Bounded Path-Loss Model for UAV-to-UAV Communications

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***Abstract*—In this paper, we focus on ultra-dense network modeling where both the Base Stations (BSs) and Mobile Terminals (MTs) are UAVs. In this case, two communication nodes can be very close to each other. However, existing cellular network analyses typically use the standard unbounded path loss model where received power decays like r^{β} over a distance r. This standard model is a good approximation for the path-loss in wireless communications over large values of r but is not valid for small values of r due to the singularity at 0. This model is often used along with a random uniform node distribution, even though in a group of uniformly distributed nodes some may be arbitrarily close to one another, thus, it will lose accuracy and may be not applicable for UAV-to-UAV communications. To tackle this problem, by using mathematical tool behind stochastic geometry, we propose tractable analytical frameworks of coverage and rate based on the novel unbounded path-loss model with a constant distance factor r_0 for analyzing the UAV-to-UAV communications.**

***Index Terms*—Cellular Networks, UAV, Stochastic Geometry, Bounded Path-Loss Model**

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) have received significant attention in wireless research as they can not only be exploited as aerial BSs, but also functioned as a new type of MTs. In most of the available literatures, the path-loss models for UAVs are devised and formulated via experimental measurement [1], [2], which may not yield a tractable analytical approach for system-level analysis. Due to the simplicity and mathematical tractability, the unbounded path-loss model $L(r) = r^{\beta}$ has been widely applied to characterize channel power gain caused by large scale fading in wireless networks [3], especially when transmission distance is large in the rural areas. However, as the network density becomes larger in the fifth generation (5G) and wireless networks, it becomes more likely that the transmission distance is small. Despite its simplicity, unbounded path-loss model fails to accurately characterize channel power gain in this case. In particular, when $r \in (0, 1)$, applying unbounded path-loss model would artificially force the received signal power to be greater than the transmitted signal power, which is physically impossible.

Therefore, a more realistic model, namely, bounded pathloss model, has been adopted to model the channel power gain caused by pathloss, especially for dense urban scenarios. Widely applied bounded path-loss models include $(1 + r)^{\beta}$, $1 + r^{\beta}$ and $\max\{1, r^{\beta}\}$. In literature, the impact of bounded path-loss model on wireless network performance has been extensively investigated, e.g., [4]. However, all the previous investigated models are based on the fixed distance factor, i.e., 1m, which is not suitable for modeling and analyzing UAV networks. The generalization to a generic bounded distance is never analyzed in the previous studies for ultra-dense UAV networks.

Motivated by these considerations, in the present paper, we propose the novel bounded path-loss model with a generic distance factor, r_0 as an approximated formulation for a measured path-loss model [5]. To evaluate the performance, tractable analytical frameworks of coverage probability and average rate are obtained with aid of the mathematical tool behind stochastic geometry. Specifically, a closed-form expression of coverage probability is derived, which could provide the potential insights for the system-level analysis and optimization.

II. SYSTEM MODEL

Consider a bi-dimensional downlink ultra-dense cellular networks with aerial BSs and MTs, i.e., drones or UAVs. The BSs are modeled as points of a homogeneous Poisson point process (PPP), denoted by Ψ_{BS}, of density λ_{BS}. The MTs are densely distributed as another independent homogeneous PPP, denoted by Ψ_{MT}, of density λ_{MT}. Each BS is assumed to emit a constant transmit power P. Without any loss of generality, the analytical frameworks are developed for the typical MT, denoted by MT_0, that is located at the origin. The BS serving MT_0 is denoted by BS_0. The subscripts 0, i and n identify the intended link, a generic interfering link, and a generic BS-to-MT (UAV-to-UAV) link. The set of interfering BSs is denoted by $\Psi_{\mathrm{BS},i}$.

For each BS-to-MT link, path-loss and fast-fading are considered. Shadowing is not explicitly taken into account because its net effect lies in modifying the density of the BSs [6]. All BS-to-MT links are assumed to be mutually independent and identically distributed (i.i.d.).

1) Path-Loss: Consider a generic BS-to-MT link of length r_n, the path-loss is $L(r_n) = k(r_0 + r_n)^{\beta}$, where k and $\beta > 2$ are the path-loss constant and the path-loss slope (exponent).

2) Fast-Fading: Consider a generic BS-to-MT link. The power gain due to small-scale fading is assumed to follow an exponential distribution with mean Ω. Without loss of generality, $\Omega = 1$ is assumed. The power gain of a generic BS-to-MT link is denoted by h.

3) Cell Association Criterion: A cell association criterion based on the highest average received power is assumed. Let $\mathrm{BS}_n \in \Psi$ denote a generic BS of the network. The serving BS, BS_0, is obtained as follows:

$$\mathrm{BS}_0 = \arg \max_{\mathrm{BS}_n \in \Psi} \{1/L(r_n)\} \quad (1)$$

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$$\mathrm{P_{cov}} = \vartheta\left(\frac{1}{k}\right)^{\frac{1}{\beta}}\frac{\eta}{4\Upsilon_2^2}\left\{2\Upsilon_2\left(\sqrt{-\Upsilon_2}+\sqrt{\Upsilon_2}\varepsilon\left(\xi\right)\right)r_0+\left(\frac{1}{k}\right)^{\frac{1}{\beta}}\left[\Upsilon_1\sqrt{-\Upsilon_2}-2\exp\left(\left(L_{r_0}\right)^{\frac{1}{\beta}}\Upsilon_1+\left(L_{r_0}\right)^{\frac{2}{\beta}}\Upsilon_2\right)\left(\Upsilon_2-\Upsilon_1\sqrt{\Upsilon_2}\mathcal{F}\left(\xi\right)\right)\right]\right\} \tag{5}$$

As for the intended link, $L_0 = \min_{r_n \in \Psi}\left\{L\left(r_n\right)\right\}$ holds.

III. Coverage Probability and Rate

In this section, we present the analytical frameworks of coverage probability and average rate of a typical MT, which are defined by [6]:

$$\mathrm{P_{cov}} = \Pr\left\{\mathrm{SIR} \geqslant \gamma_{\mathrm{th}}\right\} \tag{2}$$

$$R = \mathrm{E}\left\{\ln\left(1+\mathrm{SIR}\right)\right\} = \int_0^{\infty}\frac{\mathrm{P_{cov}}\left(t\right)}{t+1}dt \tag{3}$$

where SIR denotes the signal-to-interference ratio and γ_{th} is the reliability threshold for the successful decoding of information data. SIR is formulated by:

$$\mathrm{SIR} = \frac{P\left(h_0/L_0\right)}{P\sum\limits_{i\in\Psi_{\mathrm{BS},i}}\left(h_i/L_i\right)\mathbf{1}\left(L_i > L_0\right)} \tag{4}$$

The analytical framework of coverage probability can be derived into a closed-form, which is given by (5), and the short-hands in (5) are defined in Table I, where $\mathrm{erfi}\left(\cdot\right)$ is the imaginary error function and $\mathcal{F}\left(\cdot\right)$ is the Dawson function. The general mathematical proof of stochastic geometry background follows the steps in [6].

TABLE I: AUXILIARY FUNCTIONS.

Function definition
$\vartheta = 2\pi\lambda_{\mathrm{BS}}\exp\left\{-\pi\lambda_{\mathrm{BS}}r_0^2\right\}$, $\eta = \exp\left(-\frac{\Upsilon_1^2}{4\Upsilon_2}\right)\sqrt{\pi}$
$\xi = \frac{\Upsilon_1+2\left(kr_0^{\beta}\right)^{\frac{1}{\beta}}\Upsilon_2}{2\sqrt{\Upsilon_2}}$, $\varepsilon\left(\xi\right) = \mathrm{erfi}\left(\xi\right)$, $L_{r_0} = kr_0^{\beta}$
$\Upsilon_1 = 2\pi\lambda r_0\left(\frac{1}{k}\right)^{\frac{1}{\beta}}{}_2F_1\left(-\frac{1}{\beta},1,1-\frac{1}{\beta},-\gamma_{\mathrm{th}}\right)$
$\Upsilon_2 = -\pi\lambda\left(\frac{1}{k}\right)^{\frac{2}{\beta}}{}_2F_1\left(-\frac{2}{\beta},1,1-\frac{2}{\beta},-\gamma_{\mathrm{th}}\right)$

IV. Numerical and Simulation Results

In this section, we illustrate the numerical results of proposed analytical framework based on the bounded path-loss model with a generic distance r_0.

In Fig. 1, we evaluate the performance of coverage probability and average rate as a function of density of BSs for different values of r_0 based on (2), (3) and closed-form formulation in (5). Note that the density of BSs is represented by the cell radius R_{cell}, and $\lambda_{\mathrm{BS}} = 1/\pi R_{\mathrm{cell}}^2$. In addition, Monte-Carlo simulation results are provided to validate the accuracy of proposed analytical frameworks.

It is worth noting that the performance trends of both coverage and rate for the unbounded path-loss model are independent with cell radius or density of BSs. Nevertheless, with the bounded path-loss model, the trends of rate would be monotonically increasing when cell radius increases. In addition, it is indicated that lower r_0 value could enhance the system performance for ultra-dense scenario, and vice versa.

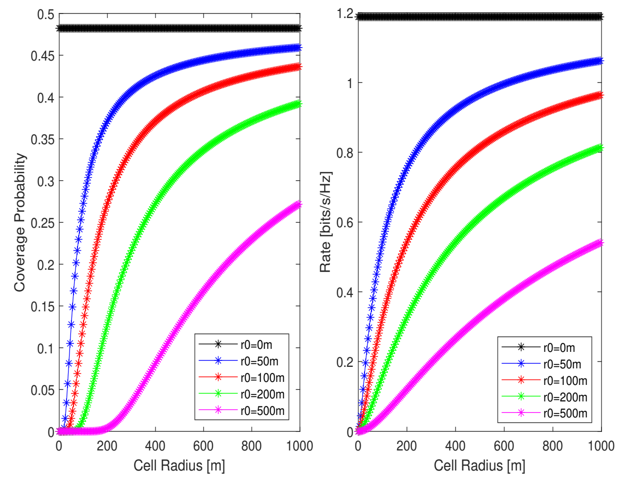

Fig. 1: Coverage probability & rate versus density of BSs. Markers: Monte-Carlo simulations. Solid lines: Analytical frameworks.

V. Conclusion

In this paper, we have introduced new analytical frameworks of coverage probability and average rate under the application of the bounded path-loss model with r_0. The proposed mathematical approach is in a good agreement with Monte-Carlo simulations. Through the performance comparisons of the conventional unbounded path-loss model, it is verified that the bounded path-loss model could provide different performance trends as a function of BSs density, which delivers potential insights and design guidelines for UAV network deployment.

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