Rayleigh fading);
- Independent zero-mean complex Gaussian $h_n^{(i,j)}$ coefficients, all of them with variance $\frac{P_{\Sigma}}{64}$, for n = 0, 1, ..., 63, and $h_n^{(i,j)} = 0$ for n = 64, 65, ..., 255 (i.e. a strongly frequency-selective Rayleigh fading).

With regard to the linear detection techniques of Sections II-B and II-C, the several performance results concerning the MU-MIMO system have been obtained by random generation of a large number of channel realizations, analytical BER computation - according to the methods of Section III - for each channel realization, and an averaging operation over the set of channel realizations. The accuracy of performance results obtained through these semi-analytical simulation methods was assessed by means of parallel conventional Monte Carlo simulations (involving an error counting procedure).

When $N_R \gg N_T$, both the MUI effects and the effects of multipath propagation (fading, ISI) tend to disappear: consequently, the BER performances for the MU-MIMO $N_T \times N_R$ Rayleigh fading channel become very close to those concerning a Single-Input Multi-Output (SIMO) $1 \times N_R$ channel with single-path propagation for all N_R TX/RX antenna pairs. The achievable performances under a "truly massive" MU-MIMO implementation can be analytically derived as shown below.

Entries of \mathbf{H}_k are i.i.d. Gaussian-distributed random variables with zero mean and variance P_{Σ} . According to the law of large numbers,

$$\lim_{N_R \to \infty} \left[\frac{1}{N_R} \sum_{i=1}^{N_R} \left| H_k^{(i,j)} \right|^2 \right] = E\left[\left| H_k^{(i,j)} \right|^2 \right] = P_{\Sigma}, \quad (25)$$

and

$$\lim_{N_R \to \infty} \begin{bmatrix} \frac{1}{N_R} & \sum_{i=1}^{N_R} & H_k^{(i,j)*} H_k^{(i,l)} \\ (l \neq j) & (l \neq j) \end{bmatrix} = \begin{bmatrix} E & \left[H_k^{(i,j)*} H_k^{(i,l)} \right] = 0.$$

Consequently, when $N_R >> N_T$, $\tilde{Y}_k^{(j)} \approx \left[S_k^{(j)}N_RP_{\Sigma} + 'Gaussian noise with variance NN_0N_RP_{\Sigma}'\right] \times C_k^{(j)}$, with

$$C_k^{(j)} \approx \frac{1}{\alpha + N_R P_{\Sigma}} = C \ (for \ any \ (j,k)). \tag{26}$$

Therefore, practically there is neither MUI nor ISI and fading at the time-domain outputs: $\tilde{y}_n^{(j)} \approx \left[s_n^{(j)} N_R P_{\Sigma} + 'Gaussian noise with variance N_0 N_R P_{\Sigma} '\right] \times C.$

The resulting BER performance becomes as follows:

$$BER \approx Q\left(\sqrt{\frac{N_R P_{\Sigma} \sigma_s^2}{N_0}}\right) = Q\left(\sqrt{2\eta N_R \frac{E_b}{N_0}}\right), \quad (27)$$

Figures 2 and 3 show the simulated BER performances for an SC/FDE-based MU-MIMO uplink and several possibilities regarding N_T and N_R , when using the linear detection techniques of Section II: optimum (MMSE) detection; reducedcomplexity detection, under $C_k^{(j)}$ coefficients given by eq. (7)



Figure 2. BER performances for SC/FDE-based MU-MIMO, with $N_T = 2$, and $N_R = 10$ (a), 50 (b) or 100 (c), under reduced-complexity (I, II) and MMSE linear detection [SIMO 1 × N_R (singe-path, multipath) reference BER performances are also included, and the five BER performances are ordered, from the worst to the best, as explained in section IV].



Figure 3. BER performances for SC/FDE-based MU-MIMO, with $N_T = 10$, and $N_R = 10$ (a), 50 (b) or 100 (c), under reduced-complexity (I, II) and MMSE linear detection [SIMO $1 \times N_R$ (singe-path, multipath) reference BER performances are also included, and the five BER performances are ordered, from the worst to the best, as explained in section IV]].

(I) or eq. (9) (II). In both figures, for the sake of comparisons, we also include SIMO $1 \times N_R$ reference performances, for both the multipath propagation channel - which implies a Rayleigh fading concerning each TX/RX antenna pair - and an ideal single-path propagation channel. For the linear detection techniques, the semi-analytical methods of Section III have been adopted; the complementary conventional Monte Carlo simulation (involving error counting) results correspond to the superposed circles in the solid lines.

Fig. 4 is dedicated to an 'SC/FDE vs OFDM' comparison of BER performances, for the strongly frequency-selective fading channel case and the MMSE linear detection. In fact, the "OFDM results" shown here could have been obtained by resorting to the SC/FDE simulation software, by replacing the strongly frequency-selective fading by a frequency-flat fading. This is due to the following reasons: under frequencyflat fading, uncoded SC/FDE and OFDM provide identical performances; uncoded OFDM performance does not depend on the frequency-selectivity of the fading effects.

In all figures, where the SIMO detection performance was analytically computed according to (22), an excellent agreement of the semi-analytical simulation results with conventional Monte Carlo simulation results can be observed.

In the simulation results concerning all subfigures of both Fig. 2 and Fig. 3, the five BER performance curves have been shown to be ordered, from the worst to the best, as follows: $N_T \times N_R$ MU-MIMO with reduced-complexity (I) linear detection; $N_T \times N_R$ MU-MIMO with reduced-complexity (II) linear detection; $N_T \times N_R$ MU-MIMO with MMSE detection; $1 \times N_R$ (multipath case) SIMO detection; $1 \times N_R$ (singlepath case) SIMO detection. These figures clearly show that the performance degradation which is inherent to the reducedcomplexity linear detection techniques (I and II) - as compared with the optimum (MMSE) linear detection - can be made quite small, by increasing N_R significantly; they also show that, under highly increased N_R values, the "MUI-free" SIMO (multipath) performance and the ultimate bound - the "MUIfree and ISI & fading-free" SIMO (single-path) performance can be closely approximated, even when adopting the reducedcomplexity (I) linear detection. These two figures emphasize a "massive MIMO" effect when $N_R \gg 1$, especially when $N_R \gg N_T$ too, which leads to BER performances very



Figure 4. SC/FDE (dashed lines) vs OFDM (solid lines) BER performances, for 2×10 (a) and 10×50 (b), under strongly frequency-selective Rayleigh fading [SIMO $1 \times N_R$ (multipath, single-path) reference performances are also included].

close to the ultimate "MUI-free and ISI & fading-free" SIMO (single-path) performance bound.

With regard to 'SC/FDE vs OFDM' (uncoded) BER performance comparisons, we can remember the significant performance advantage of SC/FDE when the Rayleigh fading is frequency-selective and $N_R = 1$ or 2 [2][3]. However, Fig. 4 clearly shows that - in spite of the strongly frequency-selective fading considered here - the performance advantage of SC/FDE practically vanishes when $N_R \gg 1$, even for a moderate $\frac{N_R}{N_T}$ (e.g., equal to 5).

Not surprisingly - having in mind the comparison depicted in Fig. 4 - Fig. 2 and Fig. 3 of this paper are very similar, respectively, to Fig. 3 and Fig. 4 of [10] (where the OFDMbased MU-MIMO alternative is considered).

V. CONCLUSIONS

This paper was dedicated to the uplink performance evaluation of a MU-MIMO system with SC/FDE transmission, when adopting a large number of antennas and linear detection techniques at the BS. The numerical performance results, discussed in detail in Section IV, show the "massive MIMO" effects provided by a number of BS antennas much higher than the number of antennas which are jointly employed in the terminals of the multiple autonomous users, even when reduced-complexity linear detection techniques are adopted.

The accuracy of performance results obtained by semianalytical means, much less time-consuming than conventional, 'error counting'-based, Monte Carlo simulations was also demonstrated. The proposed performance evaluation method can be very useful for rapidly knowing "how many antennas do we need in the BS?", so that a "massive MIMO" effect can be achievable, for a given number of antennas jointly employed in the user terminals.

The performance results of this paper also clearly show that the SC/FDE detection performance, in a MU-MIMO context with a large number of BS antennas, cannot be significantly better than that of OFDM: in fact, the SC/FDE performance advantage practically vanishes when $N_R \gg N_T$, even for a strongly frequency-selective fading channel. Nevertheless, we can say that SC/FDE is a better choice than OFDM for uplink transmission, due to its well-known "power amplification advantage" [2] and to the fact that it does not suffer from a detection performance disadvantage.

REFERENCES

- H. Sari, G. Karam, and I. Jeanclaude. An analysis of orthogonal frequency-division multiplexing for mobile radio applications. In *Vehicular Technology Conference, 1994 IEEE 44th*, pages 1635–1639 vol.3, Jun 1994.
- [2] A. Gusmao, R. Dinis, J. Conceicao, and N. Esteves. Comparison of two modulation choices for broadband wireless communications. In *Vehicular Technology Conference Proceedings, 2000. VTC 2000-Spring Tokyo. 2000 IEEE 51st*, volume 2, pages 1300–1305 vol.2, may 2000.
- [3] A. Gusmao, R. Dinis, and N. Esteves. On frequency-domain equalization and diversity combining for broadband wireless communications. *Communications, IEEE Transactions on*, 51(7):1029–1033, July 2003.
- [4] G. Foschini. "Layered Space-Time Architecture for Wireless Communication in a Fading Environment when Using Multi-element Antennas". *Bell Labs Technology Journal*, (2):1, 1996.
- [5] J. Mietzner, R. Schober, L. Lampe, W.H. Gerstacker, and P.A. Hoeher. Multiple-antenna techniques for wireless communications - a comprehensive literature survey. *Communications Surveys Tutorials, IEEE*, 11(2):87–105, Second 2009.
- [6] E.G. Larsson. Mimo detection methods: How they work [lecture notes]. Signal Processing Magazine, IEEE, 26(3):91–95, May 2009.
- [7] D. Gesbert, M. Kountouris, R.W. Heath, Chan-Byoung Chae, and T. Salzer. Shifting the mimo paradigm. *Signal Processing Magazine*, *IEEE*, 24(5):36–46, Sept 2007.
- [8] T.L. Marzetta. Noncooperative cellular wireless with unlimited numbers of base station antennas. *Wireless Communications, IEEE Transactions* on, 9(11):3590–3600, November 2010.
- [9] J. Hoydis, S. ten Brink, and M. Debbah. Massive mimo in the ul/dl of cellular networks: How many antennas do we need? *Selected Areas in Communications, IEEE Journal on*, 31(2):160–171, February 2013.
- [10] P. Torres, L. Charrua, and A. Gusmao. "Uplink Performance Evaluation of Broadband Systems which Adopt a Massive MU-MIMO Approach". *International Conference on Wireless and Mobile Communications, ICWMC*, June 2014.