

Proposal of a Null Subcarrier Allocation Method for CAZAC-OFDM Systems

Taketo Nasuno and Masahiro Muraguchi

Department of Electrical Engineering, Tokyo University of Science
6-3-1 Nijjuku, Katsushika-ku, Tokyo, 125-0051, Japan
E-mail: 4315086@ed.tus.ac.jp, murag@ee.kagu.tus.ac.jp

Abstract— A major drawback of Orthogonal Frequency Division Multiplexing (OFDM) signals is the extremely high Peak-to-Average Power Ratio (PAPR). Constant Amplitude Zero Auto Correlation (CAZAC) precoding can dramatically improve the PAPR of OFDM signals, i.e., up to the same PAPR value of single carrier signals with the same modulation scheme. One negative point of the CAZAC precoding technic is not being able to meet the current wireless standards, which usually require a null subcarrier allocated on the center of data subcarriers. In this paper, we demonstrate the CAZAC-OFDM which completely satisfies the 4G-LTE standard by using the proposed allocation method for a null subcarrier without degrading the PAPR and the Bit-Error-Rate.

Keywords- OFDM; CAZAC; null subcarrier; PAPR;

I. INTRODUCTION

In recent years, with the rapid spread of wireless LAN and mobile communication terminals, such as smart phones, tablets, etc., the demand for wireless communication is expanding. However, since the capacity of the data is also increasing, it is a problem that there are few available frequency bands. Therefore, research on Orthogonal Frequency Division Multiplex (OFDM) system is progressing because it is the modulation scheme that can provide high spectral efficiently. By using OFDM systems, high-speed communication can be realized under limited frequency bands as compared with other modulation schemes, and OFDM system is currently adopted in downlink of the mobile communication system [1]. On the other hand, OFDM has the demerits of the high Peak to Average Power Ratio (PAPR). The high PAPR causes a decrease in the average power of the transmission signal, resulting in lowering of the energy efficiency of the power amplifiers and the shortened operation time causes a serious problem in battery-powered wireless terminals.

A new PAPR reduction technique with Constant Amplitude Zero Auto Correlation (CAZAC) equalization was recently proposed [2].

Our research group have demonstrated that CAZAC precoding can reduce the PAPR without any degradation of BER performances [3], and have also provided available control procedure for PAPR and spectrum managements for the CAZAC-OFDM system [4]. Unfortunately, the CAZAC-OFDM is not able to meet the current wireless standards, which usually require a null subcarrier allocated on the center of data subcarriers. The main reason of this subcarrier which is named the DC-subcarrier is to avoid DC-offset for received

signal processing ease. In an ordinary OFDM system, each subcarrier carries independent information. On the other hand, by using CAZAC precoding, every information-data is spreading over subcarriers and every subcarrier shares all information-data. This peculiar feature, however, hinders the allocation of a null subcarrier, whereas, in case of ordinary OFDMs, it is easily allocated a null subcarrier by inputting null-data instead of information-data.

This paper proposes a method for null subcarrier allocation in the Constant Amplitude Zero Auto Correlation OFDM systems (CAZAC-OFDM systems). The method is to replace the CAZAC precoding data for the center subcarrier by null-data. This means that the information carried by the center subcarrier is dropped. Fortunately, since the CAZAC-OFDM has a frequency diversity effect, the dropped information is effectively recovered.

We will demonstrate that the CAZAC-OFDM system with proposed null subcarrier allocation method fulfills the requirements of the PAPR and the Bit-Error-Rate for 4G-LTE standard.

Section 2 describes the OFDM system and Section 3 describes the CAZAC precoding technic and proposal system. Section 4 then performs simulation verification and concludes in Section 5.

II. OFDM SYSTEM

This section describes existing OFDM systems.

A. PAPR

OFDM system is a form of multicarrier modulation and a communication system by OFDM can use frequency efficiently. In an OFDM transmitter, an Inverse Fast Fourier Transform (IFFT) is performed after primary modulation of a data sequence. As a result, the value after primary modulation is placed on each subcarrier, and a spectrum is formed in which each subcarrier overlaps. Even if each subcarrier overlaps, due to mutual orthogonality, respectively demodulation is possible, so that frequency utilization efficiency is excellent.

The N th sample with the input signal X (length N) after mapping is defined as (1) [5].

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi}{N}kn} \quad (1)$$

Where X_k is the frequency-domain signal, and N is the number of subcarriers, $j = \sqrt{-1}$. After IFFT, a guard

interval is inserted in the baseband signal. The OFDM symbol is generated by the above procedure. In particular, the value after IFFT when $n=0$ in (1) disappears the imaginary part, and only the real part remains as (2).

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \quad (2)$$

The PAPR of the OFDM signal (1) can be expressed as

$$PAPR = \frac{\max_{0 \leq n \leq N-1} |x_n|^2}{E[|x_n|^2]}, \quad (3)$$

where $E[\cdot]$ is expectation operator. PAPR represents amplitude fluctuation of each symbol.

As shown from by (1), the OFDM signal is composed of a plurality of subcarrier signals, which causes an increase in amplitude fluctuation. A high PAPR signal increases the Input Back Off (IBO) at the power amplifier in order to amplify the transmit signal without distortion. Increasing in IBO causes decreasing the efficiency of PA.

Figure 1 shows the OFDM time domain signal. While OFDM can make effectively utilize the limited frequency domain, a very large peak power is generated in the time domain. The more OFDM uses subcarriers, the more PAPR will be increased.

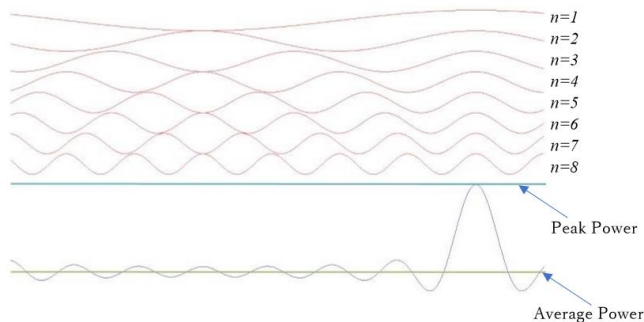


Figure 1. OFDM signals in time domain.

In order to reduce the PAPR, well-known techniques are clipping-and-filtering, partial transmit sequences (PTSS), and selected mapping (SLM)[6]. Clipping-and-filtering limits the peak amplitude of the transmission signal. However, non-linear distortion causes BER to degrade. PTS partitions input data into disjoint sub-blocks. Moreover, each sub-block are weighted by a phase factor. This technique chooses the phase factor to minimize the PAPR of combined signals. SLM generates multiple candidate data blocks. All data blocks represent the same information. Although PTS and SLM can be expected to create a certain reduction in PAPR, both techniques need side information in the receiver, which decreases spectral efficiency. The most practical solution to improving PAPR is to introduce single carrier frequency division multiplexing access (SC-FDMA). The 3GPP LTE system adopts SC-FDMA for uplink multiple access systems. However, SC-FDMA has not been considered to be suitable for next-generation high-speed communications. Therefore, we should reduce PAPR without any degradation of various performances

B. Null subcarrier

Communication standards that make use of OFDM, on the transmitting side, data is not placed on the relevant part beforehand, and zero padding is performed so that the real component is designed to be zero.

Figure 2 shows the transmission spectrum of OFDM with Null subcarrier inserted.

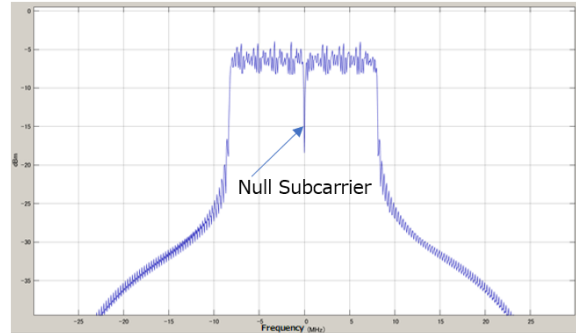


Figure 2. Null subcarrier of OFDM spectrum.

Since the data placed on the subcarrier corresponding to the part of 0Hz, which is the element which becomes the real part in the FFT processing on the receiving side is made null beforehand as the null subcarrier.

III. CAZAC-OFDM SYSTEM

This section describes the features of the CAZAC-OFDM system.

A. CAZAC precoding technique

Zadoff-Chu (ZC) sequence is one of the CAZAC sequence. The ZC sequence c_k of length L is defined as (4).

$$c_k = \begin{cases} \exp\left(\frac{j\pi k(k-1)r}{L}\right) & (L \text{ is even}) \\ \exp\left(\frac{j\pi(k-1)^2 r}{L}\right) & (L \text{ is odd}) \end{cases} \quad (4)$$

Where L is the length of the CAZAC sequence and r is the sequence number, $k = 1, 2, \dots, N^2$. L is a natural number, and r is a prime integer with respect to L .

CAZAC precoding uses a square matrix M generated from the equation in the case where L in (4) is an even number. The matrix equation is defined as (5).

$$M = \begin{bmatrix} c_1 & c_2 & \dots & c_N \\ c_{N+1} & c_{N+2} & \dots & c_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{(N-1)N+1} & \dots & \dots & c_{N^2} \end{bmatrix} \quad (5)$$

Where matrix M is the rearrangement of C_k in (4) in the row direction, and N is the number of subcarriers and $L = N^2$, $r = 1$. N is an even number, and L is also an even number.

Figures 3 and 4 show the block diagrams of CAZAC-OFDM transmitter and receiver, respectively. On the transmitting side of CAZAC-OFDM, CAZAC precoder is inserted before IFFT.

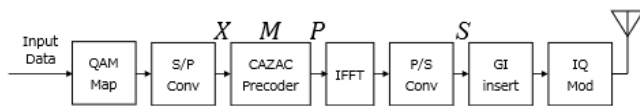


Figure 3. The transmitting side of CAZAC-OFDM

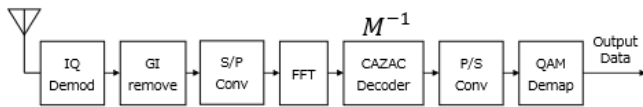


Figure 4. The receiving side of CAZAC-OFDM

CAZAC precoding signal P is generated by calculating the inner product of complex vector of QAM signal X and CAZAC matrix M like (6) [7]. S is the signal after inverse Fourier transform processing.

$$X_M = M \cdot X = \begin{bmatrix} c_1 & c_2 & \dots & c_N \\ c_{N+1} & c_{N+2} & \dots & c_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{(N-1)N+1} & \dots & \dots & c_{N^2} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} \quad (6)$$

While, on the receiving side of CAZAC-OFDM, CAZAC decoder is inserted after FFT and calculates the inner product of the received symbol Y in frequency domain and CAZAC inverse matrix M^{-1} .

According to Figure 3, X is QAM Vector, P is Modulation Vector for IFFT, M is CAZAC Matrix. Then S as

$$S(t_n) = \frac{1}{N} \sum_{k=0}^{N-1} P_k e^{j2\pi kn/N} = \sum_{k=0}^{N-1} \left\{ \sum_{m=0}^{N-1} e^{j\pi(m+kN)^2/L} X_m \right\} e^{j2\pi kn/N} = \sum_{m=0}^{N-1} e^{j\frac{\pi m^2}{N^2}} X_m \left\{ \sum_{k=0}^{N-1} e^{j\frac{2\pi k(m+n)}{N}} e^{j\pi k^2} \right\} \quad (7)$$

Where k is an integer not less than 0, and the following equation is developed.

$$\exp(j\pi k^2) = \begin{cases} 1 & (k : \text{even}) \\ -1 & (k : \text{odd}) \end{cases} \quad (8)$$

To lead the (9) from the (8).

$$\exp(j\pi k^2) = (-1)^k \quad (9)$$

Substituting the (9) into the (7) lead to the (10).

$$S(t_n) = \sum_{m=0}^{N-1} e^{j\pi m^2/N^2} X_m \left\{ \sum_{k=0}^{N-1} \{-e^{j2\pi(m+n)/N}\}^k \right\} \quad (10)$$

The inside of $\{\}$ in (10) is the sum of the geometric progression. Therefore, (11) is derived.

$$\sum_{k=0}^{N-1} \{-e^{j2\pi(m+n)/N}\}^k = \begin{cases} N & (-e^{j2\pi(m+n)/N} = 1) \\ 0 & (-e^{j2\pi(m+n)/N} \neq 1) \end{cases} \quad (11)$$

When $2\pi(m+n)/N = 1$, $2(m+n)/N$ is an integer and odd number. Also, n and m are $0 \leq n \leq N-1$, $0 \leq m \leq N-1$. (12) follows on account of these relationships.

$$m = \frac{N}{2} - n \pmod{N} \quad (12)$$

(13) is derived from (10), (11) and (12).

$$S(t_n) = X \left(\left(\frac{N}{2} - n \right)_{\text{mod } N} \right) C \left(\left(\frac{N}{2} - n \right)_{\text{mod } N} \right) \quad (13)$$

From the above, we have theoretically derived $S(t_n)$ for arbitrary n . The time waveform of CAZAC-OFDM is obtained by phase-rotating the value after mapping such as Figure 5. Therefore, PAPR can be reduced to a single carrier equivalent without using side information and without degrading BER characteristics. This is ideal as an improvement to PAPR.

In OFDM systems, cyclic prefixes are used to prevent inter-symbol interference [8][9]. And certainly, this technique can use cyclic prefix without disadvantages.

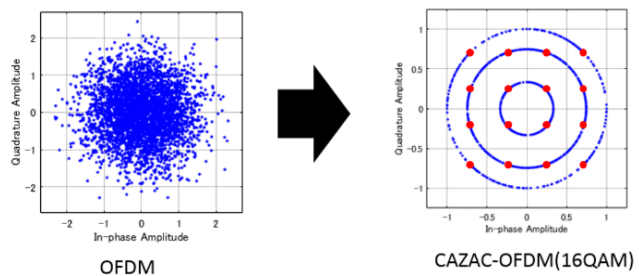


Figure 5. Time domain Signals with CAZAC precoder influenced by the value after mapping.

B. Frequency diversity effect of CAZAC-OFDM

By using CAZAC precoding for OFDM, it has a strong resistance to interference waves of a specific frequency. We consider the case where the signal of a specific subcarrier is lost due to frequency selective fading. In an ordinary OFDM system, each subcarrier has only one X component like Figure 6(a). Therefore, under the frequency selective fading, a specific subcarrier is strongly affected, and this specific data suffers serious damage like Figure 6 (b). As a result, BER deteriorates.

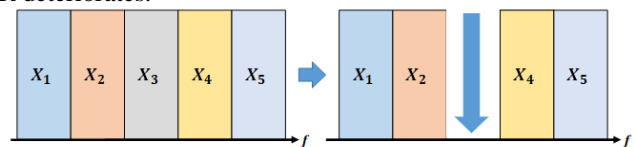


Figure 6. The example of subcarrier deterioration of ordinary OFDM.

On the other hand, in a CAZAC-OFDM system, each subcarrier is possessing all X component averagely like Figure 7(a). Even if a narrow-band interference wave strongly affects a specific subcarrier, the influence of it is uniformly distributed to each data like Figure 7(b), and BER would be improved [10].

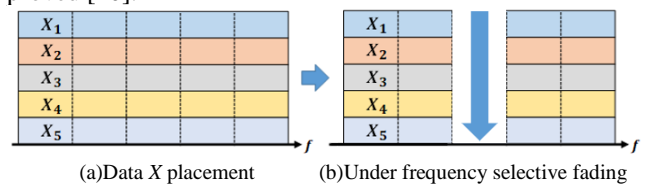


Figure 7. The example of subcarrier deterioration of CAZAC-OFDM.

Thus, it's assumed that a CAZAC-OFDM system can reduce the influence of affected subcarriers by spreading X components to all subcarriers. This effect is due to the expression of CAZAC precoding signal P before IFFT. From (6), IFFT input signal for subcarrier is shown as (14).

$$P_n = \sum_{k=0}^{N-1} C_{n(N+k)} X_k \quad (14)$$

$$= C_{nN} X_0 + C_{n(N+1)} X_1 + C_{n(N+2)} X_2 + \dots + C_{n(2N-1)} X_{N-1}$$

As shown in (14), P_n includes terms of all X components, i.e. $X_0, X_1, X_2, \dots, X_{N-1}$. This means that each subcarrier is possessing all X component. Thus, the CAZAC-OFDM has a nature of spread-spectrum just like a CDMA.

C. Null subcarrier allocation method for CAZAC-OFDM

In CAZAC-OFDM, all subcarriers have all data components averagely. At the same time, it means that the null subcarrier can't be placed at the center of the frequency spectrum. Therefore, in this state OFDM cannot be applied to the current wireless standard.

In an ordinary OFDM system, zero-padding is performed on DC subcarrier in advance to obtain a null subcarrier, thereby setting the DC component to zero as (15).

$$X' = \begin{bmatrix} 0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} \quad (15)$$

On the other hand, in CAZAC-OFDM, if an operation as shown by (6) is performed on the original data information, when zero padding is performed in the same way as in ordinary OFDM, zero can't be generated in the CAZAC precoded signal as shown in (16), and it cannot be set as a null subcarrier.

$$X'_M = M \cdot X' = \begin{bmatrix} c_1 & c_2 & \dots & c_N \\ c_{N+1} & c_{N+2} & \dots & c_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{(N-1)N+1} & \dots & \dots & c_{N^2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ X_1 \\ \vdots \\ X_{N-1} \end{bmatrix} \quad (16)$$

$$= \begin{bmatrix} c_1 \cdot 0 + c_2 \cdot X_1 + \dots + c_N \cdot X_{N-1} = X'_{m1} \neq 0 \\ c_{N+1} \cdot 0 + c_{N+2} \cdot X_1 + \dots + c_{2N} \cdot X_{N-1} = X'_{m2} \neq 0 \\ \vdots \\ c_{(N-1)N+1} \cdot 0 + \dots + c_{N^2} \cdot X_{N-1} = X'_{mN} \neq 0 \end{bmatrix}$$

Furthermore, if the size of the CAZAC matrix and the size of the IFFT do not match, the PAPR reduction effect can't be obtained. It means that IDFT must be performed including DC subcarriers. Therefore, the number of data subcarriers is increased by one compared to conventional OFDM. Thus, in order to use CAZAC-OFDM with the number of subcarriers of the current standard, it is necessary to use IDFT. Although the computational complexity $O(n^2)$ of the DFT is larger than the computational complexity $O(n \log n)$ of the FFT, it can be ignored considering the progress of the processor.

In the proposed system, DC subcarrier is set as null subcarrier while maintaining the orthogonality of the CAZAC sequence by performing the process of setting P_1 in (6) to zero. This process means that one data is lost, but CAZAC-OFDM can reduce adverse influence of affected subcarriers by

spreading influence to all subcarriers. By using this frequency diversity effect of CAZAC-OFDM, lost data is demodulated.

IV. SIMULATION RESULTS AND DISCUSSIONS

This chapter describes simulation specifications and results.

A. Simulation specification

To evaluate the performance of the proposed method, simulation was performed according to the specifications in Table 1. The simulation is performed in MATLAB using communications system toolbox. We compared the CCDF characteristics between ordinary OFDM and proposed method using the number of subcarriers used in the IEEE802.11ac standard. Next, the BER characteristics were compared while changing the value of the signal-to-noise ratio (SNR) of AWGN. At this time, fading is not taken into consideration. After that, we carried out the same comparison using the number of subcarriers used in the 4G-LTE standard.

For the forward error correction code and decoding, convolutional codes with coding rate $R = 1/2$, constraint length $K = 7$ and Viterbi decoding were applied.

Looking at Table 1, the number of data subcarriers in the conventional OFDM is one less than the number of proposed systems because transmission is performed without carrying data on DC subcarriers. On the other hand, although the DC subcarrier is set to be zero in the proposed system, it is demodulated using the frequency diversity effect, so it does not decrease the number of data subcarriers. Thus, the data rate of the proposed method is slightly improved as compared with ordinary OFDM.

TABLE I. SIMULATION SPECIFICATION

Wireless Standards	IEEE802.11ac		4G-LTE	
	Proposal	Conventional	Proposal	Conventional
Modulation	64QAM-OFDM		64QAM-OFDM	
DFT size	469		1201	
Method	Proposal	Conventional	Proposal	Conventional
Number of Data subcarriers	469	468	1201	1200
Bandwidth	160MHz		20MHz	
FEC	Convolutional code		Convolutional code	
Channel Model	AWGN		AWGN	

Figure 8 shows the configuration of the transmitter and receiver of the proposed system. One of the signals after CAZAC precoding processing is set to 0 so that only the center frequency of the signal after IDFT drops.

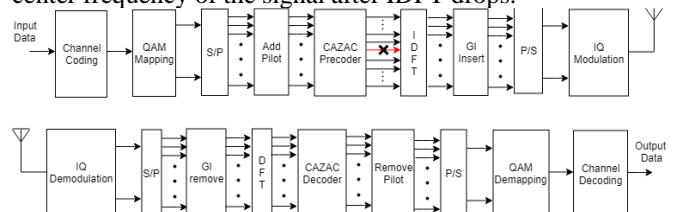


Figure 8. Transmitter and receiver configuration of the proposed system.

B. Simulation Results

First, we compare the OFDM and the proposed system for the CCDF characteristics and BER characteristics using the number of subcarriers used in the IEEE802.11ac standard as Figures 9 and 10. Here, conventional CAZAC-OFDM means it has no null subcarrier.

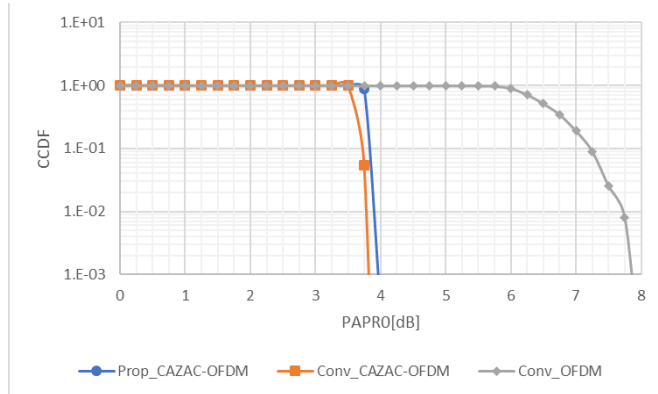


Figure 9. CCDF characteristics in the IEEE 802.11ac standard

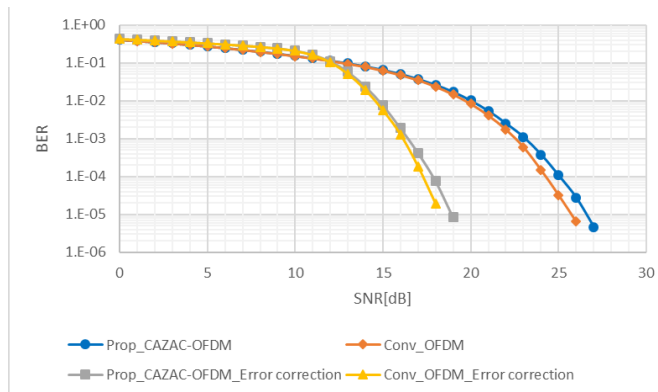


Figure 10. BER characteristics in the IEEE 802.11ac standard

Looking at Figure 9, by using the Null subcarrier allocation method like the proposed system, it is found that the proposed method can obtain CCDF characteristics almost the same as the ideal CAZAC-OFDM which does not adopt the DC subcarrier as Null. Therefore, it can be said that the proposed method can make the DC subcarrier Null without impairing the PAPR reduction effect of CAZAC-OFDM, and PAPR can be improved by about 4 dB when compared with CCDF characteristics of conventional OFDM.

Looking at Figure 10, at first, the CAZAC precoding process does not adversely affect the error correction by the convolutional code. Next, comparing the BER characteristics of ordinary OFDM and the proposed method, we can see that the BER of the proposed method is slightly deteriorating. This is because the data of the DC subcarrier is lost when the DC subcarrier is set to Null, and therefore the influence thereof appears even if the frequency diversity effect is used.

From the above results, it can be said that the proposed method in the IEEE802.11ac standard is a method which can obtain slight improvement of data rate and dramatic improvement of CCDF characteristics in exchange for slight

deterioration of BER characteristic compared to conventional OFDM.

Next, similar comparisons using the number of subcarriers used in the 4G-LTE standard are shown in Figures 11 and 12. At this time, the spectrum of the proposed system with null subcarrier arranged is shown in the figure 13.

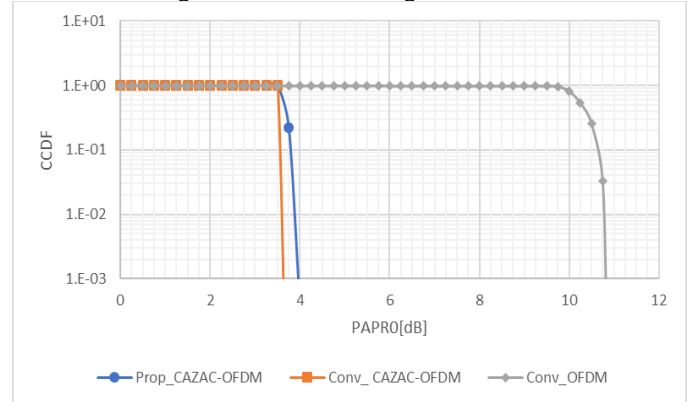


Figure 11. CCDF characteristics in the 4G-LTE standard

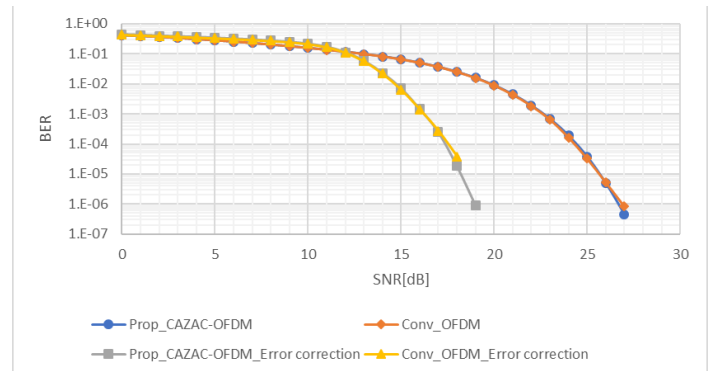


Figure 12. BER characteristics in the 4G-LTE standard

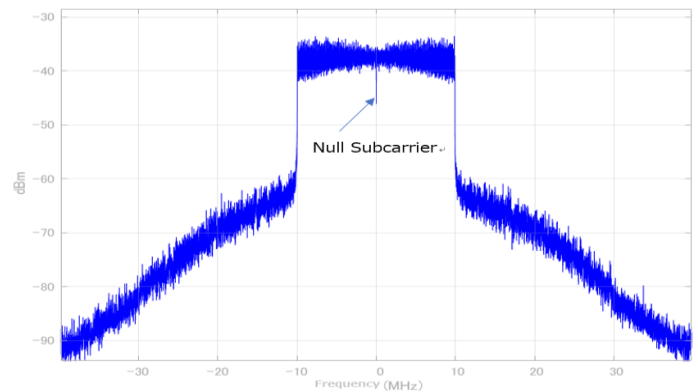


Figure 13. CAZAC-OFDM spectrum with null subcarrier arranged

Comparing Figure 11 with Figure 9, CAZAC-OFDM shows no change in CCDF characteristics even if the number of subcarriers increases. This is because CAZAC-OFDM uses the power pattern of QAM mapping as the output power after IFFT processing without depending on the number of subcarriers. On the other hand, since the conventional OFDM multiplexes a plurality of subcarriers intact, the peak power thereof increases with an increase in the number of subcarriers.

Therefore, the PAPR also increases with the number of subcarriers, which shows that the PAPR is very high in the 4G-LTE standard where the number of subcarriers is 1200. It means that PAPR can be improved as the number of subcarriers increases compared with OFDM in the proposed method, and here we can see that PAPR can be improved by about 7dB.

Looking at Figure 12, It was found that the BER performance of the proposed method is almost consistent with that of ordinary OFDM regardless of whether or not error correction is performed. Compared to Figure 10, it can be said that the frequency diversity effect by CAZAC precoding becomes more powerful as the number of subcarriers is larger, and in the 4G-LTE standard, the negative influence generated when the DC subcarrier is made Null is ignored. From the above results, it can be said that the proposed method in the 4G-LTE standard can improve the data rate slightly and dramatically improve the CCDF characteristics while maintaining the same BER characteristic as the conventional OFDM.

From these results, it was found that the BER characteristics differed depending on the number of subcarriers. Here, it is evaluated how much the deterioration degree of the BER characteristic with respect to the number of subcarriers under the simulation specification similar to Table 1 is. The difference in SNR at the error rate 10^{-3} point when only the number of data subcarriers is changed is shown in Figure 14. We evaluated using 234, 468 data subcarriers used in the IEEE802.11ac standard, 1200 subcarriers used in the 4G-LTE standard and subcarriers between them.

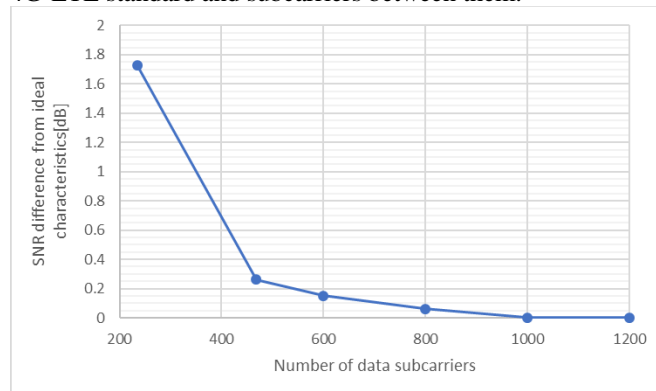


Figure 14. Degree of degradation of BER characteristics with respect to the number of data subcarriers

From this figure, when the proposed method is applied using 234 subcarriers, which is the medium-band transmission mode of the IEEE 802.11ac standard, the BER performance will be deteriorated by about 1.76 dB compared to ordinary OFDM. And it is difficult to say that it is practical. It is about 0.26 dB worse in the case of 468 subcarriers which is the broadband transmission mode, which can be said to be a practical range considering that PAPR can be greatly improved if this degree is deteriorated. And at 1000 subcarriers, the degradation of BER characteristics was not completely seen.

It has resulted that only advantages are obtained in mobile communication exceeding 1000 subcarriers and terrestrial

digital broadcasting. In recent years, communication using a large number of subcarriers tends to be performed in order to enable further high-speed communication, including a wireless LAN. This is a promising trend for the proposed system.

V. CONCLUSIONS

In CAZAC-OFDM that reduces high PAPR while taking advantage of OFDM, PAPR can be reduced while zero DC component by applying Null subcarrier placement method utilizing frequency diversity effect. It means that CAZAC-OFDM can be adapted to current wireless standards. Moreover, it is confirmed that the deterioration of the BER characteristic decreases as the number of subcarriers increases. As a result, in the 4G-LTE standard, we confirmed that the proposed system can be used without degrading BER while suppressing PAPR to the equivalent of single carrier. These results meet recent trends that are changing from narrowband transmission to broadband transmission in order to transmit data with large amount of information at high speed.

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