A New Technique of Symbol Synchronization for OFDM Systems with CAZAC Precoding

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Abstract— **As Orthogonal Frequency Division Multiplex (OFDM) signal is essentially a sum of multiple subcarrier signals aligned in frequency domain, its waveform in time domain is a waveform that changes like Gaussian noise, and thus the amplitude of time-domain waveform has high Peak-to-Average Power Ratio (PAPR). Constant Amplitude Zero Auto-Correlation (CAZAC) waveform precoding transforms from an OFDM waveform into a waveform kind of single-carrier modulation signal. In this paper, a new technique of symbol synchronization for OFDM systems with CAZAC precoding is proposed. By the CAZAC precoding, the waveform of one OFDM symbol can be changed to the waveform of Zadoff-Chu sequence by inputting proper data sequence. Here, the Zadoff-Chu sequence, known as one of CAZAC sequence, has excellent autocorrelation characteristics and is utilized as a strong tool for symbol synchronization in 4G-LTE systems.**

Keywords- OFDM; Zadoff-Chu sequence; CAZAC precorder; symbol synchronization;

I. INTRODUCTION

Currently, OFDM modulation scheme is mainly used in wireless communication systems: Wi-Fi, 4G-LTE, digital terrestrial broadcasting, etc. OFDM signals have high spectrum efficiency. However, as OFDM signal is essentially a sum of multiple subcarrier signals aligned in frequency domain, its waveform in time domain is a waveform that changes like Gaussian noise, and thus the amplitude of time domain waveform has high PAPR.

To reduce the PAPR, many techniques have been proposed: Selected Mapping (SLM), Active Constellation Extension (ACE), Partial Transmit Sequence (PTS), etc. [1]. The technique that PAPR was improved drastically in CAZAC-OFDM which used CAZAC precoding have been proposed [2]-[4]. CAZAC precoder makes the PAPR of multilevel quadrature amplitude modulation (M-QAM) OFDM signals into the PAPR of M-QAM single-carrier signals.

CAZAC-OFDM has an extremely unique time-domain waveforms of transmitted signals. The feature of the waveforms is that it consists of signals of which the phase of mapping data is rotated. Here, the mapping data means I-Q modulation data for subcarriers. In addition, the amount of phase rotation and the time ordering are uniquely determined. Thus, processing data sequence properly, the waveform of one OFDM symbol can be generated optionally.

OFDM systems require exact timing estimation on the receiver side [5]. If OFDM symbols are demodulated without exact timing, phase rotation and distortion in amplitude occurred. It results in much degradation of the bit-error rates. Therefore, most of OFDM systems employ preamble blocks and cyclic-prefix periods for synchronization.

In this paper, a new technique of symbol synchronization of OFDM systems with CAZAC precoding is proposed [6]. By using CAZAC precoding, the time-domain waveform of one OFDM symbol can be changed to the waveform of Zadoff-Chu sequence by inputting proper data sequence. Here, the Zadoff-Chu sequence, known as one of CAZAC sequence, has constant amplitude and excellent autocorrelation characteristics and is utilized as a tool for symbol synchronization in 4G-LTE systems [7]-[9]. If we accord the length of Zadoff-Chu sequence with the length of one OFDM symbol, for example, 64 Fast Fourier Transform (FFT) points, the OFDM symbol consisting of Zadoff-Chu sequence can be used for one unit of preamble sequence. Moreover, the technique is expected to be used in both of packet and continuous mode OFDM systems. By replacing data symbols with the Zadoff-Chu symbols periodically, the technique can apply to the synchronization for continues mode OFDM systems, such as digital terrestrial broadcasting systems.

The rest of this paper is organized as follows. In Section 2, we describe OFDM system, CAZAC-OFDM system and proposed system. In Section 3, we describe performance evaluation and computer simulation results. Finally, we conclude this paper in Section5.

II. PROPOSED SYSTEM

In this section, we describe OFDM system, CAZAC-OFDM system and proposed system. We first describe OFDM system and then, we explain CAZAC precoding technique. Next, we explain proposed system.

A. *OFDM System*

In OFDM system, N-point inverse Fast Fourier Transform (IFFT) is taken for the transmitted symbols $\boldsymbol{X} = [X_0, X_1, \dots, X_{N-1}]^T$, so as generate $\boldsymbol{x} = [x_0, x_1, \dots, x_{N-1}]^T$, the samples for the sum of N orthogonal subcarrier signals. Let $\boldsymbol{y}$ donate the received sample that corresponds to $\boldsymbol{x}$ with the additive noise w (i.e.,$\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{w}$). Taking the N −point FFT of the received samples, $\boldsymbol{y} = [y_0, y_1, \dots, y_{N-1}]^T$, the noisy version of transmitted symbols $\boldsymbol{Y_l} = [y_0, y_1, \dots, y_{N-1}]^T$ can be obtained in the receiver.

As all subcarriers are of the finite duration T, the spectrum of the OFDM signal can be considered as the sum of the frequency-shifted sinc functions in the frequency domain as illustrated in Figure 1, where the overlapped neighboring sinc functions are spaced by $1/T$. As the OFDM signals are orthogonal, they are inter-carrier interference (ICI)-free. The OFDM scheme also inserts a guard interval in the time domain, which mitigates the inter-symbol interference (ISI) between OFDM symbols [5].

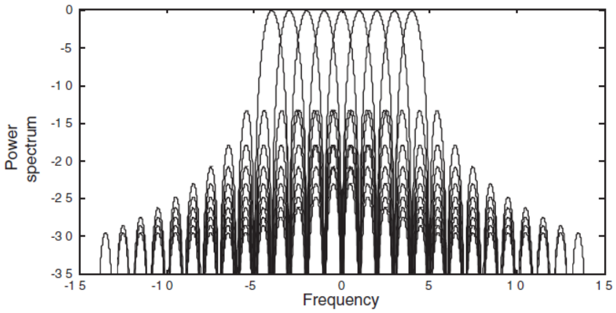


Figure 1. Power spectrum of OFDM signal

B. CAZAC-OFDM

CAZAC sequence is constant amplitude and provides a good cross-correlation property. Therefore, CAZAC sequence is used in wireless communication systems such as channel estimation and time synchronization. The Zadoff-Chu (ZC) sequence c_k which is one of the CAZAC sequences is represented as

$$c_k = e^{\frac{j\pi k^2}{N^2}} \tag{1}$$

Where $k = 1, 2, \dots, N^2 - 1$ donates the sequence index. In this paper, CAZAC $N \times N$ precoding matrix M is represented as

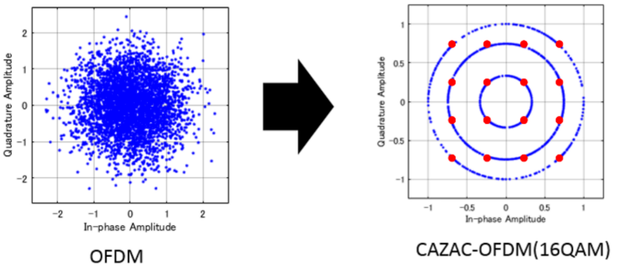
$$\boldsymbol{M} = \begin{bmatrix} c_1 & \cdots & c_N \\ c_{N+1} & \cdots & c_{2N} \\ \vdots & \ddots & \vdots \\ c_{(N-1)N+1} & \dots & c_{N^2} \end{bmatrix} \tag{2}$$

In CAZAC-OFDM system, we precode by multiplying this matrix $\boldsymbol{M}$ before IFFT.

Time signal in CAZAC-OFDM is obtained by multiplying matrix $\boldsymbol{M}$ before IFFT. The time signal x_n can be represented as

$$x_n = X\left(\left(\frac{N}{2} - n\right)_{mod\ N}\right) C\left(\left(\frac{N}{2} - n\right)_{mod\ N}\right) \tag{3}$$

where $X(n)$ is a value after data mapping. Therefore, the time signal in CAZAC-OFDM is a value after data mapping phase rotated (Figure 2).



OFDM

CAZAC-OFDM(16QAM)

Figure 2. Plots on complex phase plane of OFDM and CAZAC-OFDM

C. Zadoff-Chu Sequence

ZC sequence is currently used in the following three applications in LTE/LTE-advanced.

1). Reference sequence of uplink reference signal

It is required that the uplink reference signal has highly autocorrelation and the power fluctuation is small in the time domain. Therefore, ZC sequence having the characteristic that the power is constant in the frequency domain and the time domain is used.

2). Main Sync Signal for Cell Search

The cell search is the following before doing the terminal to communicate with LTE network.

- detect and synchronize cells in the network
- receive and decode necessary information for communication within the cell and appropriate operation.

The ZC sequence is used as the main synchronization signal to assist these cell searches.

3). A Random Access Preamble

Random access is what the terminal does in order to establish uplink synchronization. By sending the random access preamble to the base station by the terminal, the base station estimates the transmission timing of the terminal. As this random access preamble, the ZC sequence is used [7]-[9].

D. Synchronizing Symbol Timing

Synchronizing symbol timing is synchronization processing to obtain OFDM symbols of the reference. The process decides FFT window position. After establishing the synchronization, we can demodulate that received OFDM signals. In packet mode, preamble signals as short as possible is transmitted before the data to establish symbol timing synchronization. It can be roughly classified into an autocorrelation type using a repetitive signal section and a cross correlation type using a matched filter of a preamble provided in a demodulator (Figure 3) [5][10].

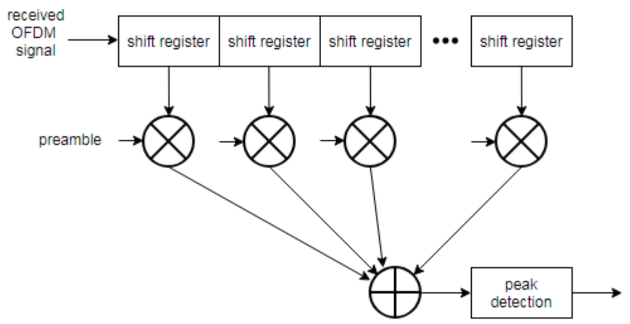


Figure 3. Cross correlation type

E. *Transmission Mode*

When the OFDM signals continue for a long time, this scheme is called continuous mode. In this case, some time is allowed for establishing frequency and symbol timing synchronization in the receiver. Therefore, establishing the synchronizations gradually is possible by using pilot signals embed in OFDM signals. A schematic of the signal in continuous mode is shown in Figure 4.

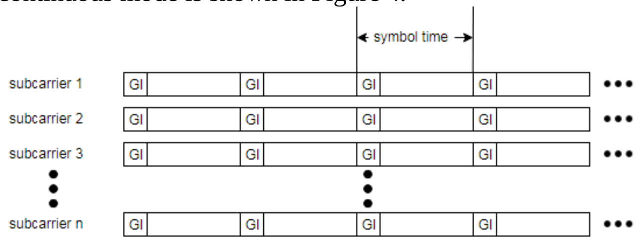


Figure 4. Continuous mode signal

On the other hand, OFDM signals separated by a certain time are transmitted and received in packet mode which are applied in wireless LAN and mobile communication. In the receiver, it is necessary to wait at all times and establish synchronizations in a short period of time. Signals to establish synchronizations called preamble are assigned in packet mode. By using this preamble for synchronizations, we can execute synchronization processing quickly and accurately, which are required for packet mode. A schematic of the signal in packet mode is shown in Figure 5.

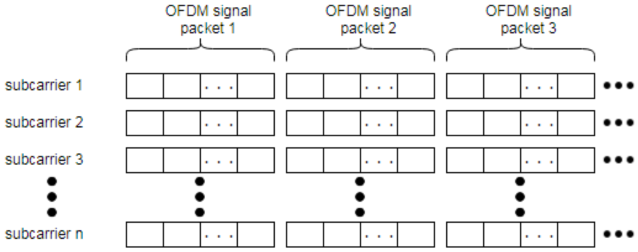


Figure 5. Packet mode signal

F. *Oversampling*

PAPR for the discrete-time base band signal $x[n]$ may not be the same as that fore the continuous-time base band signal $x(t)$. In fact, the PAPR for $x[n]$ is lower than that for $x(t)$, simply because $x[n]$ may not have all peaks of $x(t)$. In practice, the PAPR for the continuous-time baseband signal can be measured only after implementing the actual hardware, including digital-to-analog convertor (DAC). In other words, measurement of the PAPR for the continuous-time baseband signal is not straight forward. Therefore, there must be some means of estimating the PAPR from the discrete-time signal $x[n]$. Fortunately, it is known that $x[n]$ can show almost the same PAPR as $x(t)$ if it is L-times interpolated (oversampled) where $L \geq 4$.

It inserts $(L-1)$ zeros between the samples of $x[n]$ to yield $w[m]$ as follows:

$$w[m] = \begin{cases} x[m/L], & for\ m = 0, \pm L, \pm 2L, \ldots \\ 0, & elsewhere \end{cases} \quad (4)$$

A low pass filter (LPF) is used to construct the L −times-interpolated version of $x[n]$ from $w[m]$. For the LPF with an impulse response of $h[m]$, the L-times-interpolated output $y[m]$ can be represented as

$$y[m] = \sum_{k=-\infty}^{\infty} h[k]w[m-k] \quad (5)$$

Figures 6 and 7 illustrate the signals and their spectra appearing in the oversampling process with a sampling frequency of 2kHz to yield a result of interpolation with $L = 4$. Referring to these figures, the IFFT output signal $x[n]$ can be expressed in terms of the L −times interpolated version as

$$x'[m] = \frac{1}{\sqrt{L \cdot N}} \sum_{k=0}^{L \cdot N-1} X'[k] \cdot e^{\frac{j2\pi m \Delta f k}{L \cdot N}}, \quad (6)$$
$$m = 0, 1, \ldots, NL-1$$

with

$$X'[k] = \begin{cases} X[k], & for\ 0 \leq k < \frac{N}{2}\ \ and\ \ NL - \frac{N}{2} < k < NL \\ 0, & elsewhere \end{cases} \quad (7)$$

where N, Δf, and $X[k]$ donate the FFT size (or the total number of subcarriers), the subcarrier spacing and the complex symbol carried over a subcarrier k, respectively.

In the case of 4-times oversampling, the time waveform has a form obtained by interpolating between the original two signal points with three points [5].

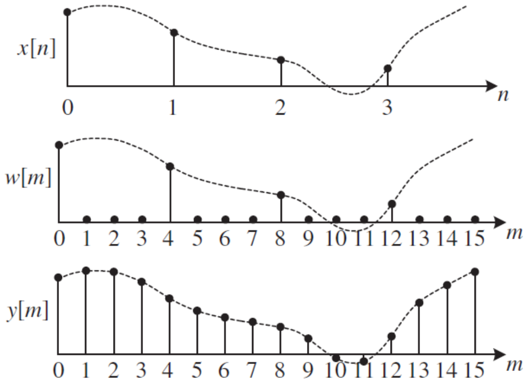


Figure 6. Interpolation with $L = 4$ in the time domain

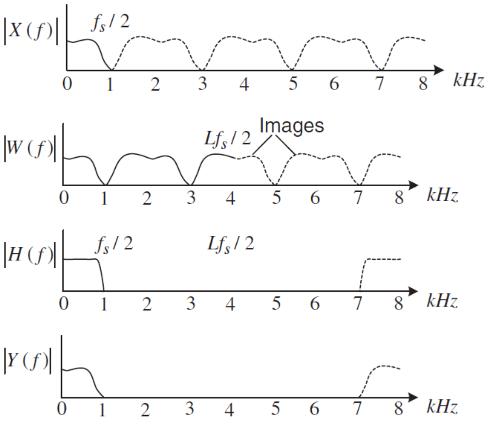


Figure 7. Interpolation with $L = 4$ in the frequency domain

G. Processed Preamble

Using the characteristics of the transmission waveform of CAZAC-OFDM shown in (1), we estimated the symbol timing by inserting a preamble which becomes the ZC sequence after CAZAC precoding and IFFT. From (1), the relation between before and after modulation of CAZAC-OFDM on the transmission side is as shown in the Figure 8. From (1) and (3), the amount of phase rotation θ at each subcarrier is represented as

$$\theta = \pi \frac{\left(\left(\frac{N}{2} - n\right)_N\right)^2}{N^2} \tag{8}$$

From these, we can shape the time waveform to desired waveform by processing CAZAC precoding and IFFT after reversing the order and giving the opposite phase rotation in advance. Then, rather than sending the preamble from the physical layer, it can treat the same way as data and send the preamble as a transmission waveform. In the conventional packet mode transmission system, when transmitting a preamble, it is transmitted using another route as shown Figure 9. Therefore, the switching operation is required at the time of transmission of data and the transmission of data. However, if the above process is used, the preamble can be transmitted without switching. Therefore, it is expected that the preamble can be embedded and used for synchronization not only in the packet mode but also in the continuous mode. In other words, it is possible to use the same system when synchronization processing and data is transmitted.

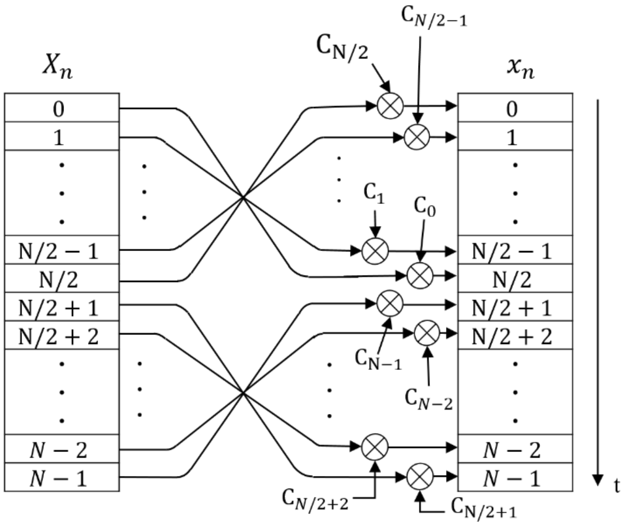


Figure 8. Schematic diagram of the symbol configuration bofore and after modulation of CAZAC-OFDM

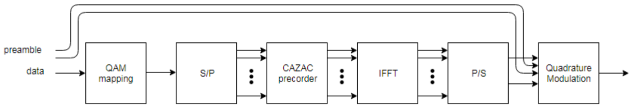


Figure 9. Conventional transmitter model

H. Proposed Flame Configuration

Proposed flame configuration is as shown in the Figure 10. The preamble is transmitted for three periods at first. Subsequently, data is transmitted. Data is demodulated by the timing decided from the preamble. Used preamble is ZC sequence and is represented as

$$ZC = \exp\left(-j\pi \frac{(k-1)^2}{64}\right) (k = 1,2,\ldots,64) \tag{9}$$

The autocorrelation property of this sequence is as shown in the shown Figure 11. In this simulation, we assume the frequency synchronization and time synchronization to be perfectly synchronized. Therefore, the preamble is used for symbol synchronization.

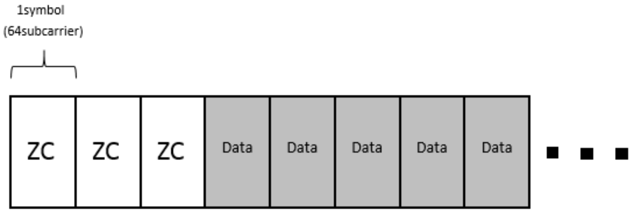


Figure 10. Proposed flame configuration(time domain)

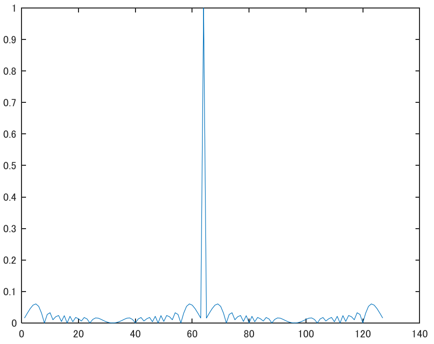


Figure 11. Autocorrelation properties of the ZC sequence

I. *Transmitter and Receiver Model*

Proposed transmitter and receiver model are as shown in the Figures 12 and 13. In the receiver, we prepared the ZC sequence and cross-correlated with the received preamble. The timing of the peak was detected, and the symbol timing was estimated.

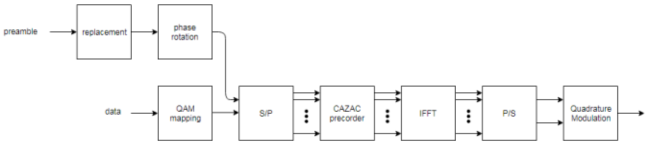


Figure 12. Transmitter model

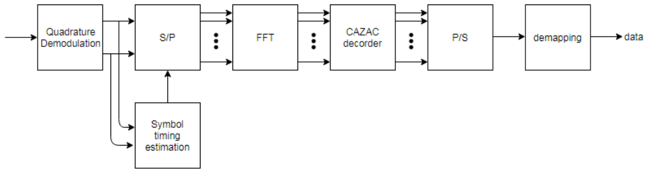


Figure 13. Receiver model

III. PERFORMANCE EVALUATION

In this section, we describe performance evaluation of proposed system. We present simulation results to discuss the performance of the proposed synchronization technique.

A. *Setup*

In order to evaluate the performance of the proposed system, we simulated by MATLAB. Table 1 summarizes the simulation specifications. Bandwidth, symbol length and guard interval are based on the specification of IEEE802.11a. We also do 4-times oversampling at the same time. Therefore, prepared the ZC sequence was expanded fourfold as in the Figure 14. With the above simulation specifications, we examined whether synchronization is established while changing the SNR.

TABLE I. SIMULATION SPECIFICATION

Modulation	16QAM-CAZAC-OFDM
Bandwidth	20MHz
Carrier frequency	600MHz
Number of subcarriers	64
Symbol time	4μs
Guard interval time	0.8μs
Channel model	AWGN
Length of the ZC sequence	64

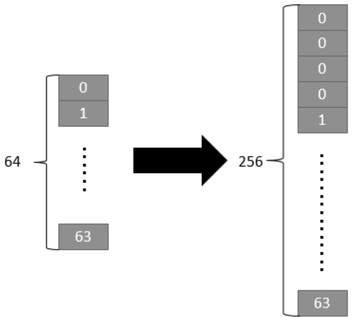


Figure 14. Expansion of the ZC sequence

B. *Simulation Results*

Figure 15 plots the probabilities of synchronizations versus SNR. In the 4-times oversample, the probability of synchronizations is 1 when the SNR is 13 dB or more. Here, the BER characteristic in conventional 16-QAM-CAZAC-OFDM and proposed system is as shown in Figure 16. The BER in proposed system has characteristics similar to that of conventional CAZAC-OFDM. The BER at which the signal can be reproduced using error correction code is 10^{-3} or less becomes reproducible in the range of 13dB or more. In that range, we confirmed that the probability of synchronization can be 1 in 4-times oversample. We confirmed that BER does not deteriorated if synchronization is established. We confirmed that we can establish synchronization using the autocorrelation property of the ZC sequence.

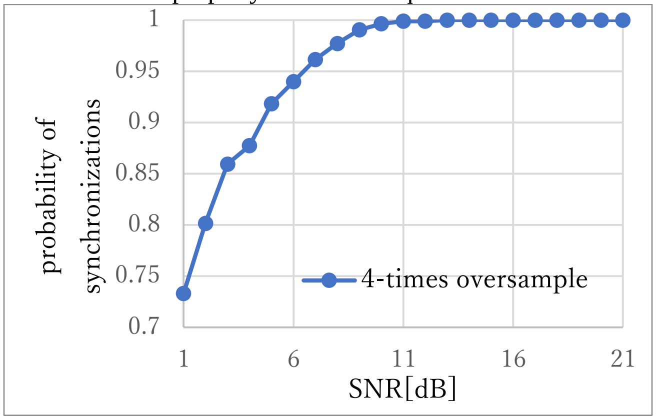


Figure 15. Probabilities of synchronizations versus SNR

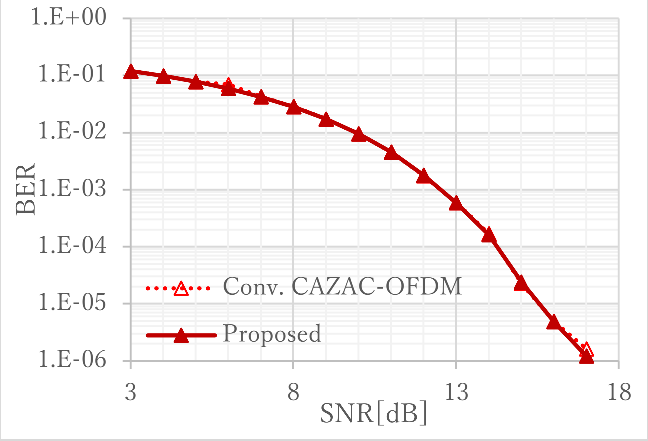


Figure 16. BER performances of the 16QAM-CAZAC-OFDM

IV. CONCLUSION

In this paper, we proposed a new technique of symbol synchronization of OFDM systems with CAZAC precoding. CAZAC-OFDM is one of the useful PAPR reduction techniques. Here, we also showed that the waveform of one OFDM symbol can be changed to the waveform of ZC sequence by processing data sequence properly. Using this technique, we demonstrated that a symbol synchronization can be established. Since the preamble can be treated as data, synchronization can be established using the preamble not only in the packet mode but also in the continuous mode. We have confirmed that once the synchronization is established, the BER does not deteriorate.

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