Raptor Code for Selecting a Receiver Antenna

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***Abstract*— Massive Multiple-Input Multiple-Output (m-MIMO) is a promising technique for operating fifth-generation wireless networks (5G). However, this technique suffers from the radio frequency chain's higher cost and processing complexity. One solution to deal with this problem is improving the antenna selection method. Nevertheless, many antenna selection methods require knowledge of channel state information (CSI) to select the best performing antenna subset. Which is impossible due to the driver contamination issue in m-MIMO. Furthermore, the exhaustive search method used in conventional multiple-Input Multiple-Output is inefficient for the m-MIMO system. Consequently, this paper proposes an optimal selection algorithm for determining the best subset of antennas at the receiver when CSI is unavailable. For this purpose, we propose a water-filling algorithm based on the mutual information maximization criterion and Raptor-decoded symbols. Numerical results show that our proposed selection algorithm attains close to optimal values as the exhaustive search method.**

Keywords-antenna selection; CSI; m-MIMO; pilot-contamination; water-filling.

I. INTRODUCTION

Massive Multiple-Input Multiple-Output (m-MIMO) is a promising technique that uses hundreds of antennas at the transmitter and receiver to improve channel performance in Fifth Generation (5G) wireless networks. m-MIMO demonstrates improved link reliability, data rate, and radiated energy efficiency than conventional systems [1]. However, the large number of antennas requires the addition of radio frequency (RF) chain elements at both links, increasing the cost and system complexity of m-MIMO. An antenna selection method is used in MIMO conventional to address this issue. A practical solution where a subset of the available antennas at the transmitter and receiver are chosen on a predefined selection criterion to minimize the system complexity and cost in MIMO [2] [3]. The successive selection method, an exhaustive search method, is the most used in these systems for its optimality to find the most performant subset of antennas. However, this method is inefficient for the massive MIMO system because of the significant number of antennas which introduce complexity in the processing. Therefore, an efficient antenna selection algorithm that performs an affordable computational cost is required in m-MIMO.

Several solutions have been proposed in the literature to fix the antenna selection methods problem in m-MIMO. One of those studies used a maximum sum-rate criterion to find the optimal number of antennas [4]. Another paper proposed the maximisation of capacity/sum-rate as the selection criteria for transmitting antennas in massive MIMO's downlink [5]. This later study performed several measurement campaigns in the 2.6 GHz frequency range and used convex optimisation to select the antenna subset that maximises the downlink's Dirty-Paper Coding (DPC) capacity. The authors of the paper assumed that perfect CSI was available at the transmitter. A third method for selecting an optimal antenna is based on a binary searching algorithm using the maximising energy criterion [6]. The authors aimed to ensure energy efficiency in the m-MIMO system and assumed there was imperfect channel estimation at the transmitter.

An algorithm that selects antennas with the highest channel gain in m-MIMO has also been proposed [7]. The selected antennas are combined with Non-Orthogonal Multiple Access (NOMA) to achieve high spectral efficiency in the 5G communication network. Antenna selection at the receiver side has also been studied [8]. In this paper, upper channel capacity bounds were statistically derived for both the Sub-Array Switching (SAS) and Full-Array Switching (FAS) systems in the large-scale limit. The authors assumed that the CSI was only available on the receiver side.

Several of these solutions are fast and optimal. However, most of the solutions that have been proposed in the literature, including those cited above, assume that the channel is perfectly known when selecting antennas. This is impossible in practice, especially when m-MIMO suffers from pilot contamination.

Motivated by these observations, we previously proposed an antenna selection method that considers pilot contamination issue [10]. For this purpose, we presented a water-filling algorithm combined with Low-Density Parity-Check (LDPC) to find the optimal subset of the antennas that maximised the ergodic capacity [10]. In this method, an LDPC decoder retrieves the received symbols. The recovered message was then used to estimate the gain H. The estimated channel was employed to select the optimal subset of antennas that satisfied the maximum capacity criterion. For more details about the channel estimation method, we refer the readers to our previous work [9]

This paper aims to enhance the performance of our previously published method [10] The Raptor codes are the most reliable among the erasure code, therefore, we include them instead of LDPC codes [10]. For more details about these codes, we refer the readers to [11] [12] [13] [14]. We, furthermore, add theoretical analysis to demonstrate how the water filling and the Raptor code will judiciously be exploited to select a performant subset of the antennas.

The proposed solution exploits the physical layer features and does not add more chain elements. In addition, the method based on Raptor decoded symbols requires less transmit power and avoids overload in the network since the symbol pilot are not sent. Furthermore, the Lagrangian and the Water filling algorithm do not require an exhaustive search, making them less complex. Consequently, the proposed solution contributes to reducing energy consumption and processing resources.

At the beginning of the process, when the decoded symbols are not yet available, we assume that the estimated channel is equal to one (that is, $\widehat{H} = 1$); moreover, no subset is selected.

This document is organised as follows: Section II presents the system models. Section III presents the simulation results, and Section IV concludes the paper.

II. SYSTEM MODEL

We consider m-MIMO system with a total of N_t transmit antennas and N_r receive antennas, $N_r \geq N_t$. For each transmission period, a set of $L_r < N_r$ receive antennas is chosen for signal reception. Here, we consider the case where $L_r > N_t$ to ensure spatial multiplexing. If $L_r < N_t$, the system will be rank-deficient [15]. The channel gains form the channel matrix $\boldsymbol{H} = [h_{ij}] \in \mathbb{C}^{N_r \times N_t}$, where $h_{ij} \sim \mathcal{CN}(0,1)$ are independent and identically distributed (i.i.d.). Moreover, $\boldsymbol{H}$ is known to the transmitter but not to the receiver. N_t Raptor-encoded symbols are sent through the channel, and the received signal is given by:

$$Y = HX + N \tag{1}$$

$\boldsymbol{X}$ contains the elements x_i,are the transmitted signals from antenna i . $\boldsymbol{Y}$ contains the entries y_j , which represent the received signals of the jth antenna where $j = 1, \ldots, N_r$.
The water-filling algorithm was used to find the optimal subset of the antennas that maximized the ergodic capacity [15]. Moreover, we used Raptor-decoded symbols to estimate the channel [9].

Gaussian noise vector $\boldsymbol{N} \in C^{N_r}$ consists of i.i.d. $CN(0, N0)$ variables so that $E[\boldsymbol{N}\boldsymbol{N}^{\dagger}] = \sigma_n^2 I_{Nr}$.

The receiver uses the belief propagation algorithm to retrieve the transmitted message with a soft decoding process. The likelihood ratio of the channel for each coded bit are expressed as follows [7]:

$$Z_0 = \frac{2\widehat{H}}{\sigma_n^2} Y \tag{2}$$

The details for Raptor encoding and decoding are provided in a previous study [7].

$\widehat{H}$ is the estimated random variable coefficients of H. The channel estimation is calculated using the Minimum Mean Squared Error (MMSE) as previously described [11] [13]. The $\widehat{H}$ channel is given by:

$$\widehat{H} = R_{HH}(R_{HH}XX^T + \sigma_n^2 I)^{-1} X^T Y \tag{3}$$

where R_{HH} is the covariance of H.

To avoid symbol pilot contamination, the Raptor-decoded symbols $\widehat{S}$ are used instead of the pilot symbols X to estimate.

the channel, as previously described [9]. Hence, X is substituted with $\hat{S}$ in (3) as follows:

$$\widehat{H} = R_{HH}(R_{HH}SS^T + \sigma_n^2 I)^{-1} S^T Y \tag{4}$$

However, to perfectly estimate H at the receiver, the average Bit Error Rate (BER) must approach zero, which means that the message must be entirely recovered (i.e., $S = X$); otherwise, the system is in an outage and H cannot be estimated.

Corresponding to this outage probability, there is a minimum received Signal-to-Noise Ratio (SNR), SNR_{min}, given by:

$$P_{out} = p(SNR < SNR_{min}) \tag{5}$$

$$BER = \frac{1}{2} erfc\sqrt{SNR}$$

$$BER_{max} = \frac{1}{2} erfc\sqrt{SNR_{min}}$$

$$BER_{max} \propto \frac{1}{SNR}$$

$$P_{out} = p(BER > BER_{max})$$

When $SNR \geq SNR_{min}$, the outage probability at the receiver reaches zero: $P_{out} \to 0$ and $BER \to 0$ .Under the fading, the channel is varying slowly. The capacity of the channel C can therefore be expressed as the maximum of mutual information using the following equation:

$$C = log_2\, det(I + SNR) \tag{6}$$

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Because $= \frac{\hat{H}\hat{H}^T}{\sigma_n^2}$, (6) can be rewritten as:

$$C = log_2\, det\left(I + \frac{\hat{H}\hat{H}^T}{\sigma_n^2}\right) \quad (7)$$

A. Antenna Selection

As discussed in the previous section, no symbols are recovered when $SNR = SNR_{min}$ the system is in an outage. In this case, the estimated channel cannot be processed using our approach .Therefore, our antenna selection method will not be applied since it is based on maximizing the capacity criterion.

However, when the BER at the receiver approaches zero, $\hat{H}$ can be calculated. Antenna selection can then be performed to find the optimal antenna subset.

As in a previous study [15], a diagonal matrix Δ of size $N_r \times N_r$ is defined as follows:
$Tr(\Delta) = \sum_i^{N_r} \Delta_i = L_r \le N_r$ represents the number of receive antennas selected at the reception. The received signal is re-written, including receive antenna selection, as:

$$Y = \Delta HX + N \quad (8)$$

The ergodic capacity function of selected antennas can be written through the matrix Δ as follows:

$$C = log_2\, det\left(I + \Delta\hat{H}\hat{H}^H\right) \quad (9)$$

The optimization problem is to pick the L_r receive antennas such that the capacity in (9) is maximized. It is equivalent to finding the matrix Δ such that:

$$C(\Delta) = arg \max_{\substack{\Delta_i \in \{0,1\} \\ \sum_i \Delta_i = L_r}} log_2\, det\left(I + \Delta\hat{H}\hat{H}^H\right) \quad (10)$$

The antenna selection problem in the massive antenna system can be expressed as:

$$\underset{\{\Delta\}}{maximize}\, C(\Delta) = log_2\, det\left(I + \Delta\hat{H}\hat{H}^H\right) \quad (11)$$

subject to:

$$0 \le \Delta \le 1 \rightarrow (Condition\ 1) \quad (12)$$

$$Trace(\Delta) = L_r \rightarrow (Condition\ 2)$$

However, the term $\hat{H}\hat{H}^H$ introduces a complexity on the order of $o(n^6)$. This complexity can be reduced using the low-rank approximation method. The key point is to use the Single Value Decomposition (SVD) method to achieve an ideal low-level estimator.

According to the signal processing theory, the channel correlation matrix can be decomposed using SVD of low-rank approximation, as previously described [16]:

$$R_{HH} = U \wedge U^H \quad (13)$$

U is a unitary matrix and ∧ is a diagonal matrix with the singular values of R_{HH}. The MMSE equation can therefore be represented by:

$$svd(\hat{H}) = U \wedge U^H (U \wedge U^H SS^T + \sigma_n^2)^{-1} SY \quad (14)$$

If taking $\Sigma = \wedge (U \wedge U^{H_P} + \sigma_n^2)^{-1}$, the eigenvalue of ∧ is $\lambda_1 \ge \lambda_2 \ge \cdots \lambda_n \ge 0$ non-zero.

$$\Sigma = \frac{\lambda_k SY}{\lambda_k SS^T + \sigma_n^2} \quad (15)$$

Only the diagonal value is considered in the low rank, so Σ could be written by:

$$\Delta_P = \begin{cases} \frac{\lambda_k SY}{\lambda_k SS^T + \sigma_n^2} & if\ k = 0;\ 1; \ldots\ldots\ldots P-1; \\ 0 & if\ k = P; P+1; \ldots\ldots\ldots N-1; \end{cases} \quad (16)$$

Then finally, the SVD algorithm can be represented as previously described [8]:

$$\Sigma = \begin{bmatrix} \Delta_P & 0 \\ 0 & 0 \end{bmatrix} \quad (17)$$

$$\Delta_P = \begin{bmatrix} \frac{\lambda_0 S^T Y}{\lambda_0 SS^T + \sigma_n^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\lambda_{P-1} S^T Y}{\lambda_{p-1} SS^T + \sigma_n^2} \end{bmatrix} \quad (18)$$

$$svd(\hat{H}\hat{H}^H) = U\Delta_p^2 U^H = U \begin{bmatrix} {\Delta_P}^2 & 0 \\ 0 & 0 \end{bmatrix} U^H \quad (19)$$

$${\Delta_P}^2 = \begin{bmatrix} \frac{\lambda_0 SYY^H S^T}{(\lambda_0 SS^T + \sigma_n^2)^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\lambda_{P-1} SYY^H S^T}{(\lambda_{P-1} SS^T + \sigma_n^2)^2} \end{bmatrix} \quad (20)$$

The simplification term in the denominator can be written as:

$$(SS^T \lambda_0 + \sigma_n^2)^2 = \lambda_0 S^T SSS^T + (\sigma_n^2)^2 \quad (21)$$

Hence,

$${\Delta_P}^2 = \begin{bmatrix} \frac{\lambda_0 SYY^H S^T}{SS^T SS^T + (\sigma_n^2)^2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\lambda_{P-1} SYY^H S^T}{SS^T SS^T + (\sigma_n^2)^2} \end{bmatrix} \quad (22)$$

In high Signal-To-Noise Ratio (SNR), the equation (22) can be rewritten as follows:

$$\varDelta_P{}^2 = \lambda_k \frac{YY^T}{SS^T} \quad (23)$$

$$\varDelta_P{}^2 = \sum_{k=1}^{p} \lambda_k \left(\frac{Y_k}{S_k}\right)^2 \quad (24)$$

And the equation (11) becomes:

$$C(\Delta) = \underset{\substack{\Delta_i \\ \sum_i \Delta_i}}{arg\,max}\, log_2\, det\left(I + \Delta(U\varDelta_P{}^2 U^H)\right) \quad (25)$$

Because $U\Delta U^H = \Pi$,

$$C(\Delta) = log_2\, det(I + \varDelta_P \Pi \varDelta_P) \quad (26)$$

The objective function is concave in Π. However, the Π are binary integer variables making the optimization problem hard for Non-deterministic Polynomial-time (NP). In time order to solve this optimization problem, as in a previous study [17], we relax the constraint that each Π must be a binary integer to the weaker constraint that:

$$0 \leq \Pi \leq 1 \quad (27)$$

The original problem thus becomes a convex optimization problem solvable with water-filling. The Lagrangian method is used to optimize the power of the selected received antennas L_r.

Let $f(\Pi) = \log_2 det(I + \Delta_P \Pi \Delta_P)$ and $(\Pi) = tr(\Pi) - L_r$. The Lagrangian equation is given as follows:

$$\mathcal{L}(\Pi, \psi) = \Delta_P \Pi \Delta_P - \psi(tr(\Pi) - L_r) \quad (28)$$

The derived form of equation (25) is given bellow:

$$\frac{\partial\mathcal{L}(\Pi,\psi)}{\partial\Pi} = \frac{\Delta_P \Delta_P}{(I + \Delta_P \Pi \Delta_P)} - \psi = 0 \Rightarrow$$

$$\psi {\Delta_P}^{-2} = (I + \Delta_P \Pi \Delta_P) \Rightarrow$$

$$\psi^{-1} {\Delta_P}^2 - 1 = \Delta_P \Pi {\Delta_P}^H \Rightarrow \Pi = \psi^{-1} - {\Delta_P}^{-2} \quad (29)$$

$$\frac{\partial\mathcal{L}(\Pi,\psi)}{\partial\psi} = -tr(\Sigma_s) + L_r = 0$$

From (27) at optimality, Π is diagonal. Then the following water filling solution can be obtained

$$\Pi = \left(\psi^{-1} - {\Delta_P}^{-2}\right)^+ \quad (30)$$

III. Simulation Results

The performance of our scheme is evaluated. The codeword length chosen for LDPC encoding is 80000 bits, the message length is 980 bits, and the code rate is 0.98. The degree of distribution of the Luby Transform (LT) encoding is the same as that used in [18]and is as follows:

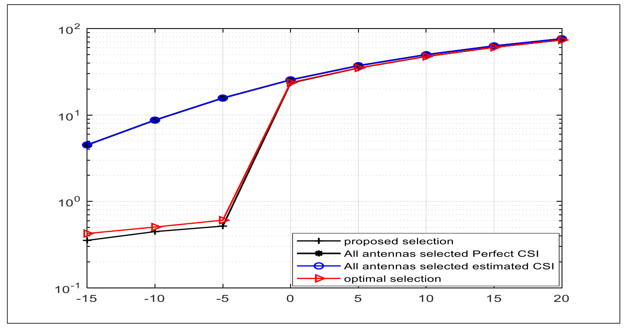

Figure1. Ergodic capacity vs. SNR

$\Omega(x) = 0.008x + 049x^2 + 0.166x^3 + 0.073x^4 + 0.083x^5 + 0.056x^8 + 0.037x^9 + 0.056x^{19} + 0.025x^{65} + 0.003x^{66}$. Furthermore, we use a massive-MIMO system involving 16 antennas at receiver and eight antennas at the transmitter and a subset of the selected antennas Lr=12.

Figure 1 shows the relationship between ergodic capacity and received SNR. The ergodic capacity allows us to select the optimal number of antennas. In this part, the simulation is performed to evaluate four scenarios:

1) Scenario 1: represents our proposed method under perfect CSI and without performing antenna selection method (shown in black with an asterisk)

2) Scenario 2: depicts our proposed method all antennas are selected, and CSI is estimated using the Raptor decoded symbols (blue with circular markers)

3) Scenario 3: describes an exhaustive method (used in conventional MiMo), the number of selected antennas Lr=12. CSI is estimated using the Raptor decoded symbols (red with a triangle pointing to the right)

4) Scenario 4: illustrates our proposed method where Lr=12. CSI estimated using Raptor decoded symbols (black with a plus sign).

Note that the graphs of the first and second scenarios are superposed because they meet the same ergodic capacity values regardless of SNRs' values. This proves our approach's efficiency. However, the ergodic capacity of the two latest scenarios remains low when the SNR is between -15dB and -5dB since the channel cannot be estimated in this interval (see section II-A).

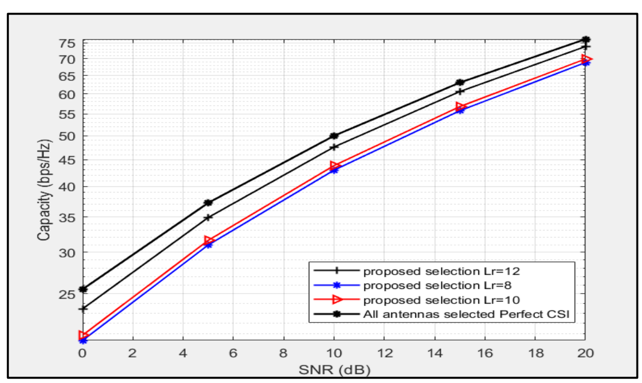

Figure 2. Ergodic capacity vs. SNR for successful decoding

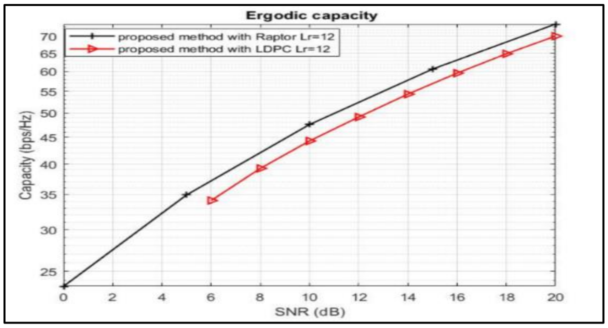

Figure 3. The capacity of Raptor and LDPC code

The ergodic capacity approaches the values of the first two scenarios for an SNR>0. Since the message is completely recovered and the channel is correctly estimated. For the following simulations, we only consider the values when SNR>0.

Figure 2. shows the ergodic capacity vs received SNR per selected antennas Lr, Lr=12, 10 and 8. The results show that Lr=12 achieves the near-to-optimal values.

Figure 3 compares the results of the Raptor-based antenna optimizations and LDPC- based antenna optimizations method proposed in previous work [10]. The channel estimated with Raptor code attains higher optimal capacity than the channel estimated with LDPC code.

IV. CONCLUSION

This paper proposes an antenna selection method performed under imperfect CSI. Our solution combines an antenna selection method based on mutual information maximization and the Raptor decoded information symbols. The Raptor decoded message is used to estimate the channel, and then the water-filling algorithm uses the estimated channel to select the highest-performing subset of antennas. This method requires less transmit power and avoids overload in the network since the symbol pilot are not sent. Which contributes to reducing energy consumption and processing resources Simulation results show that the ergodic capacity reaches near to optimal values using Raptor code than LDPC. Future work can include other methods of antenna selection.

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