International Journal on Advances in Internet Technology, vol 5 no 1 & 2, year 2012, http://www.iariajournals.org/internet_technology/

Quality Analysis of a Chaotic Proven Keyed Hash Function

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Abstract—**Hash functions are cryptographic tools, which are notably involved in integrity checking and password storage. They are of primary importance to improve the security of exchanges through the Internet. However, as security flaws have been recently identified in the current standard in this domain, new ways to hash digital data must be investigated. In this document an original keyed hash function is evaluated. It is based on asynchronous iterations leading to functions that have been proven to be chaotic. It thus possesses various topological properties as uniformity and sensibility to its initial condition. These properties make our hash function satisfies established security requirements in this field. This claim is qualitatively proven and experimentally verified in this research work, among other things by realizing a large number of simulations.**

***Keywords*-Keyed Hash Function; Internet Security; Mathematical Theory of Chaos; Topology.**

I. Introduction

The security and the privacy of data exchanged through the Internet are guaranteed by protocols that make an adequate use of a few cryptographic tools as secure pseudorandom number generators or hash functions. Hash functions are applications that map words of any lengths to words of fixed lengths (often 256 or 512 bits). These hash functions allow, for instance, to store passwords in a secure manner or to check whether a download has occurred without any error. They be designed to depend from a given parameter, called a key. According to their field of application, the requirements an hash function has to satisfy can change. They need at least to be very fast, so that the diffusion of the digest into the set of hash values occurs (whatever the bias into the inputted message), and so that a link between a message and its digest is impossible to establish in practice (confusion). The possibility to use a key or to distribute the computation on numerous threads must often be offered in several applications. Finally, in the computer security field, stringent complexity properties have to be proven, namely the collision, preimage, and second-preimage resistances, the unpredictability, and the pseudo-randomness properties. Each of the latter one have a rigorous formulation in terms of polynomial indistinguishability.

Several hash functions have been proposed as candidates to be standards in computer science. Such standards are designed by the scientific community and selected, after peer studies, by administrations as the NIST one (National Institute for Standards and Technologies of the US government). SHA-1 is probably the most widely used hash function. It is present in a large panel of security applications and protocols through the Internet.

However, in 2004, MD5 and SHA-0 have been broken. An attack over SHA-1 has been achieved with only 2^{69} operations (CRYPTO-2005), that is, $2,000$ times faster than a brute force attack (that requires 2^{80} operations). Even if 2^{69} operations still remain impossible to realize on common computers, such a result, based on a previous attack on SHA-0, is a very important one: as the SHA-2 variants are algorithmically close to SHA-1 and eventually produce message digests on principles similar to the MD4 and MD5 message digest algorithms, a new hash standard based on original approaches is then eagerly awaited. This is why a SHA-3 contest has been launched these last few years, to find a new, more secure standard for hash functions. So new original hash functions, or improvements for existing ones, must be found.

In this context, we have proposed a new hash function in [1], [2]. Being designed by using discrete dynamical systems, and taking benefits from various established topological properties, this new family of hash functions is thus based on a completely different approach. Among other things, in our proposal, an ingredient of chaos is added to existing hash functions, in order to reinforce their properties. Introducing chaos into the design of hash functions has been already addressed in [3], [4], [5], [6]. These methods usually transform the initial message into its padded fixed length version and then translate it into a real number. Next, with a chosen chaotic map (some chaotic functions of real variables like logistic, tent, or Arnold's cat maps, for instance [7]), methods set the initial algorithm parameters according to the secret key and start iterations. Methods are then left to extract some bits from the iterations results and to juxtapose them to get the hash value. It is then supposed that the final hash function preserves the properties of chaos. However, the idea of chaotic hash functions has been controversially discussed in the community [8], [9]. Moreover, even if these algorithms are themselves proven to be chaotic, their implementations on finite machines can result into the loss of chaos property. Among other things, the main reason is that chaotic functions (embedded in these researches) only manipulate real numbers, which do not exist in a computer. In [2], the hash function we have proposed does not simply integrate chaotic maps into algorithms hoping that the result remains chaotic; we have conceived an algorithm and have mathematically proven that it is chaotic. To do both, our theory and our implementation are based on finite integer domains and finite states iterations, where only one randomly chosen element is modified at each step. This iteration mode is further referred to as asynchronous mode.

These studies lead to the conclusion that the chaos of asynchronous iterations is very intense [10]. As this mode only manipulates binary digits or integers, we have shown that truly chaotic computer programs can be produced. They can thus be applied to pseudorandom number generators [11] and to a complete class of information hiding schemes [12], for instance. In this paper, the complete chaotic behavior of asynchronous iterations is capitalized to produce a truly chaotic keyed hash function.

This research work is an improvement of a previous article accepted at the Third International Conference on Evolving Internet, INTERNET11 (June 19-24, 2011, Luxembourg) [1]. Compared to this research work, the proposed hash function (Section III) has been completely rethought. It appears now more as a post-treatment on existing hash functions, to improve their security (Sections III, IV), than as a hash function designed from scratch. Moreover, the second-preimage resistance has been proven in Section IV-B and the strict avalanche criterion has been statistically studied (Section V-C). All these improvements lead to obviously better scores for the proposed hash functions, when experimentally evaluating its security.

The remainder of this research work is organized in the following way. In Section II, basic notions concerning asynchronous iterations and Devaney's chaos are recalled. Our keyed hash function is presented in Section III. Performance analyses are presented in the next two sections: in the first one a qualitative evaluation of this function is outlined, whereas in the second one it is evaluated experimentally. This research work ends by a conclusion section, in which our contribution is summarized and intended future work is mentioned.

II. Background Section

In this section, we first give definitions of Secure Keyed One-Way Hash Functions and of the Strict Avalanche Criterion (SAC), which is a property that such a function has to verify. Next we give some recalls on Boolean discrete dynamical systems and link them with topological chaos. Finally, we establish relations between the algorithm properties inherited from topological results and the requirements of Secure Keyed One-Way Hash Function.

A. Secure Keyed One-Way Hash Function

Definition 1 (Secure Keyed One-Way Hash Function [13]) *Let Γ and Σ be two alphabets, let $k \in K$ be a key in a given key space, let l be a natural number, which is the length of the output message, and let $h : K \times \Gamma^+ \to \Sigma^l$ be a function that associates a message in Σ^l for each pair of key, word in $K \times \Gamma^+$. The set of all functions h is partitioned into classes of functions $\{h_k : k \in K\}$ indexed by a key k and such that $h_k : \Gamma^+ \to \Sigma^l$ is defined by $h_k(m) = h(k, m)$, i.e., h_k generates a message digest of length l.*

A class $\{h_k : k \in K\}$ is a Secure Keyed One-Way Hash Function *if it satisfies the following properties:*

1) *the function h_k is* keyed one-way. *That is,*
    a) *Given k and m, it is easy to compute $h_k(m)$.*
    b) *Without the full knowledge of k, it is*
        - *difficult to find m when $h_k(m)$ is given; this property is referred to as* preimage resistance*;*
        - *difficult to find $h_k(m)$ when only m is given.*
2) *The function h_k is the* keyed collision resistant*, that is, without the knowledge of k it is difficult to find two distinct messages m and m' s.t. $h_k(m) = h_k(m')$. A weaker version of this property is the* second preimage resistance*, which is established if for a given m it is difficult to find another message m', $m \neq m'$, such that $h_k(m) = h_k(m')$.*
3) *Images of function h_k have to be uniformly distributed in Σ^l in order to counter statistical attacks.*
4) *Length l of the produced image has to be larger than 128 bits in order to counter birthday attacks [14].*
5) *Key space size has to be sufficiently large in order to counter exhaustive key search.*

Finally, hash functions have to verify the *strict avalanche criterion* defined as follows:

Definition 2 (Strict Avalanche Criterion [15]) *Let x and $\overline{x}^i$, two n-bit, binary vectors, such that x and $\overline{x}^i$ differ only in bit i, $1 \leqslant i \leqslant n$. Let f be the cryptographic transformation (hash function applied on vector of bits for instance). Let $\oplus$ be the exclusive or operator. The f function meets the* strict avalanche criterion *if and only if the following property is established;*

$$\forall n. \forall i, j. 1 \leqslant i \leqslant n \wedge 1 \leqslant j \leqslant m \Rightarrow \\ P\left(\left(f(x) \oplus f(\overline{x}^i)\right)_j = 1\right) = 1/2$$

This means that for any length message, each bit of the digest is independent of modifying one bit in the original message. In other words, a difference of one bit between two given medias has to lead to completely different digests.

B. Boolean Discrete Dynamical Systems

Let us first discuss the domain of iterated functions. As far as we know, no result rules that the chaotic behavior of a function that has been theoretically proven on $\mathbb{R}$ remains valid on the floating-point numbers, which is the implementation domain. Thus, to avoid the loss of chaos this research work presents an alternative, namely to iterate Boolean maps: results that are theoretically obtained in that domain are preserved during implementations. This section recalls facts concerning Boolean discrete-time dynamical Systems (BS) that are sufficient to understand the background of our approach.

Let us denote by $[\![a;b]\!]$ the interval of integers: $\{a, a+1, \ldots, b\}$, where $a \leqslant b$. Let n be a positive integer. A Boolean discrete-time system is a discrete dynamical system defined from a *Boolean map* $f : \mathbb{B}^n \to \mathbb{B}^n$ s.t.

$$x = (x_1, \ldots, x_n) \mapsto f(x) = (f_1(x), \ldots, f_n(x)),$$

and an iteration scheme: parallel, asynchronous... With the *parallel* iteration scheme, the dynamics of the system are described by $x^{t+1} = f(x^t)$ where $x^0 \in \mathbb{B}^n$. Let thus $F_f : [\![1;n]\!] \times \mathbb{B}^n$ to $\mathbb{B}^n$ be defined by

$$F_f(i, x) = (x_1, \ldots, x_{i-1}, f_i(x), x_{i+1}, \ldots, x_n),$$

with the *asynchronous* scheme, the dynamics of the system are described by $x^{t+1} = F_f(s^t, x^t)$ where $x^0 \in \mathbb{B}^n$ and s is a *strategy*, *i.e.*, a sequence in $[\![1;n]\!]^{\mathbb{N}}$. Notice that this scheme only modifies one element at each iteration.

Let G_f be the map from $\mathcal{X} = [\![1;n]\!]^{\mathbb{N}} \times \mathbb{B}^n$ to itself s.t.

$$G_f(s, x) = (\sigma(s), F_f(s^0, x)),$$

where $\sigma(s)^t = s^{t+1}$ for all t in $\mathbb{N}$. Notice that the parallel iteration of G_f from an initial point $X^0 = (s, x^0)$ describes the "same dynamics" as the asynchronous iteration of f induced by the initial point x^0 and the strategy s.

The state-vector $x^t = (x_1^t, \ldots, x_n^t) \in \mathbb{B}^n$ of the system at discrete time t (also said at *iteration* t) is further denoted as the *configuration* of the system at time t.

In what follows, the dynamics of the system is particularized with the negation function $\neg : \mathbb{B}^n \to \mathbb{B}^n$ such that $\neg(x) = (\overline{x_i}, \ldots, \overline{x_n})$ where $\overline{x_i}$ is the negation of x_i. We thus have the function $F_\neg$ that is defined by:

$$\begin{array}{rcl} F_\neg : [\![1;n]\!] \times \mathbb{B}^n & \to & \mathbb{B}^n \\ F_\neg(s,x)_j & = & \left\{ \begin{array}{l} \overline{x_j} \text{ if } j = s \\ x_j \text{ otherwise.} \end{array} \right. \end{array}$$

With such a notation, configurations are defined for times $t = 0, 1, 2, \ldots$ by:

$$\left\{ \begin{array}{l} x^0 \in \mathbb{B}^n \text{ and} \\ x^{t+1} = F_\neg(S^t, x^t) \end{array} \right. . \tag{1}$$

In the space $\mathcal{X} = [\![1;n]\!]^{\mathbb{N}} \times \mathbb{B}^n$ we define the distance between two points $X = (S, E), Y = (\check{S}, \check{E}) \in \mathcal{X}$ by

$$\begin{array}{rcl} d(X,Y) & = & d_e(E,\check{E}) + d_s(S,\check{S}), \text{ where} \\ d_e(E,\check{E}) & = & \displaystyle\sum_{k=1}^{n} \delta(E_k, \check{E}_k), \text{ and} \\ d_s(S,\check{S}) & = & \displaystyle\frac{9}{n} \sum_{k=1}^{\infty} \frac{|S^k - \check{S}^k|}{10^k}. \end{array}$$

If the floor value $\lfloor d(X,Y) \rfloor$ is equal to j, then the systems $E, \check{E}$ differ in j cells. In addition, $d(X,Y) - \lfloor d(X,Y) \rfloor$ is a measure of the differences between strategies S and $\check{S}$. More precisely, this floating part is less than 10^{-k} if and only if the first k terms of the two strategies are equal. Moreover, if the k^{th} digit is nonzero, then the k^{th} terms of the two strategies are different.

In his PhD thesis [10], Guyeux has already proven that:

- The function G_f is *continuous* on the metric space $(\mathcal{X}, d)$.
- The parallel iterations of $G_\neg$ are *regular*: periodic points of $G_\neg$ are dense in $\mathcal{X}$.
- The function $G_\neg$ is *topologically transitive*: for all $X, Y \in \mathcal{X}$, and for all open balls B_X and B_Y centered in X and Y respectively, there exist $X' \in B_X$ and $t \in \mathbb{N}$ such that $G_\neg^t(X') \in B_Y$.
- The function $G_\neg$ has *sensitive dependence on initial conditions*: there exists $\delta > 0$ such that for any $X \in \mathcal{X}$ and any open ball B_X, there exist $X' \in B_X$ and $t \in \mathbb{N}$ such that $d(G_\neg^t(X), G_\neg^t(X')) > \delta$.

To put it differently, a system is sensitive to initial conditions if any point contains, in any neighborhood, another point with a completely different future trajectory. Topological transitivity is established when, for any element, any neighborhood of its future evolution eventually overlaps with any other open set. On the contrary, a dense set of periodic points is an element of regularity that a chaotic dynamical system has to exhibit.

We have previously established that the three conditions for Devaney's chaos hold for asynchronous iterations. They thus behave chaotically, as it is defined in the mathematical theory of chaos [16], [17]. In other words, quoting Devaney in [16], a chaotic dynamical system "is unpredictable because of the sensitive dependence on initial conditions. It cannot be broken down or simplified into two subsystems, which do not interact because of topological transitivity. And in the midst of this random behavior, we nevertheless have an element of regularity".

Intuitively, the topologically transitivity and the sensitivity on initial conditions respectively address the preimage resistance and the avalanche criteria. Section IV formalizes this intuition.

The next section presents our hash function that is based on asynchronous iterations.

III. Chaos-Based Keyed Hash Function Algorithm

The hash value is obtained as the last configuration resulting from iterations of $G_\neg$. We then have to define the pair $X^0 = ((S^t)^{t \in \mathbb{N}}, x^0)$, *i.e.*, the strategy S and the initial configuration x^0.

A. Computing x^0

The first step of the algorithm is to transform the message in a normalized $n = 256$ bits sequence x^0. Notice that this size n of the digest can be changed, *mutatis mutandis*, if needed. Here, this first step is close to the pre-treatment of the SHA-1 hash function, but it can easily be replaced by any other compression method.

To illustrate this step, we take an example, our original text is: "*The original text*".

Each character of this string is replaced by its ASCII code (on 7 bits). Following the SHA-1 algorithm, first we append the character "1" to this string, which is then

```
10101001 10100011 00101010 00001101 11111100
10110100 11100111 11010011 10111011 00001110
11000100 00011101 00110010 11111000 11101001.
```

Next we append the block 1111000, which is the binary value of this string length (120) and let R be the result. Finally another "1" is appended to R if and only if the resulting length is an even number.

```
10101001 10100011 00101010 00001101 11111100
10110100 11100111 11010011 10111011 00001110
11000100 00011101 00110010 11111000 11101001
1111000.
```

The whole string is copied, but in the opposite direction:

```
10101001 10100011 00101010 00001101 11111100
10110100 11100111 11010011 10111011 00001110
11000100 00011101 00110010 11111000 11101001
11110000 00111110 01011100 01111101 00110010
11100000 10001101 11000011 01110111 00101111
10011100 10110100 11111110 11000001 01010011
00010110 010101.
```

The string whose length is a multiple of 512 is obtained, by duplicating the string obtained above a sufficient number of times and truncating it at the next multiple of 512. This string is further denoted by D. Finally, we split our obtained string into two blocks of 256 bits and apply to them the exclusive-or (further denoted as XOR) function, from the first two blocks to the last one. It results a 256 bits sequence, that is in our example:

```
00001111 00101111 10000010 00111010 00001110
01100111 01111000 10011101 01010111 00110101
11010100 01101001 11111001 00011011 01001110
00110000 11000111 00101101 10001001 11111001
01100010 10111010 11001110 10101011 10010001
11101110 01100111 00000101 11000100 00011111
01001111 00001100.
```

The configuration x^0 is the result of this pre-treatment and is a sequence of $n = 256$ bits. Notice that many distinct texts lead to the same string x^0. The algorithm detailed in [1] always appends "1" to the string R. However such an approach suffered from generating the same x^0 when R's length is 128. In that case the size of its reverse is again 128 bits leading a message of length 256. When we duplicate the message, we obtain a message of length 512 composed of two equal messages. The resulting XOR function is thus 0 and this improvement consequently allows us to avoid this drawback.

Let us build now the strategy $(S^t)^{t\in\mathbb{N}}$ that depends on the original message and on a given key.

B. Computing $(S^t)^{t\in\mathbb{N}}$

To obtain the strategy S, the chaotic proven pseudorandom number generator detailed in [18] is used. The seed of this PRNG is computed as follows: first the ASCII code (on 7 bits again) of the key is duplicated enough and truncated to the length of D. A XOR between D and this chain gives the seed of the PRNG, that is left to generate a finite sequence of natural numbers S^t in $[\![1, n]\!]$ whose length is $2n$.

C. Computing the digest

To design the digest, asynchronous iterations of $G_\neg$ are realized with initial state $X^0 = ((S^t)^{t\in\mathbb{N}}, x^0)$ as defined above. The result of these iterations is a $n = 256$ bits vector. Its components are taken 4 per 4 bits and translated into hexadecimal numbers, to obtain the hash value:

```
AF71542C90F450F6AE3F649A0784E6B1
6B788258E87654B4D6353A2172838032.
```

As a comparison if we replace "*The original text*" by "*the original text*", the hash function returns:

```
BAD8789AD6924B6460F8E7686A24A422
8486DC8FDCAE15F1F681B91311426056.
```

We then investigate the qualitative properties of this algorithm.

IV. Quality Analysis

We show in this section that, as a consequence of recalled theoretical results, this hash function tends to verify desired informal properties of a secure keyed one-way hash function.

A. The Strict Avalanche Criterion

In our opinion, this criterion is implied by the topological properties of sensitive dependence to the initial conditions, expansivity, and Lyapunov exponent. These notions are recalled below.

First, a function f has a constant of expansivity equal to ε if an arbitrarily small error on any initial condition is *always* magnified till ε. In our iteration context and more formally, the function $G_\neg$ verifies the *expansivity* property if there exists some constant $\varepsilon > 0$ such that for any X and Y in $\mathcal{X}$, $X \neq Y$, we can find a $k \in \mathbb{N}$ s.t. $d(G_\neg^k(X), G_\neg^k(Y)) \geqslant \varepsilon$. We have proven in [19] that, $(\mathcal{X}, G_\neg)$ is an expansive chaotic system. Its constant of expansivity is equal to 1.

Next, some dynamical systems are highly sensitive to small fluctuations into their initial conditions. The constants of sensibility and expansivity have been historically defined to illustrate this fact. However, in some cases, these variations can become enormous, can grow in an exponential manner in a few iterations, and neither sensitivity nor expansivity are able to measure such a situation. This is why Alexander Lyapunov has proposed a new notion able to evaluate the amplification speed of these fluctuations we now recall:

Definition 3 (Lyapunov Exponent) *Let be given an iterative system $x^0 \in \mathcal{X}$ and $x^{t+1} = f(x^t)$. Its* Lyapunov exponent *is defined by:*

$$\lim_{t\to+\infty} \frac{1}{t} \sum_{i=1}^{t} \ln \left| f'\left(x^{i-1}\right) \right|$$

By using a topological semi-conjugation between $\mathcal{X}$ and $\mathbb{R}$, we have proven in [10] that, for almost all X^0, the Lyapunov exponent of asynchronous iterations $G_\neg$ with X^0 as initial condition is equal to $\ln(n)$.

We can now justify why, in our opinion, the topological properties of the proposed hash function lead to the avalanche effect. Indeed, due to the sensitive dependence to the initial condition, two close media can possibly lead to significantly different digests. The expansivity property implies that these similar medias mostly lead to very different hash values. Finally, a Lyapunov exponent greater than 1 leads to the fact that these two close media will always end up by having very different digests.

B. Preimage Resistance

1) Topological Justifications: Let us now discuss about the preimage resistance of our keyed hash function denoted by h. As recalled previously, an adversary given a target image D should not be able to find a preimage M such that $h(M) = D$. One reason (among many) why this property is important is that, on most computer systems, users passwords are stored as the cryptographic hash of the password instead of just the plain-text password. Thus an attacker who gains access to the password file cannot use it to then gain access to the system, unless it is able to invert target message digest of the hash function.

We now explain why, topologically speaking, our hash function is resistant to preimage attacks. Let m be the message to hash, (S, x^0) its normalized version (*i.e.*, the initial state of our iteration scheme), and $M = h(m)$ the digest of m by using our method. So iterations with initial condition (S, M) and iterate function $G_\neg$ have x^0 as final state. Thus it is impossible to invert the hash process with a view to obtain the normalized message by using the digest. Such an attempt is equivalent to try to forecast the future evolution of asynchronous iterations of the $\neg$ function by only using a partial knowledge of its initial condition. Indeed, as M is known but not S, the attacker has an uncertainty on the initial condition. He/she only knows that this value is into an open ball of radius 1 centered at the point M, and the number of terms of such a ball is infinite.

With such an incertainty on the initial condition, and due to the numerous chaos properties possessed by our algorithm (as stated in the previous Section), this prediction is impossible. Furthermore, due to the transitivity property, it is possible to reach all of the normalized medias, when starting to iterate into this open ball. These qualitative explanations can be formulated more rigorously, by the proofs given in the next section.

2) Proofs of the Second-Preimage Resistance: We will focus now on a rigorous proof of the second-preimage resistance: an adversary given a message m should not be able to find another message m' such that $m \neq m'$ and $h(m) = h(m')$.

More precisely, we will show that a more general instance of the proposed post-treatment described below preserves this character for a given hash function.

Let

- k_1,k_2,n, all in $\mathbb{N}^*$, where k_1 is the size of the key, k_2 is the size of the seed, and n is the size of the hash value,
- $h : (k, m) \in \mathbb{B}^{k_1} \times \mathbb{B}^* \longmapsto h(k, m) \in \mathbb{B}^n$ a keyed hash function,
- $S : k \in \mathbb{B}^{k_2} \longmapsto S(k) \in [\![1, n]\!]$ a cryptographically secure pseudorandom number generator,
- $\mathcal{K} = \mathbb{B}^{k_1} \times \mathbb{B}^{k_2} \times \mathbb{N}$ called the *key space*,
- and $f : \mathbb{B}^n \longrightarrow \mathbb{B}^n$ a bijective map.

We define the keyed hash function $\mathcal{H} : \mathcal{K} \times \mathbb{B}^* \longrightarrow \mathbb{B}^n$ by the following procedure

Inputs: $K = (K_1, K_2, N) \in \mathcal{K}$
$m \in \mathbb{B}^*$
Runs: $X = h(K_1, m)$
for $i = 1, \ldots, N$:
  $X = G_f(S^i(K_2), X)$
return X

where K_1 is the key of the inputted hash function, K_2 the seed of the strategy used in the post-treatment iterations, where N is for the size of this strategy. We have the following result.

Theorem 1 *If h satisfies the second-preimage resistance property, then it is the case for $\mathcal{H}$ too.*

To achieve the proof, we introduce the two following lemmas.

Lemma 1 *If $f : \mathbb{B}^n \longrightarrow \mathbb{B}^n$ is bijective, then for any $S \in [\![1, n]\!]$, the map $G_{f,S} : x \in \mathbb{B}^n \rightarrow G_f([S, 1, \ldots, 1], x)_2 \in \mathbb{B}^n$ is bijective too where $G_f(\_, \_)_2$ is the second term of the pair $G_f(\_, \_)$.*

Proof: Since $\mathbb{B}^n$ is a finite set, it is sufficient to prove that $G_{f,S}$ is surjective. Let $y = (y_1, \ldots, y_n) \in \mathbb{B}^n$ and $S \in [\![1, n]\!]$. Thus $G_{f,S}\left((y_1, \ldots, y_{S-1}, f^{-1}(y_S), y_{S+1}, \ldots, y_n)\right)_2 = G_f\left([S, 1, \ldots, 1], (y_1, \ldots, y_{S-1}, f^{-1}(y_S), y_{S+1}, \ldots, y_n)\right)_2 = \left([1, \ldots, 1], (y_1, \ldots, y_{S-1}, y_S, y_{S+1}, \ldots, y_n)\right)_2 = (y_1, \ldots, y_{S-1}, y_S, y_{S+1}, \ldots, y_n) = y$. So $G_{f,S}$ is a surjective map between two finite sets and thus bijective. ■

Lemma 2 *Let $S \in [\![1, n]\!]^{\mathbb{N}}$ and $N \in \mathbb{N}^*$. If f is bijective, then $G_{f,S,N} : x \in \mathbb{B}^n \longmapsto G_f^N(S, x)_2 \in \mathbb{B}^n$ is bijective too.*

Proof: Indeed, $G_{s,f,n} = G_{f,S^n} \circ \ldots \circ G_{f,S^0}$ is bijective as a composition of bijective maps (as stated in Lemma 1). ■

We are now able to prove theorem 1.

Proof: Let $m, k \in \mathbb{B}^* \times \mathcal{K}$. If a message $m' \in \mathbb{B}^*$ can be found such that $\mathcal{H}(k, m) = \mathcal{H}(k, m')$, then, according to Lemma 2, $h(k_1, m) = h(k_1, m')$: a second-preimage for h has thus been found. ■

C. Algorithm Complexity

In this section the complexity of the above hash function is evaluated for a size l of the media (in bits).

Theorem 2 *Let l be the size of the message to hash and n be the size of its hash value. The algorithm detailed along these lines requires $\mathcal{O}(l) + \mathcal{O}(n^2)$ elementary operations to produce the hash value.*

Proof: In the x^0 computation stage only linear operations over binary tables are achieved. More precisely it first executes one ASCII translation yielding a message of length $7l$, a length computation that increases the message length of $\log_2(7l)$. One bit is possibly added. Thus a reversed copy that leads to a message of length l' that is $14l + 2 + 2\log_2(7l)$. The number of duplication steps to get a message whose length is greater than a multiple of $2n$ is formally given by

$$\min_{k \geqslant 1}\{k \mid \exists p \, . \, p \geqslant 1 \wedge k \times l' \geqslant 2np\} \tag{2}$$

This number is bounded by

$$k' = \max\{1, n\}.$$

If $14l + 2 + 2\log_2(7l)$ is greater than $2n$ it is sufficient to duplicate the message once. Otherwise, $\lfloor 1 + \frac{2n}{14l + 2 + 2\log_2(7l)} \rfloor$ is greater than $\frac{2n}{14l + 2 + 2\log_2(7l)}$ and thus $l' \times \frac{2n}{14l + 2 + 2\log_2(7l)}$ is greater than $2n$ and there exists a p ($p = 1$) such that $k \times l' \geqslant 2np$. Thus the minimum of the set given in Eq.(2) is less than $\lfloor 1 + \frac{2n}{14l + 2 + 2\log_2(7l)} \rfloor$, which is less than n.

To sum up, the initialization of x^0 requires at most $k' + l'$ elementary operations.

Let us now detail the S computation step. The number of elementary operations to provide the seed is bounded by $k' + l'$. Next, the embedded PRNG [18], that combines the XORShift, xor128, and XORWow PRNGs requires 35 elementary operations (17 XOR, 13 rotations, and 5 arithmetic operations) for generating a 32 bits number and thus $35\frac{2n}{32}$ to get a number on $2n$ bits. Furthermore, since the strategy length is $2n$, the computation of S requires at most $k' + l' + 2n \times 35\frac{2n}{32}$, which is less than $k' + l' + 5n^2$.

At least, since each iteration modifies only one bit, iterations require $2n$ elementary operations.

Finally, we have at most $2k' + 2l' + 5n^2 + 2n$ elementary operations to provide a hash value of size n. This bound is in $\mathcal{O}(l + n^2)$. ■

V. Experimental Evaluations

Let us now give some examples of hash values before statistically studying the quality of hash outputs.

A. Examples of Hash Values

Let us consider the proposed hash function with $n = 256$. We consider the key to be equal to "*my key*". To illustrate the confusion and diffusion properties [20], we use this function to generate hash values in the following cases:

Case 1. The original text message is the poem *Ulalume* (E.A.Poe), which is made of 104 lines, 667 words, and 3, 754 characters.
Case 2. We change *serious* by *nervous* in the verse "*Our talk had been serious and sober*"
Case 3. We replace the last point '.' with a coma ','.
Case 4. In "*The skies they were ashen and sober*", skies becomes Skies.
Case 5. The new original text is the binary value of Figure 1.

Figure 1: The original plain-image.

Case 6. We add 1 to the gray value of the pixel located in position (123,27).

Case 7. We subtract 1 to the gray value of the pixel located in position (23,127).

The corresponding hash values in hexadecimal format are:

Case 1. 0B4730459FBB5E54A18A9CCD676C8396 365B0104407D98C866FDAA51A07F0E45,

Case 2. 752E28088150B98166D870BC24177342 23A59463D44B83E9808383B30F8B8409,

Case 3. C10EED0A9D44856847F533E5647D0CCD 2C58A08643E4D3E5D8FEA0DA0E856760,

Case 4. 52BF23429EC3AD16A0C9DE03DF51C420 4466285448D6D73DDFB42E7A839BEE80,

Case 5. 5C639A55E2B26861EB9D8EADDF92F935 5B6214ADC01197510586745D47C888B8,

Case 6. E48989D48209143BAE306AC0563FFE31 EAB02E5E557B49E3442A840996BECFC1,

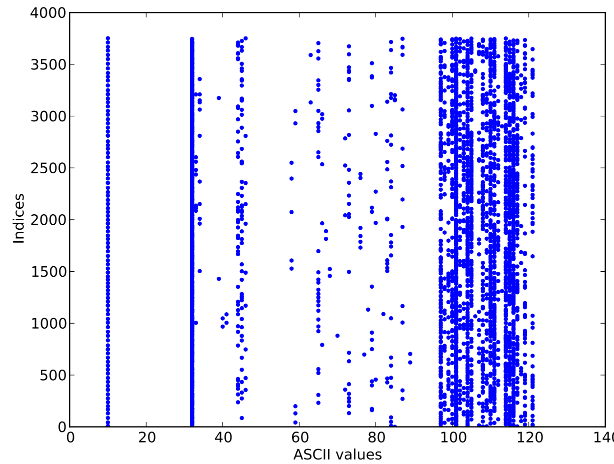
Case 7. EC850438A2D8EA95E691C746D487A755 12BEE63F4DDB4466C11CD859671DFBEB,

These simulation results are coherent with the topological properties of sensitive dependence to the initial condition, expansivity, and Lyapunov exponent: any alteration in the message causes a substantial difference in the final hash value.

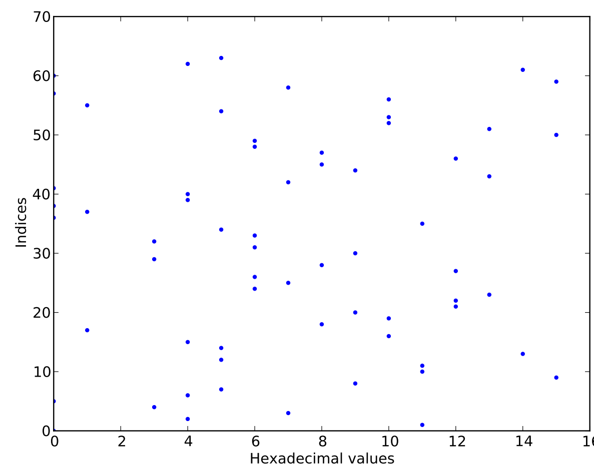
B. Statistical Evaluation of the Algorithm

We focus now on statistical studies of diffusion and confusion properties. Let us recall that confusion refers to the desire to make the relationship between the key and the digest as complex and involved as possible, whereas diffusion means that the redundancy in the statistics of the plain-text must be "dissipated" in the statistics of the cipher-text. Indeed, the avalanche criterion is a modern form of the diffusion, as this term means that the output bits should depend on the input bits in a very complex way.

1) Uniform repartition for hash values: To show the diffusion and confusion properties verified by our scheme, we first give an illustration of the difference of characters repartition between a plain-text and its hash value, when the original message is again the Ulalume poem. In Figure 2a, (resp. in Figure 2b) the X-axis represents ASCII numbers (resp. hexadecimal numbers) whereas the Y-axis gives for each X-value its position in the original text (resp. in the digest). For instance, in Figure 2b, the point $(1, 17)$ means that the character 1 is present in the digest at position 17 (see Case 1, Section. V-A). We can see that ASCII codes are localized within a small area (e.g., the ASCII “space” code and the lowercase characters), whereas in Figure 2b the hexadecimal numbers of the hash value are uniformly distributed.



(a) Original text



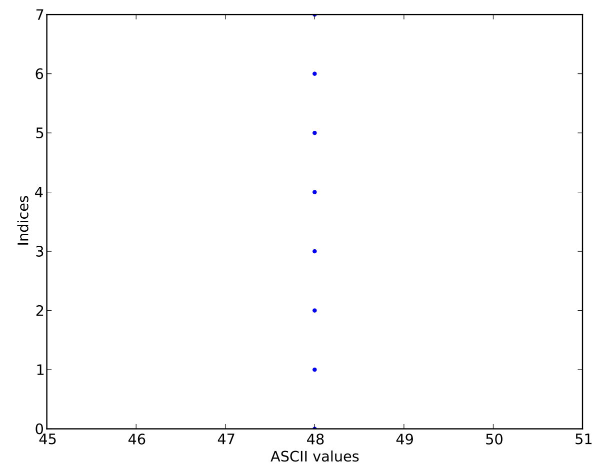
(b) Digest

Figure 2: Values repartition of Ulalume poem

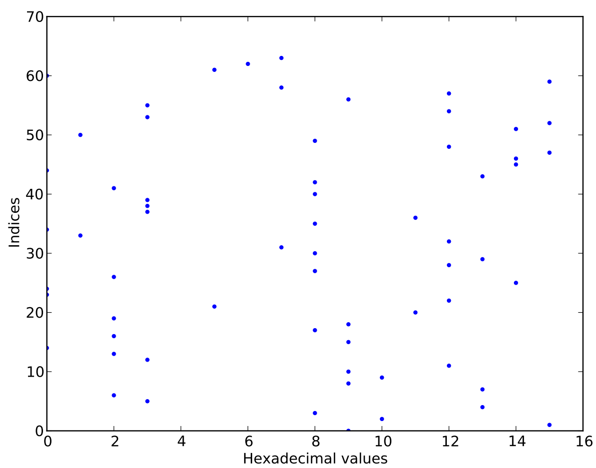
A similar experiment has been realized with a message having the same size, but which is only constituted by the character “*0*”. The contrasts between the plain-text message and its digest are respectively presented in Figures 3a and 3b. Even under this very extreme condition, the distribution of the digest still remains uniform. To conclude, these simulations tend to indicate that no information concerning the original message can be found into its hash value, as it is recommended by the Shannon’s diffusion and confusion requirements.

2) Behavior through small random changes: We now consider the following experiment. A first message of 1000 bits is randomly generated, and its hash value of size $n = 256$ bits is computed. Then one bit is randomly toggled into this message and the digest of the new message is obtained. These two hash values are compared by using the hamming distance, to compute the number B_i of changed bits. This test is reproduced $t = 10,000$ times. The corresponding distribution of B_i is presented in Figure 4.

As desired, Figure 4 shows that the distribution is centered around 128, which reinforces the confidence put into the good



(a) Original text



(b) Digest

Figure 3: Values repartition of the "*00000000*" message

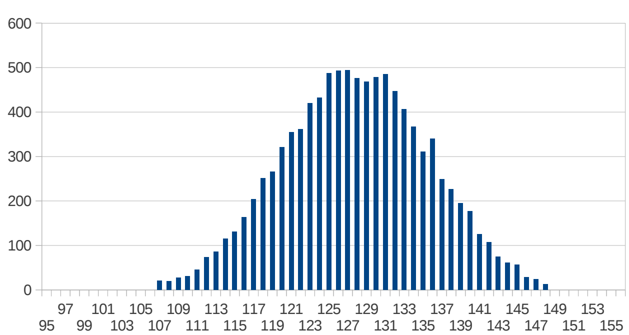


Figure 4: Histogram

capabilities of diffusion and confusion of the proposed hash algorithm. To analyse these results, the following common statistics are used.

- Mean changed bit number

$$\overline{B} = \frac{1}{t}\sum_{i=1}^{t} B_i.$$

	B_{min}	B_{max}	B	$P(\%)$	ΔB	$\Delta P(\%)$
$n = 256$	87	167	127.95	49.98	8.00	3.13
$n = 512$	213	306	255.82	49.97	11.29	2.21
$n = 1024$	446	571	511.54	49.96	15.97	1.56

Table I: Statistical performances of the proposed hash function

- Mean changed probability

$$P = \frac{\overline{B}}{n}.$$

- $\Delta B = \sqrt{\frac{1}{t}\sum_{i=1}^{t}(B_i - \overline{B})^2}.$
- $\Delta P = \sqrt{\frac{1}{t}\sum_{i=1}^{t}(\frac{B_i}{n} - P)^2}.$

The obtained statistics are listed in Table I where n belongs to $\{256; 512; 1,024\}$. In that study, starting from a message of length $1,000$ and its digest, all the messages that have one bit of difference are further generated and the digest of the new message is obtained. Obviously, both the mean changed bit number $\overline{B}$ and the mean changed probability P are close to the ideal values ($\frac{n}{2}$ bits and 50%, respectively), which illustrates the diffusion and confusion capability of our algorithm. Lastly, as ΔB and ΔP are very small, these capabilities are very stable.

C. Strict Avalanche Criterion Evaluation

This section focuses on checking whether the developed hash function verifies the strict avalanche criterion, as given in Definition 2. Quoting remarks of [15], "*Unless n is small, it would be an immense task to follow this procedure for all possible vector pairs x and $\overline{x}^i$*". The authors propose thus the alternative method of computing a dependence matrix J of size $m \times n$ between the j-th, $1 \leqslant j \leqslant m$, element of the digest and i-th, $1 \leqslant i \leqslant n$, element of the original message. A simulation consists in first randomly choosing the size n of the message to hash (100 values in $[\![1, 1000]\!]$ for us). Next, a set of large size r ($r = 1,000$ in our case) of messages x is randomly computed. For each of them, the set $\{\overline{x}^1, \ldots, \overline{x}^n\}$ is formed such that x and $\overline{x}^i$ only differ in bit i. The set of m-bit vectors

$$\{f(x) \oplus f(\overline{x}^1), \ldots, f(x) \oplus f(\overline{x}^n)\}$$

is thus computed where f is the hash function applied on vector of bits. The value of bit i (either a 1 or a 0) in $\big(f(x) \oplus f(\overline{x}^i)\big)_j$ is added to J_{ij}. Finally each element of J is divided by r. If every J_{ij} are close to one half, the strict avalanche criterion is established. For all these experiments, the average value of J_{ij} is 0.5002, the minimal value is 0.418, the maximal value is 0.585, and the standard deviation is 0.016.

VI. Conclusion

In this research work, the hash function proposed in the Third International Conference on Evolving Internet, INTERNET11 (June 19-24, 2011, Luxembourg) [1] has been completely rethought. The second-preimage resistance has been proven, leading to better experimental results for the proposed hash function. Moreover, we have shown that this function has a complexity that can be expressed as a polynomial function of the message length and of the digest size. Finally, we have statistically established that our function verifies the SAC.

If we now consider our approach as an asynchronous iterations post-treatment of an existing hash function. The security of this hash function is reinforced by the unpredictability of the behavior of the proposed post-treatment. Thus, the resulting hash function, a combination between an existing hash function and asynchronous iterations, satisfies important properties of topological chaos such as sensitivity to initial conditions, uniform repartition (as a result of the transitivity), unpredictability, and expansivity. Moreover, its Lyapunov exponent can be as great as needed. The results expected in our study have been experimentally checked. The choices made in this first study are simple: initial conditions designed by using the same ingredients as in the SHA-1, negation function for the iteration function, *etc.* But these simple choices have led to desired results, justifying that such a post-treatment can possibly improve the security of the inputted hash function. And, thus, such an approach should be investigated more largely.

This is why, in future work, we will test other choices of iteration functions and strategies. We will try to characterize topologically the diffusion and confusion capabilities. Other properties induced by topological chaos will be explored and their interest for the realization of hash functions will be deepened. Furthermore, other security properties of resistance and pseudo-randomness will be proven. We will thus compare the results of this post-treatment on several hash functions, among other things with the SHA-3 finalists [21].

ACKNOWLEDGMENT

The authors are grateful to the anonymous reviewers for their suggestions to improve the quality of the paper.

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