On Type II Hybrid-ARQ with Decode and Forward Relay using Non-Binary Rate-Compatible Punctured LDPC Code on MIMO SC-FDMA up-link

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Abstract— **In this paper, Non-Binary Rate-Compatible Punctured Low Density Parity Check (NB RCP LDPC) code is designed over the extended Galois Field. The designed NB RCP LDPC code is applied to the type II Hybrid Automatic Repeat reQuest (HARQ) with Decode and Forward (DF) relay on Multiple Input Multiple Output (MIMO) Single Carrier-Frequency Division Multiple Access (SC-FDMA) up-links. The designed code enables us to decrease the coding rate with incremental redundancy for each retransmission in HARQ. The retransmission is done from the DF relay after the successful decoding in the relay. We have verified through computer simulations that the proposed type II HARQ scheme with DF relay greatly improves the throughput and average retransmission characteristics compared with the scheme without DF relay. Multiple relay cases are also considered.**

Keywords-NB RCP LDPC code; Hybrid-ARQ; Decode and Forward Relay; MIMO SC-FDMA; Symbol-LLR.

I. INTRODUCTION

An LDPC code which suits the flexible coding rate design and has the high error correcting capability through iterative decoding can be constructed on arbitrary extended Galois field. The Non-Binary (NB) LDPC code constructed on extended Galois field generally exhibits the better BER performance than the binary LDPC codes [1][2]. There also exist Rate-Compatible Punctured (RCP) LDPC codes with variable coding rate obtained by properly puncturing the mother LDPC code [3]. The RCP LDPC codes enable us to use the same decoder as the mother code and suit the ARQ error correcting schemes [4] with the incremental redundancy. By combining the NB LDPC codes with the RCP codes, the NB RCP LDPC codes were designed and the designed NB RCP LDPC codes were applied to the type II HARQ [5]. On the other hand, the Decode and Forward (DF) relay schemes [6] are useful for HARQ schemes. By using the DF relay, the source node can be replaced by the relay, once the relay correctly decodes the LDPC encoded packet from the source. This replacement from the source to the relay effectively reduces the number of retransmissions and improves the throughput. The NB RCP LDPC coded type II HARQ with DF relay is applied to the MIMO-OFDM modulation in [7]. The incremental redundancy in HARQ with DF relay is especially suited to the up-link transmission like in Long Term Evolution (LTE) or 4G. Due to the necessities of low Peak to Average Power Ratio (PAPR) and the high power efficiency in the amplification, MIMO SC-FDMA [8] is usually adopted to the up-links in cellular networks. Among SC-FDMA, interleaved SC-FDMA is especially useful because of its very low PAPR nature and excellent frequency diversity effect [8]. In [9], NB LDPC coding with NB repletion codes is applied to multiple relay case for flat fading channel. However, the application of NB RCP LDPC codes to MIMO interleaved SC-FDMA with multiple DF relays has not been reported yet. In this paper, we have investigated the NB RCP LDPC coded type II HARQ with DF relays on MIMO interleaved SC-FDMA up-links. We have verified through computer simulations that the proposed up-link scheme greatly improves the throughput and the average number of retransmission characteristics compared with the case of no DF relay. Moreover, we considered multiple relay cases, i.e., serial or parallel arrangement of two relays.

The paper is organized as follows. In Section II, RCP LDPC code is introduced. In Section III, NB LDPC coded Type II HARQ scheme is described. In Section IV, we propose the DF relaying scheme. In Section V, we present the symbol LLR generation in interleaved SC-FDMA demodulation. In Section VI, computer simulation results are shown. The paper concludes with Section VII.

II. RCP LDPC CODE

The encoding and decoding procedure of RCP LDPC code is as follows. We call the code before puncture and the code after puncture as the mother code and the punctured code, respectively. In RCP LDPC code, the encoder and decoder of mother code can also be applied to the punctured code. When the parity check matrix of mother code is given by $\boldsymbol{H}_M (n_M \times n_N)$ and the generator matrix by $\boldsymbol{G}_M (n_N \times n_K)$ with $n_K = (n_N - n_M)$, the coding rate of mother code becomes $R_M = (1 - n_M / n_N) = n_K / n_N$. The coding rate after the puncture of n_P symbols from the mother code is given by $R_E = n_K / \left(n_N - n_P\right)$. We denote the message vector as $\boldsymbol{m} = (m_1, m_2, \ldots, m_{n_K})$, the code word of mother code as $\boldsymbol{C_M} = (c_{M1}, c_{M2}, \ldots, c_{Mn_N})$, the index of position to be punctured as $\boldsymbol{P} = (p_1, p_2, \ldots, p_{n_P})$ and the code word of punctured code as $\boldsymbol{C}_P = (c_{P1}, c_{P2}, \cdots, c_{Pn_P})$. The encoding procedure is first to generate the mother code by $\boldsymbol{C_M} = \boldsymbol{mG_M}$ which is systematic, and next, to puncture the position using $\boldsymbol{P}$ to obtain $\boldsymbol{C}_P$. The decoding procedure is to produce the symbol LLR from the receive signal and it is fed to the mother code decoder as the initial value for the sum-product algorithm. The symbol LLR for the position $\boldsymbol{P}$ is initially set to 0, because there is no available symbol LLR corresponding to the position $\boldsymbol{P}$.

III. NB RCP LDPC CODED TYPE II HARQ SCHEME

In Fig. 1, we show the transmitter and receiver block diagram of NB RCP LDPC coded Type II HARQ using 2×2 MIMO SC-FDMA. At the transmitter, the data bits are firstly encoded by the Cyclic Redundancy Check (CRC)-16 error detecting code and secondly encoded by the NB LDPC code on GF(4) or GF(16). The encoded LDPC code word is divided into the transmission packets and they are modulated by QPSK or 16QAM depending on GF(4) or GF(16), respectively. Matching GF(Q) to the modulation level Q is preferable in calculating the symbol LLR and reduces the complexity compared with the use of bit LLR calculation. The modulated QAM symbols are then N-point FFT transformed at each antenna stream and the subcarrier mapping is done to make the interleaved SC-FDMA spectrum as shown in Fig. 2. The interleaved spectrum is then M-point IFFT transformed, where $M = U \times N$ and U is the number of users. Cyclic Prefix (CP) is added to the time domain complex samples of an IFFT block. After the interpolation filtering and the up-conversion to carrier frequency, the RF signal is transmitted from each antenna. At the base station, after the down-conversion to baseband and the sampling, CP is removed and the M-point FFT is done to obtain the frequency domain signal. The frequency domain signal is then subcarrier-de-mapped to aggregate the interleaved spectrum of each user back to the N sample spectrum again. The Frequency Domain Equalization (FDE) is made to compensate the channel frequency response and separate the multiple spatial streams of each user. The FDE weight $\boldsymbol{G}_u(i)$ in MMSE criterion is expressed as

$$\boldsymbol{G}_u(i) = \boldsymbol{H}_u(i)^H \left\{ \boldsymbol{H}_u(i)\boldsymbol{H}_u(i)^H + n_T\sigma^2\boldsymbol{I}_{n_T} \right\}^{-1} \qquad (1)$$

where $\boldsymbol{H}_u(i)$ is $n_T \times n_T$ channel matrix at subcarrier i of user u , n_T is the number of transmit antennas, σ^2 is the noise variance of each subcarrier and $\boldsymbol{I}_{n_T}$ is the identity matrix of size n_T . After the FDE, N-point IFFT is made to obtain the time domain signal of each stream in each user. The symbol LLR defined in section V is then calculated and fed to the NB LDPC decoder. Using Sum-Product Algorithm (SPA), the LDPC code word is decoded and the hard decision is made to obtain the data bits. The dada bits are then CRC-checked. NACK or ACK is returned to the transmitter of each user corresponding to the error or no error detection.

In type II HARQ, like in Fig. 3, at the first transmission, only uncoded information symbols are transmitted, and at the second transmission and after, the parity check symbols are retransmitted at the incremental redundancy policy. Accordingly, when the channel condition is good, the first uncoded transmission is successful enough and it achieves high throughput. On the other hand, when the channel is bad, by decreasing the coding rate at each retransmission, the error correction capability increases gradually. The generation of RCP LDPC code is done only once at the transmitter and there is no need of regeneration of code word when the coding rate is decreased. Therefore there is no increase of complexity of RCP LDPC code compared with the fixed rate LDPC code. Also at the receiver side, the complexity of RCP LDPC decoder does not increase compared with the fixed rate LDPC decoder, because the same and only one LDPC decoder can be used for various coding rates of RCP LDPC code.

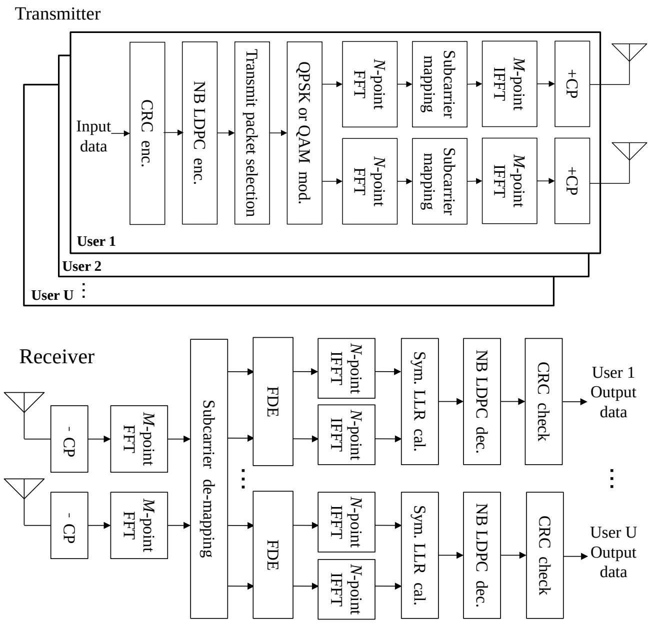


Fig. 1 Transmitter and receiver structure of NB RCP LDPC coded type II HARQ scheme using 2×2 MIMO interleaved SC-FDMA

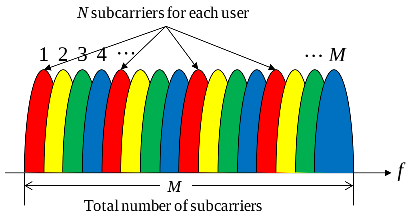


Fig. 2 Sub-carrier mapping in interleaved SC-FDMA
(Red, yellow, green and blue spectrum show the subcarriers of user 1, user 2, user 3 and user 4, respectively.)

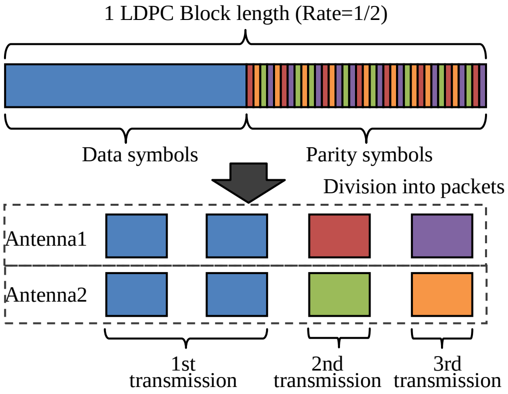


Fig. 3 Division of an LDPC code word into transmission packets

IV. DECODE AND FORWARD RELAYING SCHEME

The Decode and Forward relay model is shown in Fig. 4. We consider the relay arrangement where the relay locates at the middle point between the source (transmitter at each user) and the destination (receiver at base station).

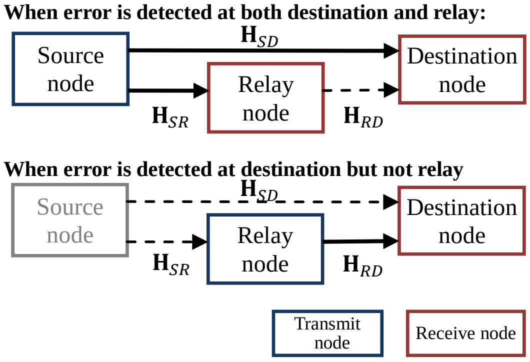
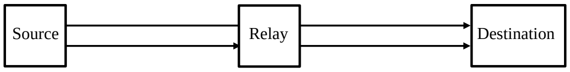


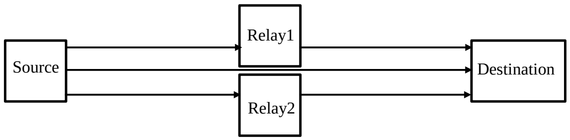
Fig. 4 DF (Decode and Forward) relaying model

At the first transmission, the source broadcasts the uncoded information packet to the relay and destination simultaneously. The relay and destination independently detect the transmission errors using CRC-16 code. The relay and destination independently return (broadcast) ACK or NACK to the source. This ACK or NACK is shared among source, relay and destination. If the destination returns ACK, the transmission finishes at the first transmission and this condition is equivalent to no relay. Otherwise, retransmission is made. The source sends parity check packets with incremental redundancy. The relay and destination receive the parity check packet and combine it with already received packet. The LDPC decoding and CRC error detection is done both at relay and destination. ACK or NACK is returned and shared among source, relay and destination. At this point, if destination returns NACK but relay does ACK, then the relay sends the parity check packet hereafter instead of source, i.e., the source is replaced by the relay which locates closer to destination. The transmission from relay to destination is more successful than source to destination due to the near distance between relay and destination. Also, as the source and relay do not simultaneously retransmit the parity check packet, the total transmission power is the same between with and without relay. This saves the total transmit energy in the case where the same power as source is allocated to the relay.

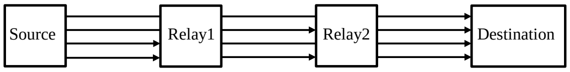
Next, we consider the two relay cases where two relays are allocated in a serial or parallel manner as shown in Fig. 5(b) or (c), respectively. Fig. 5 (a) is the arrangement of single relay already discussed. In Fig. 5 (b), two relays are allocated in the middle point between source and destination in parallel. In Fig. 5(c), relay 1 and relay 2 are allocated serially with equal distance interval between source and destination. When the power attenuation exponent is given by α and the distance between source and destination is defined as 1, the relay at the middle point between source and destination in Fig. 5 (a) and (b) receives 2^{α} times more power than the direct link between source and destination. Similarly, the relay 1 and relay 2 in Fig. 5 (c) receives 3^{α} times more power than the direct link.



(a) Single relay



(b) Parallel arrangement of 2 relays



(c) Serial arrangement of 2 relays

Fig. 5 DF relay arrangement in case of multiple relays

V. SYMBOL LLR GENERATION IN MIMO INTERLEAVED SC-FDMA DEMODULATION

The received signal in time domain after *N*-point IFFT in Fig. 1 is expressed as

$$y_k = h_k x_k + n_k,\ k = 1,\cdots N,\qquad \xi^2 = (1/2)\cdot E\{|n_k|^2\} \tag{2}$$

where h_k is the complex gain for the symbol x_k after the equalization and spatial de-multiplexing, and ξ^2 is the variance of receive noise. When the modulation level of QAM is given by Q and the probability of occurrence of Q symbols are all equal, i.e., $p(s_1) = \cdots = p(s_Q) = 1/Q$, the symbol LLR is defined and calculated as

$$\begin{aligned}\lambda_a &= \log\left\{\frac{p(s_a \mid y_k)}{p(s_1 \mid y_k)}\right\} = \log\left\{\frac{\left(p(y_k \mid s_a)p(s_a)\right)/p(y_k)}{\left(p(y_k \mid s_1)p(s_1)\right)/p(y_k)}\right\}\\ &= \log\left\{\frac{p(y_k \mid s_a)}{p(y_k \mid s_1)}\right\} = \log\left\{\frac{\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{|y_k - h_k s_a|^2}{2\xi^2}\right)}{\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{|y_k - h_k s_1|^2}{2\xi^2}\right)}\right\}\\ &= \frac{-|y_k - h_k s_a|^2 + |y_k - h_k s_1|^2}{2\xi^2},\quad a = 1,\cdots,Q\end{aligned} \tag{3}$$

VI. COMPUTER SIMULATION RESULTS

The BER characteristics of NB RCP LDPC code for each coding rate on GF(4) or GF(16) are examined. The simulation conditions are listed in the shaded area in Table I and the simulation results are shown in Fig. 6 and Fig. 7. C++ language is utilized for programming. We can see that each punctured code having different coding rate is obtained from the mother code with rate 1/2. Each punctured code shows the BER characteristics corresponding to its coding rate.

Next, from Fig. 8~Fig. 15, we show the throughput and the average number of retransmission characteristics of NB RCP LDPC coded type II HARQ on 2 × 2 MIMO interleaved SC-FDMA. We have shown the simulation results for the single relay case of Fig. 5 (a) in Figs. 16 and 17 and the serial two relay case of Fig. 5 (c) in Figs.9, 11, 13, 15, 16 and 17. The simulation conditions are also listed in Table I.

When QPSK modulation is employed, one GF(4) LDPC mother code word is divided into 16 packets. As the coding rate of mother LDPC code is 1/2, the former 8 packets contain only information symbols and the latter 8 packets consist of parity check symbols. For the 1st transmission, 8 information packets are transmitted from two antennas. For the 2nd retransmission and after, two parity check packets are retransmitted at each retransmission resulting in lowering the coding rate at receiver from 4/5 to 4/6, 4/7, 4/8. After all the parity check packets are retransmitted and the coding rate at destination reaches 1/2, if error is still detected at destination, the same RCP LDPC code transmission is repeated two times and each time the symbol LLR's are summed up at destination by symbol LLR addition. Thus, the total 3 transmissions of RCP LDPC code word are done before the final discard of RCP LDPC code in case of failure of error correction at destination.

As a comparative scheme, we also considered the LDPC coded type I HARQ scheme where the coding rate is fixed for each retransmission. The maximum number of retransmissions is limited to 15 and the symbol LLR addition is employed at the destination. For QPSK and GF(4), the throughput characteristics and the average number of retransmission characteristics are shown in Fig. 8 and Fig. 9 and in Fig. 12 and Fig. 13 respectively.

When 16QAM modulation is employed, one GF(16) LDPC mother code word is divided into 8 packets. The former 4 packets contain only information symbols and the latter 4 packets consist of parity check symbols. The coding rate decreases from 2/3 to 2/4 at each retransmission. After all the packets are retransmitted and the coding rate at destination reaches 1/2, if error is still detected at destination, the same RCP LDPC transmission is repeated 3 times in total, which is the same as the case of QPSK modulation as mentioned in the above. For 16QAM and GF(16), the throughput characteristics and the average number of retransmission characteristics are shown in Fig. 10 and Fig. 11 and in Fig. 14 and Fig. 15, respectively.

For the two serial relay case, we also show the throughput and the average number of retransmission characteristics in Fig. 16 and Fig. 17, respectively.

Regarding the simulation results, we first compare the type I HARQ with the type II HARQ. The throughput characteristic of type I HARQ saturates in the high average E_b/N_0 region, because the coding rate is fixed. As the coding rate of type I HARQ increases, the throughput also increases in the high average E_b/N_0 region. On the other hand, the throughput of type II HARQ approaches to 4 (bps/Hz) and 8 (bps/Hz) in case of QPSK and 16QAM, respectively, in the high E_b/N_0 region. This is because type II HARQ can change the coding rate adaptively and it can use the coding rate of 1 for high SNR region. The slight decrease of throughput in type II HARQ is due to the use of CRC-16 error detection code. We also observe that for entire E_b/N_0 region, the throughput of type II HARQ is optimized and is superior to type I HARQ. However, the average number of retransmission of type II HARQ is worse than type I HARQ. This is because parity check packets are sent sequentially with several time slots in type II HARQ, while the parity check packet is sent at a time in type I HARQ.

Next, we compare the case with relay and without relay. When the average E_b/N_0 is high, the throughput with relay is the same as without relay. This is because, both in the case with relay and without relay, the average number of retransmission is almost 0 for the high average E_b/N_0 region, and there makes no difference between the two. On the other hand, when the average E_b/N_0 is low, we see that the throughput and the average number of

TABLE I. SIMULATION CONDITIONS OF NB RCP LDPC CODED TYPE II HARQ WITH DF RELAY ON 2×2 MIMO INTERLEAVED SC-FDMA

NB LDPC mother code	Size of Galois field	GF(4)	GF(16)
	Size of parity check matrix	(512,1024)	(256,512)
	Average weight	(2.66,5.32)	(2.41,4.82)
	Coding rate	4/8	2/4
Punctured code	Information bit length	1024	
	Coding rate	4/4,4/5,4/6, 4/7,4/8	2/2,2/3,2/4
Max SPA iteration		20	
Number of users U		4	
Transmit and receive antennas		2×2	
modulation		QPSK	16QAM
Number of subcarriers / user		$N = 64$	
Number of total subcarriers		$M = 256$	
CP length (T_s :QAM symbol length)		$16\times(T_s/4)=4T_s$	
Channel model between each transmit and receive antenna		Quasi-static Rayleigh fading with 16 delay paths having equal average power	
Interval of delay paths		$T_s/4$	
Channel State Information		Known at receiver	
Error detecting code		CRC-16	
Power attenuation exponent		$\alpha = 3$	

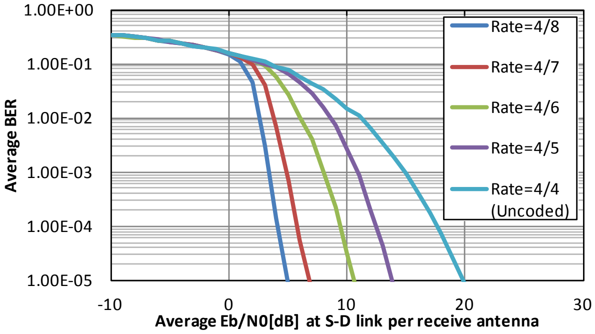


Fig. 6 BER characteristics of NB RCP LDPC GF(4) code using 2×2 MIMO interleaved SC-FDMA (QPSK)

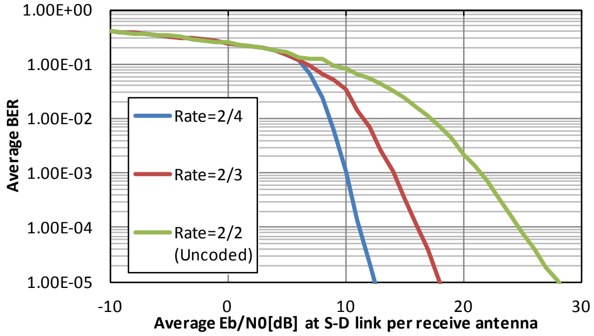


Fig. 7 BER characteristics of NB RCP LDPC GF(16) code using 2×2 MIMO interleaved SC-FDMA (16QAM)

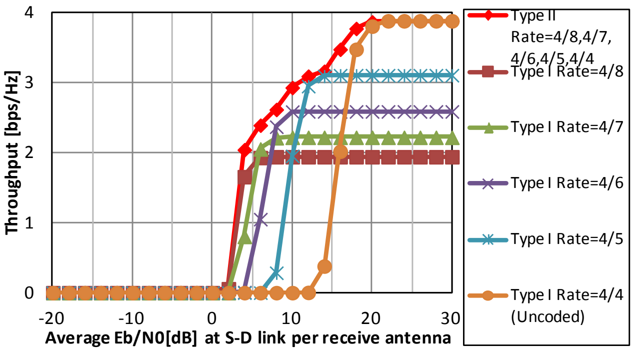


Fig. 8 Throughput characteristics of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, QPSK)

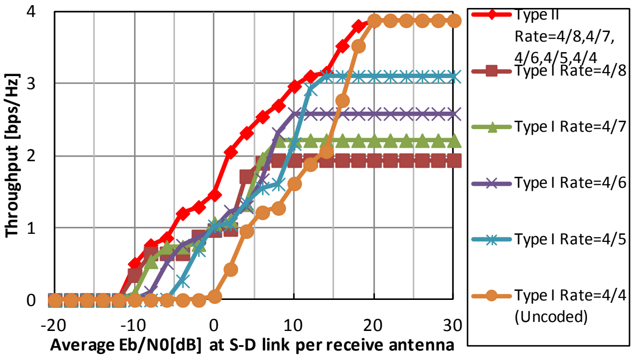


Fig. 9 Throughput characteristics of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with 2 serial relays, 2×2, QPSK)

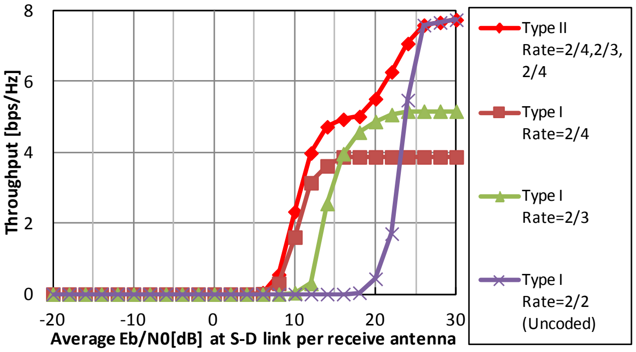


Fig. 10 Throughput characteristics of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, 16QAM)

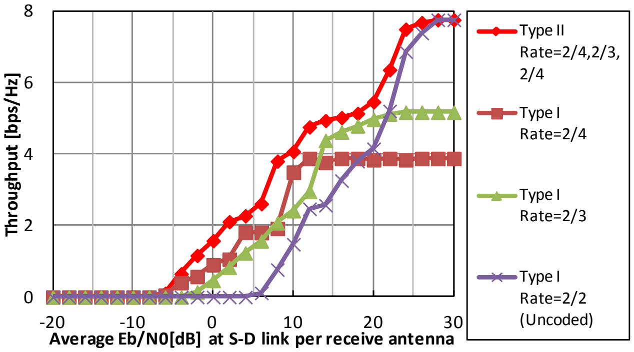


Fig. 11 Throughput characteristics of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with 2 serial relays, 2×2, 16QAM)

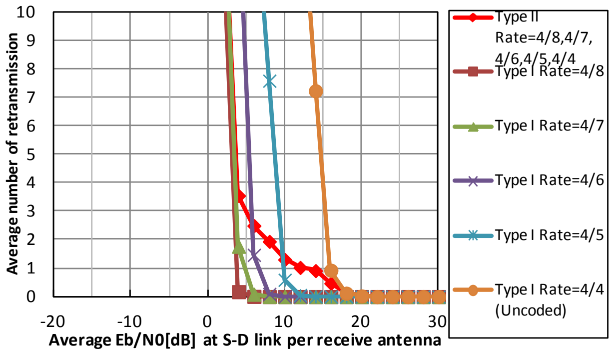


Fig. 12 Average number of retransmission of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, QPSK)

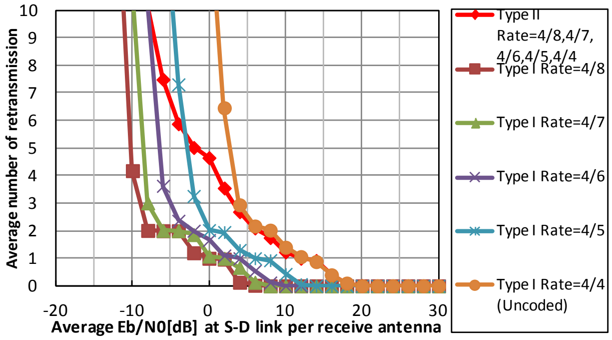


Fig. 13 Average number of retransmission of NB GF(4) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with 2 serial relays, 2×2, QPSK)

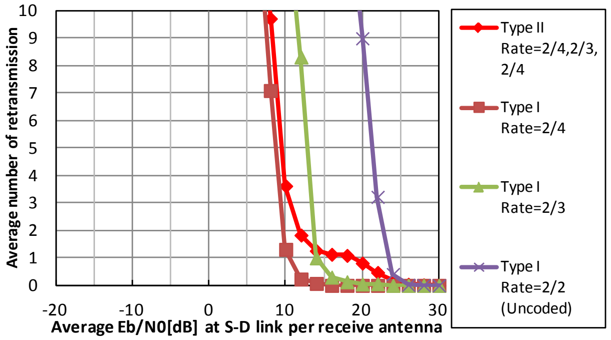


Fig. 14 Average number of retransmission of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (without relay, 2×2, 16QAM)

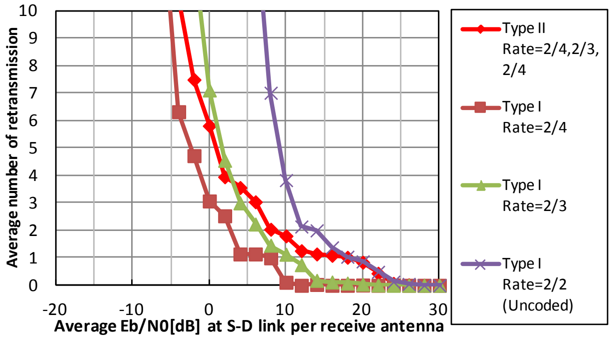


Fig. 15 Average number of retransmission of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy and type I HARQ with fixed coding rate (with 2 serial relays, 2×2, 16QAM)

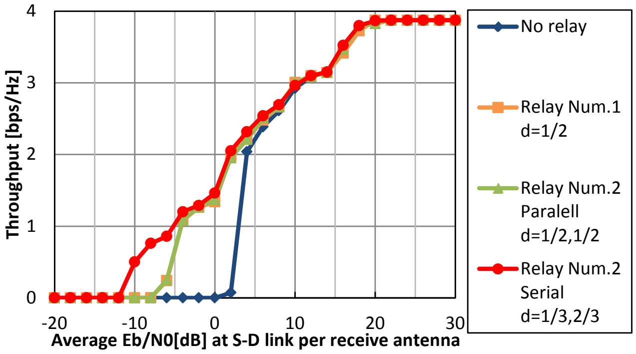


Fig. 16 Comparison of throughput characteristics of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy for three relay arrangements (2×2 , 16QAM)

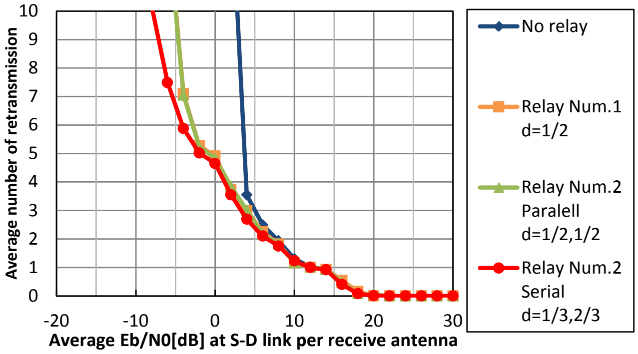


Fig. 17 Comparison of average number of retransmission of NB GF(16) RCP LDPC coded type II HARQ scheme with incremental redundancy for three relay arrangements (2×2 , 16QAM)

retransmission characteristics with relay are much better than the ones without relay. This is because for low average E_b/N_0 region, although the destination frequently fails to decode the code word correctly, the relay succeeds in decoding with high probability. Accordingly, as the retransmission is executed from the relay to the destination instead of the source to the destination, the probability of successful decoding at destination is increased. We also observe that when the number of average retransmission is greater than 0, i.e., the retransmission is done and the total number of transmission is more than 2, the throughput with relay is largely improved compared with the one without relay. This improvement happens below $E_b/N_0 \approx 20$ (dB) for GF(4) and QPSK in Figs. 8,9,12 and 13, and below $E_b/N_0 \approx 25$ (dB) for GF(16) and 16QAM in Figs. 10,11,14 and 15.

As for the relay arrangements in Fig. 5, from Fig. 16 and Fig. 17, we see that the throughput and the average number of retransmission characteristics for the serial arrangement in Fig. 5 (c) show the best. The parallel arrangement in Fig. 5 (b) exhibits almost the same performance as the one relay case in Fig. 5 (a). This observation comes from the fact that the receive power at relay or destination in the serial arrangement becomes larger than the parallel arrangement.

VII. CONCLUSIONS AND FUTURE WORKS

In this paper, assuming the up-link transmission in cellar wireless networks, we have investigated the throughput and the average number of retransmission characteristics of the proposed NB RCP LDPC coded type II HARQ with DF relays using MIMO interleaved SC-FDMA. We have verified the effectiveness of the proposed scheme through computer simulation. In the proposed scheme, for the first transmission, only uncoded information packet is broadcasted to both for relay and destination. If error is detected at destination, parity check packets are retransmitted for the 2nd and the subsequent retransmission. The error correction decoding is done both at relay and destination. When the destination fails in decoding, but the relay succeeds, the relay replaces the source hereafter. The relay retransmits the remaining packets instead of source. The destination receives the parity check packets with incremental redundancy till the coding rate reaches 1/2. We made clear that by using DF relay the throughput and the average number of retransmission characteristics are improved for low receive SNR region. We also clarified that two DF relays serially arranged between source and destination improve the characteristics further. MIMO interleaved SC-FDMA seems to be a promising candidate for up-link transmission in 4G and after 4G, because of its low PAPR and frequency diversity effect. Considerations on other channel models among nodes, the improvement of BER characteristic of MMSE nulling receiver in SC-FDMA, etc., will be future studies.

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