Derivation of Some Analytical Expressions in a Model of Overall Telecommunication System with Queue

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***Abstract*—In this paper, we consider a conceptual model in which, for the first time, a queuing system is included in an overall telecommunication system including users' behavior. On the basis of this model, analytical expressions for some of the important parameters of the model are derived. The results obtained allow definitions of new overall performance indicators of human-cyber-physical-systems, including interaction among human users, telecommunication network (including its protocols and management rules) and the nature-socio-economic environment.**

***Keywords*–Network performance modeling; Overall telecommunication system; Queuing systems; Conceptual modeling; Human-computer interaction.**

I. INTRODUCTION

The classical model of overall telecommunication system is described in [1] and developed in more detail in [2]. It considers users' behavior, finite number of homogenous users and terminals, losses due to abandoned and interrupted dialing, blocked and interrupted switching, unavailable intent terminal, blocked and abandoned ringing and abandoned communication. The traffic of the calling (denoted by A) and the called (denoted by B) terminals and user's traffic are considered separately, but in their interrelation.

At the bottom of the structural model presentation, we consider base virtual devices that do not contain any other virtual devices.

The parameters of a base virtual device named x are the following (see [3] for terms definitions): Fx - intensity or incoming rate (frequency) of the flow of requests (i.e., the number of requests per time unit) to device x; Px - probability of directing the requests towards device x; Tx - service time (duration of servicing of a request) in device x; Yx - traffic intensity [Erlang]; Vx - traffic volume [Erlang - time unit]; Nx - number of lines (service resources, positions, capacity) of device x.

We consider an extension of the classical model of overall telecommunication system in which a queuing system is included in the switching stage. It is proposed and described in detail in [4]. The graphical representation of the model is shown in Figure 1. Two types of virtual devices are included in the model: base and comprising base devices. The graphic representations of the base virtual devices together with their names and types are also shown in Figure 1 (see [2]). Each base virtual device belongs to one of the following types: Generator, Terminator, Modifier, Server, Enter Switch, Switch, Queue and Graphic connector.

The names of the virtual devices are concatenations of the first letters of the branch exit, branch and stage, in that order (see Figure 1). For example, **ad** stands for the virtual device "abandoned dialling" while **rad** – for "repeated abandoned dialling".

For better understanding of the model and for a more convenient description of the intensity of the flow, a special notation including qualifiers (see [3]) is used. For example, $dem.F$ stands for demand flow; $inc.Y$ for incoming traffic; $ofr.Y$ for offered traffic; $rep.Y$ for repeated traffic.

The following comprising virtual devices denoted by **a, b, s** and **ab** are considered in the model.

- **a** comprises all calling terminals (A-terminals) in the system. It is not shown in Figure 1, but includes the four shown stages: dialing, switching, ringing and communication;
- **b** comprises all called terminals (B-terminals) in the system. It is shown in a bold line box in Figure 1;
- **ab** comprises all the terminals (calling and called) in the system. It is not shown in Figure 1;
- **s** virtual device corresponding to the switching system.

In our model, the queuing system in the switching stage of the telecommunication network in Kendall's notation (see [5]) is represented as $M|M|Ns|Ns+Nq|Nab|FIFO$, where M stands for exponential distribution, Ns is the capacity of the Switching system (number of equivalent internal switching lines), Nq is the buffer length and Nab is the total number of active terminals which can be calling and called. This is related to the derivation of the analytical model of the system.

The queuing system in the model differs from other well known and studied queuing systems [6]–[8] in that: it has more than one exit to the server; the duration of service of the requests in the server depends on the overall state of the telecommunication system; there is a feedback in terms of call attempts.

II. MAIN ASSUMPTIONS AND PREVIOUS RESULTS

We consider the conceptual model of overall telecommunication system with queue shown in Figure 1 and described briefly in the previous section. Parameters with known values

 ISBN: 978-1-61208-732-0

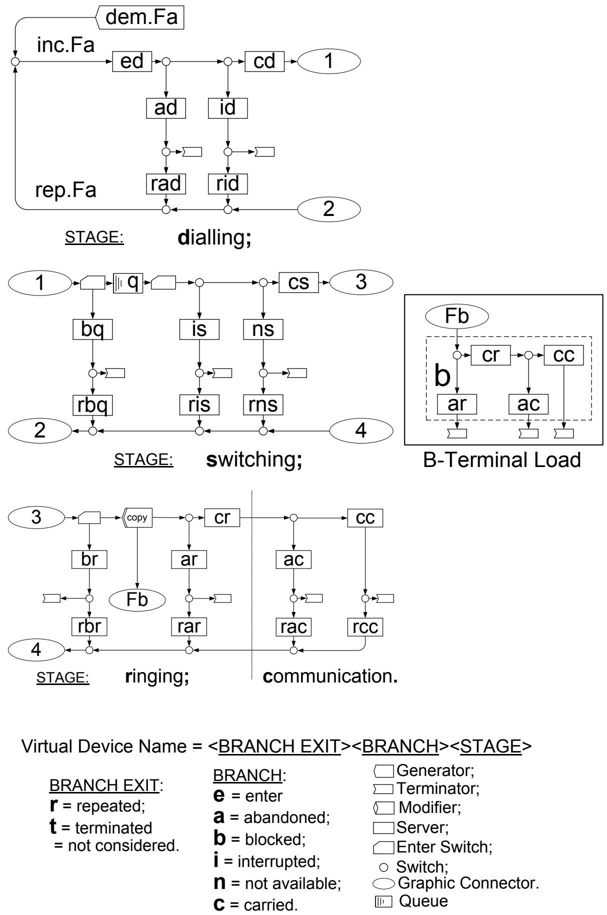

Figure 1. Conceptual model of an overall telecommunication system with a queue in the switching stage.

are all probabilites for directing the call to a device (the P–parameters), the holding time parameters of the base virtual devices (T – parameters) and the values of the intensity of the incoming calls flow – $inc.Fa = Fa$. The unknown parameters are the parameters of the comprising virtual devices except Fa and Nab.

To obtain simple analytical expressions in the process of solving different teletraffic tasks, as in [1], we need to state the following assumptions:

1) We consider a closed telecommunication system which is represented graphically and functionally in Figure 1.
2) All base virtual devices except the Queue device have unlimited capacity. The Queue has capacity Nq, which is the buffer size. The comprising virtual devices have limited capacity: the **ab** device contains all active terminals $Nab \in [2, \infty)$. The switching system (**s**) has capacity Ns. One internal switching line can carry only one call for both incoming and outgoing calls.
3) Every call from the incoming flow to the system ($inc.Fa$) occupies only a free terminal which becomes a busy A-terminal.
4) The system is in a stationary state and the Little's theorem [5] can be applied for every device.
5) Every call occupies one place in a base virtual device independently from the other devices.
6) Any calls in the telecommunication network's environment (outside the **a** and **b** devices) do not occupy any of the telecommunication system's devices.
7) The probabilities of directing the calls to the base virtual devices and the holding time in the devices are independent of each other and of the intensity of the incoming flows $inc.Fa$. Their values are determined by the users' behavior and the technical characteristics of the telecommunication system. Exception to this assumption are the devices of type Enter Switch corresponding to Pbq and Pbs, and Pbr (see Figure 1).
8) For the base virtual devices **ar, cr, ac** and **cc**, the probabilities of directing the calls to them and the duration of occupation of the device are the same for the **a** and the **b** comprise virtual devices.
9) The variables in the model are random with fixed distributions. The Little's theorem allows us to use their mean values.
10) Every call occupies simultaneously all base virtual devices through which it has passed, including the device where it is at the current moment of observation. When a call leaves the comprising devices **a** or **b**, the places occupied by it in all base virtual devices are released.

 ISBN: 978-1-61208-732-0

The following propositions proved for the classical conceptual model of overall telecommunication system in [1] can be used without proof due to analogy. They obviously hold for the conceptual model of overall telecommunication network with queue, considered in the present paper.

Proposition 1: The traffic intensity of all terminals (Yab) is a sum of the traffic intensities of the calling terminals (Ya) and the called terminals (Yb):

$$Yab = Ya + Yb\,. \tag{1}$$

Proposition 2: (Total terminal traffic (Yab) is restricted)

$$0 \leq Yab \leq Nab\,, \tag{2}$$

and here Nab is the total number of all active terminals.

Proposition 3: The calls flow intensity occupying all A and B terminals (Fab) is a sum of the intensities of the calls flow occupying the A-terminals (Fa) and the B-terminals (Fb):

$$Fab = Fa + Fb\,. \tag{3}$$

The Little's formula for the comprised virtual devices **a** and **b** gives dependences for the intensities of the calling (Ya) and called (Yb):

Proposition 4:

$$Ya = Fa\,Ta\,. \tag{4}$$

Proposition 5:

$$Yb = Fb\,Tb\,. \tag{5}$$

In [4], with the help of the above assumptions, analytical expressions for the parameters of the queuing system – expected length of the queue (Yq), mean time of service in the q device (Tq) and the probability of blocked queuing (Pbq) are derived. Here, we present them without proof, but using a more appropriate notation.

Proposition 6:

$$Yq = \frac{p_0 r^{Ns}\rho}{Ns!(1-\rho)^2}[(\rho-1)\rho^{Nq}(Nq+1)+1-\rho^{Nq+1}]\,. \tag{6}$$

Proposition 7:

$$Tq = \frac{p_0 r^{Ns}\rho}{Ns!(1-\rho)^2}\,\frac{[(\rho-1)\rho^{Nq}(Nq+1)+1-\rho^{Nq+1}]}{\lambda(1-Pbq)}\,. \tag{7}$$

Proposition 8:

$$Pbq = \frac{\lambda^{Ns+Nq}}{Ns^{Nq}Ns!\mu^{Ns+Nq}}p_0\,. \tag{8}$$

In the above three propositions, we have used the following notation:

$$p_0^{-1} = \begin{cases} \sum_{n=0}^{Ns-1}\frac{r^n}{n!} + \frac{r^{Ns}}{Ns!}\frac{1-\rho^{Nq+1}}{1-\rho} & \text{for } \rho \neq 1\,. \\ \sum_{n=0}^{Ns-1}\frac{r^n}{n!} + \frac{r^{Ns}}{Ns!}(Nq+1) & \text{for } \rho = 1\,, \end{cases} \tag{9}$$

$$\frac{1}{\mu} = Ys\left[\frac{Yis+Yns+Ycs}{Fis+Fns+Fcs} + \frac{Ybr+Yar+Ycc+Yac}{Fbr+Far+Fac+Fcc}\right],$$

$$\lambda = ofr.Fq\,, r = \frac{\lambda}{\mu}\,, \rho = \frac{r}{Ns}\,.$$

III. Derivation of analytical expressions for some of the parameters of overall telecommunication system with queue

Here, we demonstrate how the conceptual model shown in Figure 1 can be used for derivation of analytical expressions for some of the parameters of the model. In particular, we shall derive expressions for the mean intensity of the **B**-terminals (Fb), the mean service time of the **A**-terminals (Ta) and the intensity of the repeated flow of the **A**-terminals ($rep.Fa$).

Theorem 1:

$$Fb = Fa(1-Pad)(1-Pid)(1-Pbq)(1-Pis)(1-Pns) \cdot(1-Pbr)\,. \tag{10}$$

Proof: Before occupying the intent **B**-terminals with mean intensity Fb, the **a**-calls have to avoid all six types of losses considered in the model in Figure 1 with probability 1 minus the corresponding probabilities for directing the calls to each of the devices **ad, id, bq, is, ns, br**. These probabilities are multiplied since we have independent events. Here, we use assumptions 1, 5, 6, 7, 8 and 9. ■

Theorem 2:

$$Ta = Ted + Pad\,Tad + (1-Pad)[Pid\,Tid + (1-Pid)[Tcd + Pbq\,Tbq + (1-Pbq)\,[Tq + Pis\,Tis + (1-Pis)[Pns\,Tns + (1-Pns)[Tcs + Pbr\,Tbr + (1-Pbr)Tb]]]]]\,. \tag{11}$$

Proof: From the conceptual model on Figure 1, the parameters' independence (Assumption 7) and the virtual channel switching (Assumption 10), we have that Ya is a sum of the traffics of all base virtual devices included in the comprising **a** device.

$$Ya = Yed + Yad + Ycd + Yid + Ybq + Yq + Yis + Ycs + Yns + Ybr + Yar + Ycr + Yac + Ycc\,. \tag{12}$$

Using Little's formula, we can express the traffic intensities in the following way:

$$Yed = Fa\,Ted\,, \tag{13}$$

$$Yad = Fad\,Tad = Fa\,Pad\,Tad\,, \tag{14}$$

$$Yid = Fid\,Tid = Fa(1-Pad)Pid\,Tid\,, \tag{15}$$

$$Ycd = Fcd\,Tcd = Fa(1-Pad)(1-Pid)Tcd\,, \tag{16}$$

$$Ybq = Fbq\,Tbq = Fa(1-Pad)(1-Pid)Pbq\,Tbq\,, \tag{17}$$

$$Yq = Fq\,Tq = Fa(1-Pad)(1-Pid)(1-Pbq)Tq\,, \tag{18}$$

$$Yis = Fis\,Tis = Fa(1-Pad)(1-Pid)(1-Pbq)Pis\,Tis\,, \tag{19}$$

$$Yns = Fns\,Tns = Fa(1-Pad)(1-Pid)(1-Pbq)$$

$$\cdot(1-Pis)Pns\,Tns\,, \tag{20}$$

$$Ycs = Fcs\,Tcs = Fa(1-Pad)(1-Pid)(1-Pbq)$$
$$\cdot(1-Pis)(1-Pns)Tcs\,, \tag{21}$$

$$Ybr = Fbr\,Tbr = Fa(1-Pad)(1-Pid)(1-Pbq)$$
$$\cdot(1-Pis)(1-Pns)Pbr\,Tbr\,. \tag{22}$$

From Assumption 8, we have that Yar, Ycr, Yac, and Ycc are the same for the **A** and **B** terminals. Therefore, using Theorem 1, we obtain

$$Yb = Yar + Ycr + Yac + Ycc = Fa(1-Pad)(1-Pid)$$
$$\cdot(1-Pbq)(1-Pis)(1-Pns)(1-Pbr)Tb\,. \tag{23}$$

After substitution of (13)-(23) in (12) and using (4) we obtain (11). ■

Theorem 3:

$$rep.Fa = Fa\{Pad\,Prad + (1-Pad)[Pid\,Prid + (1-Pid)$$
$$\cdot[Pbq\,Prbq + (1-Pbq)[Pis\,Pris + (1-Pis)$$
$$\cdot[Pns\,Prns + (1-Pns)[Pbr\,Prbr + (1-Pbr)$$
$$\cdot[Par\,Prar + (1-Par)[Pac\,Prac+$$
$$+(1-Pac)Prcc]]]]]]]\}\,. \tag{24}$$

Proof: Using Figure 1 and Assumption 1, $rep.Fa$ can be expressed as a sum of intensities of repeated attempts flows in all branches:

$$rep.Fa = Frad + Frid + Frbq + Fris + Frns + Frbr$$
$$+Frar + Frac + Frcc\,. \tag{25}$$

Using the graphical representation in Figure 1, the intensities of the reapeated attempts flows in all branches can be expressed as a function of Fa in the following ways:

$$Frad = Fa\,Pad\,Prad\,, \tag{26}$$

$$Frid = Fa(1-Pad)Pid\,Prid\,, \tag{27}$$

$$Frbq = Fa(1-Pad)(1-Pid)Pbq\,Prbq\,, \tag{28}$$

$$Fris = Fa(1-Pad)(1-Pid)(1-Pbq)Pis\,Pris\,, \tag{29}$$

$$Frns = Fa(1-Pad)(1-Pid)(1-Pbq)(1-Pis)$$
$$\cdot Pns\,Prns\,, \tag{30}$$

$$Frbr = Fa(1-Pad)(1-Pid)(1-Pbq)(1-Pis)$$
$$\cdot(1-Pns)Pbr\,Prbr\,, \tag{31}$$

$$Frar = Fa(1-Pad)(1-Pid)(1-Pbq)(1-Pis)$$
$$\cdot(1-Pns)(1-Pbr)\,Par\,Prar\,, \tag{32}$$

$$Frac = Fa(1-Pad)(1-Pid)(1-Pbq)(1-Pis)$$
$$\cdot(1-Pns)(1-Pbr)(1-Par)Pac\,Prac\,, \tag{33}$$

$$Frcc = Fa(1-Pad)(1-Pid)(1-Pbq)(1-Pis)(1-Pns)$$
$$\cdot(1-Pbr)(1-Par)(1-Pac)Prcc\,. \tag{34}$$

Adding equations (26)-(34) and performing elementary operations, we obtain the right-hand side of (24). This completes the proof. ■

Using Theorem 3, we can determine the intensity of the input flow to the telecommunication system ($Fa = inc.Fa$). From the graphical representation of the system shown in Figure 1, we have that the intensity of the incoming flow can be represented as a sum of the intensities of the demand calls ($dem.Fa$) and the repeated attempts ($rep.Fa$):

$$Fa = dem.Fa + rep.Fa\,. \tag{35}$$

As in the classical model of overall telecommunication system [1], we use the following equation for the intensity of the demand calls:

$$dem.Fa = Fo(Nab + M\,Yab)\,, \tag{36}$$

where Fo is the intensity of the input flow from one idle terminal, Nab is the number of active terminals and M is a parameter characterizing Bernoulli-Poisson-Pascal (BPP) flow of demand calls [1]. When $M = -1$, demand flow corresponds to Bernoulli (Engset) distribution. When $M = 0$ – to Poisson (Erlang) and when $M = 1$ – to Pascal (Negative binomial) distribution. In general, M can take every value in the interval $[-1, 1]$.

The following classification of the parameters is proposed in [4]:

- Static parameters: $M, Nab, Ns, Ted, Pad, Tad, Prad, Pid, Tid, Prid, Ted, Pis, Tis, Pris, Pns, Tns, Tes, Prns, Tbr, Prbr, Par, Tar, Prar, Tcr, Pac, Tac, Prac, Tcc, Prcc, Nq, Tbq, Trbq, Prbq$. Their values are considered independent of the system state Yab (see [9]) but may depend on other factors. For the model time interval, they are considered constants.
- Dynamic parameters: $Fo, Yab, Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofr.Fq, crr.Fs, Tq, Pbq$. Their values are mutually dependent.

Using this classification, we can express $rep.Fa$ as a function of the dynamic parameters Fa, Pbr and Pbq.

Theorem 4: The intensity of the calls of repeated attempts $rep.Fa$ can be determined by

$$rep.Fa = Fa[R_1 + R_2(1-Pbq)Pbr + R_3Pbq]\,, \tag{37}$$

where

$$R_1 = PadPrad + (1 - Pad)[PidPrid + (1 - Pid)[PisPris + (1 - Pis)[PnsPrns + (1 - Pns)[ParPrar + (1 - Par)[PacPrac + (1 - Pac)Prcc]]]]] , \quad (38)$$

$$R_2 = (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)[Prbr - ParPrar - (1 - Par)[PacPrac + (1 - Pac)Prcc]] , \quad (39)$$

$$R_3 = (1 - Pad)(1 - Pid)[Prbq - [PisPris + (1 - Pis)[PnsPrns + (1 - Pns)[ParPrar + (1 - Par)[PacPrac + (1 - Pac)Prcc]]]]] . \quad (40)$$

Proof: Adding equations (26)-(34), taking into account (25) and separating static from dynamic parameters, we obtain

$$rep.Fa = Fa\{PadPrad + (1 - Pad)[PidPrid + (1 - Pid)[PisPris + (1 - Pis)[PnsPrns + (1 - Pns) \cdot [ParPrar + (1 - Par)[PacPrac + (1 - Pac)Prcc]]]]] + (1 - Pbq)Pbr(1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)[Prbr - ParPrar - (1 - Par)[PacPrac + (1 - Pac)Prcc]] + Pbq(1 - Pad)(1 - Pid)[Prbq - [PisPris + (1 - Pis)[PnsPrns + (1 - Pns)[ParPrar + (1 - Par)[PacPrac + (1 - Pac)Prcc]]]]]\} . \quad (41)$$

Now, we denote the expression in (41) which does not contain dynamic parameters by R_1, the coefficient of $(1 - Pbq)Pbr$ by R_2 and the coefficient of Pbq by R_3 and we obtain (37). This proves the theorem. ■

Similarly, after separating static from dynamic parameters in (11) and using the Little's formula for the A-terminals' traffic intensity (Ya), we obtain the following representation:

Theorem 5:

$$Ya = Fa[Sa_1 + Sa_2(1 - Pbq)Tq + Sa_3(1 - Pbq)Pbr + Sa_4Pbq] , \quad (42)$$

where

$$Sa_1 = Ted + PadTad + (1 - Pad)[PidTid + (1 - Pid)[Tcd + PisTis + (1 - Pis)[PnsTns + (1 - Pns)[Tcs + Tb]]]] , \quad (43)$$

$$Sa_2 = (1 - Pad)(1 - Pid) , \quad (44)$$

$$Sa_3 = (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)(Tbr - Tb) . \quad (45)$$

$$Sa_4 = (1 - Pad)(1 - Pid)[Tbq - PisTis - (1 - Pis)[PnsTns + (1 - Pns)[Tcs + Tb]]] . \quad (46)$$

Proof: We denote by Sa_1 the sum of all expressions in (11) that do not include dynamic parameters:

$$Sa_1 = Ted + PadTad + (1 - Pad)[PidTid + (1 - Pid)[Tcd + PisTis + (1 - Pis)[PnsTns + (1 - Pns)[Tcs + Tb]]]] . \quad (47)$$

We denote by Sa_2 the coefficient of $(1 - Pbq)Tq$ in (11):

$$Sa_2 = (1 - Pad)(1 - Pid) . \quad (48)$$

We denote by Sa_3 the coefficient of the expression $(1 - Pbq)Pbr$ in (11) :

$$Sa_3 = (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)(Tbr - Tb) . \quad (49)$$

We denote by Sa_4 the coefficient of Pbq in (11):

$$Sa_4 = (1 - Pad)(1 - Pid)[Tbq - PisTis - (1 - Pis)[PnsTns + (1 - Pns)[Tcs + Tb]]] . \quad (50)$$

In this way, for Ta we obtain the representation:

$$Ta = Sa_1 + Sa_2(1 - Pbq)Tq + Sa_3(1 - Pbq)Pbr + Sa_4Pbq . \quad (51)$$

After substitution of (51) in (4), we confirm the validity of (42). ■

Theorem 6: The traffic of all simultaneously busy terminals Yab as a function of Fa and other parameters is given by

$$Yab = Fa[S_1 + S_2(1 - Pbq)Tq + S_3(1 - Pbq)Pbr + S_4Pbq] , \quad (52)$$

where

$$S_1 = Ted + PadTad + (1 - Pad)[PidTid + (1 - Pid)[Tcd + PisTis + (1 - Pis)[PnsTns + (1 - Pns)[Tcs + 2Tb]]]] , \quad (53)$$

$$S_2 = (1 - Pad)(1 - Pid) , \quad (54)$$

$$S_3 = (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)(Tbr - 2Tb) , \quad (55)$$

$$S_4 = (1 - Pad)(1 - Pid)[Tbq - PisTis - (1 - Pis)[Pns + (1 - Pns)[Tcs + 2Tb]]] . \quad (56)$$

Proof: From (1), (4), (5) and Theorem 1 we have

$$Yab = Ya + Yb = FaTa + FbTb = Fa\{Ta + (1 - Pad)(1 - Pid)(1 - Pbq)(1 - Pis)(1 - Pns)(1 - Pbr)Tb\} . \quad (57)$$

After substitution of Ta in the above equation with its equal expression from (51), we obtain:

$$Yab = Fa\{Sa_1 + Sa_2(1 - Pbq)Pbr + Sa_3(1 - Pbq)Pbr + Sa_4Pbq - (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns) \cdot (1 - Pbq)PbrTb + (1 - Pad)(1 - Pid)(1 - Pis) \cdot (1 - Pns)(1 - Pbq)Tb\} . \quad (58)$$

In the above expression, we denote by S_1 the part which does not contain T_q, Pbq and Pbr:

 ISBN: 978-1-61208-732-0

$$S_1 = Sa_1 + (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)Tb =$$
$$Ted+PadTad+(1-Pad)[PidTid+(1-Pid)[Tcd+PisTis$$
$$+(1 - Pis)[PnsTns + (1 - Pns)[Tcs + 2Tb]]]] \,. \qquad (59)$$

By S_2 we denote the coefficient of $(1-Pbq)Tq$ and it is equal to Sa_2:

$$S_2 = Sa_2 = (1 - Pad)(1 - Pid) \,. \qquad (60)$$

By S_3 we denote the coefficient of $(1 - Pbq)Pbr$:

$$S_3 = Sa_3 - (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)Tb$$
$$= (1-Pad)(1-Pid)(1-Pis)(1-Pns)(Tbr-2Tb) \,. \qquad (61)$$

Finally,we denote the coefficient of Pbq in (58) by S_4:

$$S_4 = Sa_4 - (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)Tb =$$
$$(1 - Pad)(1 - Pid)[Tbq - PisTis - (1 - Pis)[PnsTns$$
$$+(1 - Pns)[Tcs + 2Tb]]] \,. \qquad (62)$$

This proves Theorem 6. ■

Now, we can express Fa as a function of the intensity of the input flow of one idle terminal (Fo), Pbr, Pbq and Tq.

Theorem 7:

$$Fa[1 - Fo\,M[S_1 + S_2(1 - Pbq)Tq + S_3(1 - Pbq)Pbr$$
$$+S_4Pbq] - R_1 - R_2(1 - Pbq)Pbr - R_3] = Fo\,Nab \,. \qquad (63)$$

Proof: From (35) and (36), we have

$$Fa = Fo\,Nab + Fo\,M\,Yab + rep.Fa \,. \qquad (64)$$

After substitution of Yab and $rep.Fa$ with their equal expressions from (52) and (37), respectively, and regrouping we obtain (63). ■

IV. CONLUSIONS AND FUTURE WORK

The analytical expressions derived here are the first step towards the construction of a complete analytical model of the overall telecommunication system with queue. The analytical model can be used for solving different teletraffic tasks such as Quality of Service (QoS) prediction task, technical characteristics task (e.g., network dimensioning / re-dimensioning for guaranteeing the target QoS), human behavior task (investigate possible effects of users' behavior and prepare recommendations and administrative limitations, e.g., of the maximal duration of ringing and busy tone), etc. For the purpose of constructing an analytical model that is easier to work with, in the expression for Ta derived here, the static and dynamic parameters are separated. The presented results allow definitions of new overall telecommunication system performance indicators, including human users' characteristics, following the approach described in [10]. A problem will be the numerical solution of the derived system of equations, because it is non-linear due to comprising Erlang-type queuing blocking formula. The current investigations confirm our belief in a successful numerical solution of the system, using the methods discussed in [1] and [11].

ACKNOWLEDGMENT

The work of Velin Andonov was partially funded by the Bulgarian NSF under grant DM 12/2 – "New Models of Overall Telecommunication Networks with Quality of Service Guarantees".

The work of Emiliya Saranova was supported by the National Scientific Program "Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICT in SES)" financed by the Bulgarian Ministry of Education and Science.

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