Monitoring of Non-stationary Health-Recovery Processes with Control Charts

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***Abstract*—The paper presents a statistical process control method for monitoring health-recovery processes described by short non-stationary time series. The Shewhart control chart for residuals, based on model averaging approach, is built for differences between values of consecutive observations. The practical applicability of this new approach has been demonstrated using a real-life example of a recovery from a mild hypertension episode.**

Keywords–E-health; Control chart for residuals; Short time series; Non-stationary process; Stability of recovery process.

I. INTRODUCTION

This paper is a significant extension of the conference paper "Monitoring of Health-Recovery Processes with Control Charts" presented in the Proceedings of ACCSE'2017 conference, held in Venice, Italy [1], and is focused on monitoring of the stability of non-stationary processes. Stability is an important feature of many processes. A process is considered stable, or under control, when its uncontrolled variation is purely random (e.g., due to random measurement errors). In 1924 W. Shewhart introduced a simple tool for monitoring stable processes - a control chart. In its initial stage, which is assumed to be in-control, monitored process characteristics are measured, and their mean value and standard deviation are recorded. These recorded values are used for the design of a control chart, known as the Shewhart control chart, which consists of control lines: central, and two (or one, when only deviation of a process level in one direction is interesting) control. The central line represents the mean value of the process level (or a certain target value for this process), and control lines are located at three standard deviations from a central line. The process is considered stable when its future observations are located inside control lines (limits). When an observation falls outside the control lines, an alarm signal is generated, and the process is considered as being possibly out of control (unstable). When a monitored process goes out of control, it is recommended to look for the reason of this, and take appropriate actions with the aim to revert it to the in-control state.

Within this paper, we discuss the construction of a control chart for autocorrelated and non-stationary processes. In the experimental part, we extend the results presented in [1], and related to a special kind of medical data, namely data describing health-recovery processes.

A. Monitoring of health-recovery processes

For many years, physicians have been prescribing certain treatments, and advances in the health recovery of a treated patient have been monitored during visits, e.g., in health care units. Therefore, possible failures of applied treatments were usually disclosed with delay. In many cases, such delays have had detrimental effects on patient's health. However, with the development of e-health systems based on telemedicine this situation has been dramatically changed. Nowadays, it is possible to monitor the state of patient's health even continuously. However, the main problem now is not related to measurements and transmission of data, but to processing of available information. When human's life is endangered, very expensive systems, e.g., in intensive care hospital units, are used. However, in many cases, the usage of all those sophisticated Information Technology (IT) systems is not necessary. It is quite sufficient to process data off-line, and to signal only these cases when consultancy or intervention of a physician is really needed. What is important in this context, it is the stability of health-recovery processes, understood as non-existence of abnormal and unpredictable changes of the monitored process. It has to be noted here, that an unstable process may be still inside some "normal limits", pre-established by physicians, but its revealed instability suggests the possibility of going beyond such limits. Monitoring of such stability can be achieved by the usage of appropriately designed control charts. The proposal of such monitoring processes, based on a control chart for so called residuals, is the main purpose of this paper.

It has to be noted here that the design of monitoring procedures, such as these related to health care, has to fulfill two conflicting requirements. A procedure has to be designed using an appropriate mathematical model of a monitored process, but on the other hand, it has to be simple to use, and the results of this procedure have to be easily interpreted, even by users without any special statistical training. This conflict is especially visible in monitoring health-related processes. For example, processes continuously monitored in intensive care units, or processes monitored by wearable sensors, are very complex, because their mathematical models have to take into account many different circumstances, such like natural 24-hours variability or drug administration. Unfortunately, monitoring procedures based on such complex mathematical models, if not fully automated, are very difficult to interpret. In this paper we consider a particular case when a mathematical model is still relatively simple, but the results of its application are much easier for interpretation. By taking this assumption we restrict applicability of the proposed procedure to cases, in which measurements are made in comparable circumstances, e.g., once a day at the same time.

B. Real-life example of a health-recovery process

The experimental part of this paper is a case study. Daily recordings of blood pressure (BP) for a period of 480 days of a patient who is under treatment against mild blood hypertension

are analyzed and discussed. In Figure 1, we present the considered process of one-a-day BP measurements.

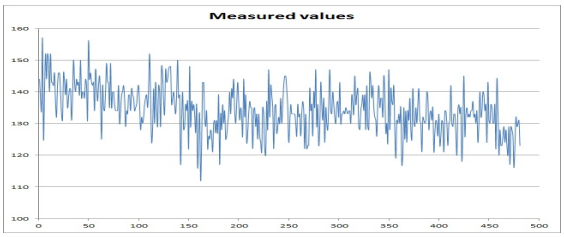

Figure 1. Measurements of blood pressure.

In the case of health-related time series data underlying processes are often non-stationary and correlated, and therefore, proper establishment of control limits is challenging. This paper investigates this problem.

C. Related work

Basic control charts, used in over 90% practical applications, are designed under two main assumptions: statistical independence of consecutive observations, and the normal distribution of measured characteristics. However, in many practical cases, especially when individual process observations are monitored, these assumptions are not fulfilled. Thus, in the recent 40 years, many inspection procedures that do not rely on these assumptions have been proposed. They are usually described in scientific journal papers or in a few textbooks on statistical quality control, such as a famous book of Montgomery [2]. They have been applied in many areas, but only some of them have been applied in health-related services, and similar applications, where their applicability seems to be quite natural. One of these natural applications, monitoring of health-recovery processes, has been described in [1]. A comprehensive review of other applications of control charts in health-care and public-health surveillance, written from a perspective of a quality control specialist, can be found in the paper by Woodall [3]. Similar review papers, but written from a perspective of a health care specialist, were written by Tennant et al. [4], Thor et al. [5], and Winkel and Zhang [6]. Since the time of the publication of these papers, many other papers on this topic have been published, mainly in journals related to medicine.

Despite real popularity of control charts in many areas, such as industry, finance and business, the number of their applications in health care is relatively small. Probably the main reason for this situation was given by the authors of [4] who wrote in their conclusions "Control charts appear to have a promising but largely under-researched role in monitoring clinical variables in individual patients". However, the situation has been changed during the last few years. In their editorial titled "Statistical process control in healthcare improvement - new kid on the block?", and published in a recent issue of Acta Anaesthesiologica Scandinavica, Møller and Anhøj [7] have written that the yearly number of PubMed citations including the term "statistical process control" in any field has increased from a level of 10 in 2010 to the level of 90 in 2018. If we take a perspective of a statistician, a probable reason of this situation is incompatibility of basic assumptions used for construction of statistical process control (SPC) tools, such as control charts, and the reality of health care. For example, consecutive observations of health-related characteristics are seldom independent. Moreover, they are often described by non-stationary random processes, and the runs of interesting observations are short. Therefore, control charts described in popular textbooks, and in the great majority of scientific papers, are not appropriate for monitoring such processes, and the results of using such charts may be unsatisfactory. Some new, more appropriate, approaches have been investigated quite recently. For example, the properties of control charts used for short runs for autocorrelated, but stationary, data have been discussed in [8].

First research papers on applications of statistical process control in cardiology have been published already in the 1990's. For example, G. Cornélissen et al. [9] use control charts to monitor blood pressure and heart rate for individualized assessment of a patient's response to a drug. Another application of control charts in monitoring of blood pressure has been proposed recently in the paper by Albloushi et al. [10]. Furthermore, benefits of applications of control charts in cardiology are constantly being confirmed by several papers on this topic published each year. For example, see the recent paper of Jung and Kim [11], who introduced an electrocardiographic (ECG) monitoring procedure for diagnosing PVC beats using a wavelet-based statistical process control methodology, or the paper of Lambeth et al. [12], who adapted the statistical process control methods to monitor the stability of admission temperature and glucose-level processes for the very low birthweight infants within first hour of birth at a neonatal intensive care unit (NICU).

The paper is organized as follows. In Section II, we describe a mathematical model of a stochastic process (a time series) that may be useful for the description of health-recovery data. Then, in Section III, we propose a control chart based procedure that may be used for monitoring non-stationary health-recovery processes. The problem of the monitoring of short time series using the sXWAM chart, proposed by us in [13], is considered in Section IV. In Section V, results for the real-life health-recovery process are presented and discussed. The paper is concluded in its last section, where we discuss limitations of the proposed methodology, and indicate some possible future applications.

II. Mathematical model of a monitored process

Series of dependent observations may be described by many mathematical models. When the expected value (the mean), the variance, and the covariances of the underlying process are constant in time, we call such processes stationary. When these statistical characteristics vary in time (e.g., according to a certain trend function) we call such processes non-stationary. Statistical methods of the analysis of time series, both stationary and non-stationary, can be found in many textbooks, such as, e.g., the book by Brockwell and Davis [14].

Simple visual analysis of the considered data (Fig. 1) reveals the existence of a visible trend. Thus, this time series cannot be described by a simple mathematical model. There are formal ways, namely statistical tests, to verify stationarity of a time series. One of the most powerful tests, the Kwiatkowski-Philips-Schmidt-Shin (KPSS) test, was introduced by Kwiatkowski et al. [15], and enables testing the

null hypothesis of stationarity, either around a level or around a linear trend, against the alternative of a unit root. For the considered exemplary time series, the value of the KPSS statistic amounts to 2.9737 and its related p-value is smaller than 0.01. Basing on this result, we reject the null hypothesis of stationarity, concluding that the considered time series is non-stationary.

However, there exists a large class of non-stationary processes that can be transformed to stationary processes, which are much easier for analysis. An important member of this class of non-stationary time series can be described by the Autoregressive Integrated Moving Average (ARIMA) model, introduced in the seminal book by Box and Jenkins [16]. For an ARIMA non-stationary process of first order, differences between consecutive observations are described by a stationary Autoregressive Moving Average (ARMA) process. The time series $\{X_t\}$ is ARMA(p,q) process if $\{X_t\}$ is stationary and if for every t [14],

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = Z_t + \theta_1 Z_{t-1} + \cdots + \theta_q Z_{t-q}, \tag{1}$$

where $\{Z_t\}$ is a purely random (white noise) process of uncorrelated observations having null expected values and the same finite variances, and the polynomials $(1-\phi_1 z-\cdots-\phi_p z^p)$ and $(1+\theta_1 z+\cdots+\theta_q z^q)$ have no common factors. The special case of the ARMA(p,q) model is the autoregression model AR(p) defined by the following equation

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = Z_t. \tag{2}$$

When the process $\{Z_t\}$ has a certain non-zero, and constant in time, expected value μ, the model described by (2) has to be slightly modified, and in this case has the following form

$$X_t - \mu - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = Z_t. \tag{3}$$

Now, let us look at Figure 2, where differences between the values of consecutive observations of the process presented in Figure 1 are displayed.

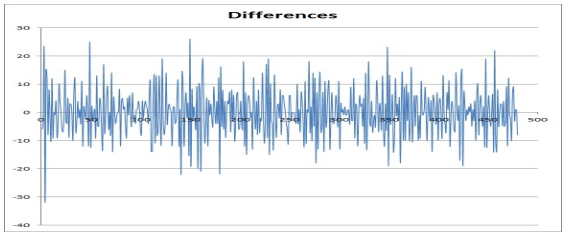

Figure 2. Differences between consecutive measurements.

Basing on visual inspection, the process displayed in Figure 2 looks stationary. The statistical tests also confirm its stationarity. The KPSS statistic is 0.0107, and the p-value amounts to 0.1, thus there are no reasons to reject the null hypothesis of stationarity. Hence, we can conclude that this time series is stationary, and may be well described by an autoregression process of the fourth order $AR(-0.862, -0.713, -0.358, -0.207)$. The constant term μ in this case is close to zero, and equal to -0.085. Therefore, this real-life example has motivated us to consider in this paper time series described by models of a similar type.

Let $X_1, X_2, \ldots, X_n$ be a series of measurements obtained during a period of time when a monitored process may be considered (e.g., according to a physician who supervises the treatment) as stable. The process of first differences is now defined as follows: $D_i = X_{i+1} - X_i, i = 1, \ldots, n-1$. We assume the ith difference is related to the previous observations according to the equation

$$D_i = \mu + a_1 * d_{i-1} + a_2 * d_{i-2} + \cdots + a_p * d_{i-p} + \epsilon_i, i = p+1, \ldots, \tag{4}$$

where $\epsilon_i, i = p+1, \ldots$ are normally distributed independent random variables with the expected value equal to zero, and the same finite standard deviation.

Estimation of the model $AR(p)$, given by (4), is relatively simple when we know the order of the model p. In order to find estimators $\hat{a}_1, \ldots, \hat{a}_p$, we have to calculate first p sample autocorrelations $r_1, r_2, \ldots, r_p$, defined as

$$r_i = \frac{n \sum_{t=1}^{n-i} (d_t - \hat{\mu})(d_{t+i} - \hat{\mu})}{(n-i) \sum_{t=1}^{n} (d_t - \hat{\mu})^2}, i = 1, \ldots, p, \tag{5}$$

where n is the number of observations in the sample (usually, it is assumed that $n \geq 4p$), and $\hat{\mu}$ is the sample average. Then, the parameters $a_1, \ldots, a_p$ of the $AR(p)$ model are calculated by solving the Yule-Walker equations (see, [14])

$$\begin{aligned} r_1 &= a_1 + a_2 r_1 + \ldots + a_p r_{p-1} \\ r_2 &= a_1 r_1 + a_2 + \ldots + a_p r_{p-2} \\ &\ldots \\ r_p &= a_1 r_{p-1} + a_2 r_{p-2} + \ldots + a_p \end{aligned} \tag{6}$$

When a more general model of the AR process (3) has to be used, the constant term μ may be estimated using the following equation

$$\hat{\mu} = \bar{x}(1 - a_1 - \cdots - a_p), \tag{7}$$

where $\bar{x}$ denotes the average of the observed process values.

The estimators obtained by solving the Yule-Walker equations are, unfortunately, not numerically stable, especially for small sample sizes. A better method was proposed by Burg. A good description of Burg's algorithm can be found in [17]. Burg's algorithm is used to solve the following optimization problem: for the set of observations $x_1, \ldots, x_N$ find the values $a_1^*, \ldots, a_k^*$ that minimize F_k defined as

$$F_k = \sum_{n=k}^{N} (x_n - (-\sum_{i=1}^{k} a_i x_{n-i}))^2 \tag{8}$$

The estimators of the $AR(p)$ model given by (2) are obtained by setting $k = p, N = n$, $x_i = d_i, i = 1, \ldots, n-1$, and $\hat{a}_i = -a_i^*, i = 1, \ldots, p$. Note, that in this formulation of the optimization problem we do not use a constant term μ. Therefore, the models estimated using the Yule-Walker equations (moment estimators), and the models estimated using Burg's algorithm, may be different.

In both presented methods for the estimation of the AR model we have assumed that the model order p is known in advance. In practice, however, we do not know the order of the autoregression process, so we need to estimate p from data. In order to do this, we define a transformed random variable,

called the *residual*. In the case of autoregression processes, considered in this paper, the residual is defined as

$$Z_i = D_i - (\hat{\mu} + a_1 d_{i-1} + \ldots + a_p d_{i-p}), i = p+1, \ldots, n. \quad (9)$$

When we know exactly the autoregression model, the probability distribution of residuals is the same as the distribution of random variables $\epsilon_i, i = 1, \ldots$ in (2), and its variance can be used as a measure of the accuracy of predictions of future values of the process. For given sample data of size n, the variance of residuals is decreasing with the increasing values of p. However, the estimates of p model's parameters $a_1, \ldots, a_p$ become less precise, and thus the overall precision of prediction with future data deteriorates. As the remedy to this effect, several optimization criteria with a penalty factor, which discourages the fitting of models with too many parameters, have been proposed. In this research we have used the criterion proposed by Akaike [18], and defined as

$$\begin{aligned} BIC = \quad & (n-p)\ln[n\hat{\sigma}^2/(n-p)] + n(1+\ln\sqrt{2\pi}) + \\ & p\ln[(\textstyle\sum_{t=1}^{n} d_t^2 - n\hat{\sigma}^2)/p], \end{aligned} \quad (10)$$

where d_t are our transformed process observations centered in such a way that their expected values are equal to zero, and $\hat{\sigma}^2$ is the observed variance of residuals. The fitted model, i.e., the estimated order p and parameters of the model $\hat{a}_1, \ldots, \hat{a}_p$ (and $\hat{\mu}$, if a constant term is used) minimizes the value of BIC calculated according to (10). We will use this model in the construction of a control chart for monitoring health-recovery processes.

III. Control Chart for Process Monitoring with Autocorrelated Data

A. Design of a chart

The design of a simple Shewhart control chart, in the case of a sufficiently large number of individual and mutually independent observations, is extremely simple. One has to collect data (a sample) from a period when the monitored process is stable, calculate average value $\bar{x}$ and standard deviation σ_x, and set the control limits, upper (CUP) and lower (CLO), to

$$\begin{aligned} CUP &= \bar{x} + 3\sigma_x \\ CLO &= \bar{x} - 3\sigma_x. \end{aligned} \quad (11)$$

When process deterioration is related only to increase (decrease) of a process level, one can use one-sided control charts with respective upper (lower) control limits. Usually, it is assumed that the monitored characteristic is normally distributed, and in this case the probability of observing the observation outside one control limit when the monitored process is stable (i.e., observing a false alarm) is very low, and equals 0.00135. It means, that the expected number of observations between consecutive false alarms is equal approximately to 740 (for a one-sided chart), or to 370 (for a two-sided chart).

When consecutive observations of a monitored process are statistically dependent, the situation becomes much more complicated. For example, when sample data are autocorrelated, the properties of a control chart designed using a simple algorithm described above may be completely different from those observed for independent data. Consider, for example, a Shewhart control chart constructed using the results of first 20 measurements, and presented in Figure 3.

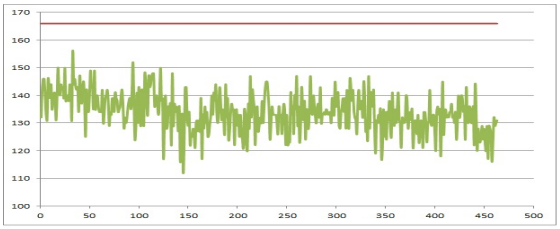

Figure 3. Control chart for original measurements.

In this case we use the chart with only an upper control limit because alarms should be triggered only by abrupt increases of blood tension. If we look at Figure 3 we may immediately notice that the charted values of measurements are rather far from the upper control limit. Moreover, the difference between the observed process level and the control limit increases in time. Therefore, the ability of the chart to trigger an alarm deteriorates in time, and for this reason the chart constructed using a classical way becomes useless.

The problem presented in Figure 3 is frequently observed when charted data are autocorrelated, as it is the case of our measurements. To cope with this problem, statisticians have proposed two general approaches. In the first one, we chart the original data, but control limits are adjusted using the knowledge about the type of dependence. In the second general approach, originally introduced by Alwan and Roberts [19], a control chart is used for monitoring residuals. Their methodology is applicable for any class of processes, so it is also applicable for the autoregression process of differences D_i considered in this paper. According to the methodology proposed by Alwan and Roberts [19], the deterministic part of (2) or (3) is estimated from sample data of n elements, and used for the calculation of residuals according to (9). Then, these residuals are used for the construction of our control chart according to the algorithm described above.

It is worth noticing that the Shewhart control chart for individual observations, also known as the X chart, is not the only control chart used for monitoring stability of monitored processes. However, it is the simplest one. Moreover, it is easy to interpret by non-specialists. This second feature seems to us very important if we have to use it in a simple health-care monitoring procedure.

B. Operating procedure

Operating procedure of the proposed control chart for residuals, applied for differences between consecutive observations of the monitored process, is the same as in the case of a classical Shewhart control chart. Using the estimated process model, we calculate the predicted value of the difference between the next two observations of the monitored process. An alarm signal is generated when an observed residual (difference between an observed and predicted values) falls beyond control lines. In Figure 5, we present a one-sided (with an upper control limit) control chart for residuals calculated for the process of differences between consecutive measurements of blood pressure displayed in Figure 1. The model of the process of differences D_i was estimated using first 20 observations of the monitored process of blood pressure measurements. Let us start our analysis from the case when residuals are calculated

using the model estimated from the Yule-Walker equations. The estimated model is the AR model of the fourth order $AR(-0.9062, -0.6613, -0.1419, -0.0955)$ with the constant μ equal to 0.2953. The sample residuals calculated using this model have the mean value equal to 0.63, and standard deviation equal to 6.575. Hence, the upper control limit of the Shewhart control chart is equal to 20.356, and this chart is presented in Figure 4. The residuals displayed in Figure 4 are only slightly autocorrelated ($\rho(1) = 0.087$), so the estimated model well describes the considered model. However, sample residuals are moderately autocorrelated ($\rho(1) = 0.431$). Therefore, the parameters of the designed Shewhart control chart may be improved.

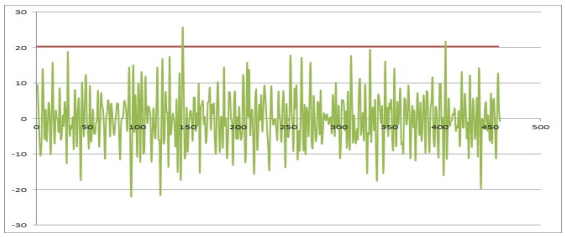

Figure 4. Control chart for first differences.

When the process model is estimated using Burg's algorithm the designed control chart is very similar. We have found that in this case the considered autoregression process is represented by the set of four parameters $(-0.987, -0.805, -0.217, -0.133)$. Then, residuals calculated for differences $D_5, \ldots, D_{19}$ have been used for the design of a control chart with the upper control limit equal to 20.29. The estimated model has been used for the calculation of residuals related to the next 80 observations. These residuals are displayed on the control chart. We can see that the monitored process seems to be under control, as all calculated residuals are located below the upper control limit.

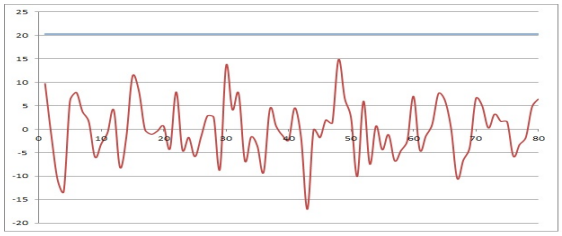

Figure 5. Control chart for residuals of differences of the first order related to measurements of blood pressure (Burg's method of prediction).

In comparison to a classical control chart for original observations, a control chart for residuals of differences has one important disadvantage: self-adaptation to a changed pattern of data. In order to explain this feature, let us transform our exemplary data by adding 20 to each observation starting from the 10th. The control chart in this case is presented in Figure 6.

From Figure 6, we can see that starting from the 10th point until the 12th point on the chart the values of displayed residuals sharply increase, but do not exceed the control limit. Later on, the process has returned to the previous level. It means that our chart is able to detect shifts of the monitored process only immediately after the jump. This is in sharp contrast to the classical Shewhart control chart (if it can be applied), where all data points observed after the shift indicate the deterioration of the monitored process. Thus, if the alarm is not generated immediately, it will be generated in the future quite randomly, despite the obvious deterioration of the monitored process. Therefore, we have to add an additional mechanism that will increase the probability of detection just after the shift.

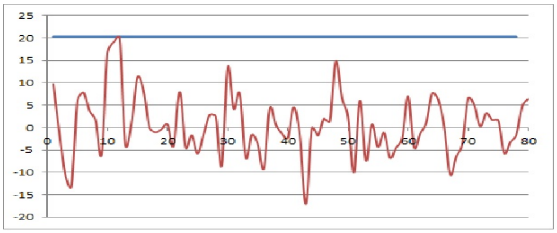

Figure 6. Control chart for residuals of differences of the first order related to measurements of blood pressure with a shifted process level,(Burg's method of prediction).

One of possible solutions of the problem mentioned above is to use an additional control chart. It can be a control chart for residuals calculated for second order differences defined as $D2_i = X_{i+2} - X_i$. The methodology for the design of this chart is exactly the same as that already described in this paper. Additional advantage of this approach is due to a fact that differences of the second order decrease or even cancel the impact of short cycles in the observed time series. A "weak" alarm signal is generated if it is generated on only one of these two charts. A "strong" alarm signal, that detects possible persistent deterioration, is generated when two consecutive points on the second chart are located beyond its control limits.

In our numerical example of shifted data, the model of the time series of differences of the second order, estimated from the sample of 20 observations, is the autoregression process of the second order $AR(-0.444, -0.555)$. Using this model, we can calculate residuals and design a respective control chart, presented in Figure 7. We can see that in the case of this control chart, deterioration of the process has been revealed with a delay of one measurement. Thus, if we have used both charts, we would detect the change of the process.

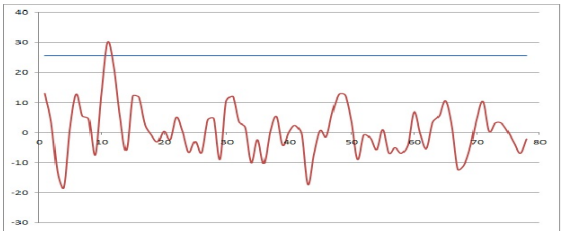

Figure 7. Control chart for residuals of differences of the second order related to measurements of blood pressure with a shifted process level (Burg's method of prediction).

Another possible solution, which is simpler for implementation, but theoretically less justified, is to calculate an additional residual as the difference between the observed difference of the second order and the predicted difference

of the first order, but calculated for the previous observation, and to plot the maximum of these two residuals on the chart designed for the case of differences of the first order. A "weak" alarm is generated when a point on the chart is located beyond the control limits. For a "strong" alarm it is necessary to observe at least two consecutive points on the chart situated beyond the control limits.

It has to be stressed here, that the proposed procedures are based on rather heuristic reasoning, based on observations of a particular series of measurements. Unfortunately, closed mathematical formulae that describe statistical properties of a control chart when observed values of measurements are statistically dependent, as for now, do not exist (except for the simplest cases). Therefore, the properties of the proposed procedures have to be investigated in the future using complex simulation experiments.

IV. Using the sXWAM control chart for short process runs

One of the most important characteristics of a control chart is its rate of false alarms. An alarm is considered false if it is generated in a period of time when a monitored process is stable. False alarm rate is usually accompanied with good abilities to detect process disorders, so if this falsity does not lead to serious consequences, higher false alarm rates may be considered acceptable. However, when an alarm cannot be neglected because of its serious consequences, the false alarm rate should be very low. For example, in certain pharmaceutical production processes an alarm should trigger a stop of a process, and this may be very costly if the triggering alarm is false. In the case of a stable process, described by the model $AR(-0.987, -0.805, -0.217. - 0.133)$ estimated from a sample of $n = 20$ observations, a chart presented in Figure 8 exhibits two false alarms.

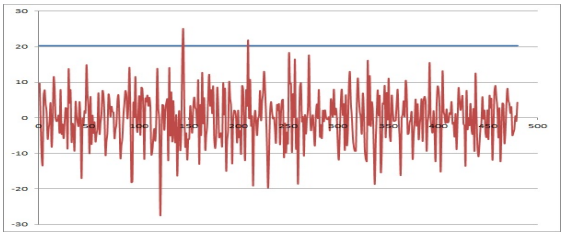

Figure 8. Control chart for residuals with two false alarms.

It has been observed by many authors (see [8], for more information) that control charts for autocorrelated data, especially those designed using small samples of observations, have elevated false alarm rates. Hryniewicz and Kaczmarek-Majer [8] have noted that this rather unfavorable property is somewhat related to the problem of bad predictability in short time series. Inspired by the very good properties of their prediction algorithm for short time series [20], they proposed in [8] a new control chart for residuals, named the XWAM control chart, based on the concept of model averaging.

Let us denote by M_0 the model of a monitored process estimated from a (usually) small sample, and describe its parameters by a vector $(a_{1,0}, \ldots .a_{p_0,0})$. We assign to this estimated model a certain weight $w_0 \in [0, 1]$. We also consider k alternative models $M_j, j = 1, \ldots, k$, each described by a vector of parameters $(a^0_{1,j}, \ldots, a^0_{p_j,j})$. In general, any model with known parameters can be used as an alternative one, but in this paper we restrict ourselves to the models chosen according to an extended version of the algorithm described in [13]. Let $w_1^{'}, \ldots, w_k^{'}$ denote the weights assigned to models $M_1, \ldots, M_k$ by this algorithm when only alternative models are considered. Because the total weight of the chosen alternative models is $1 - w_0$, to the estimated model we assign the weight w_0, and to each chosen alternative model we will assign a weight $w_j = (1 - w_0)w_j^{'}, j = 1, \ldots, k$.

When we model our process using $k+1$ models (one estimated from data, and k alternatives) each process observation generates $k+1$ residuals. In the case of differences of the first order considered in this paper, they are calculated using the following formula

$$z_{i,j} = \begin{array}{l} d_i - (\mu + a_{1,j}d_{i-1} + \ldots + a_{p_j,j}d_{i-p_j}), \\ j = 0, \ldots, k; i = p_j + 1, \ldots . \end{array} \quad (12)$$

In (12), we have assumed that for a model with $p_j, j = 0, \ldots, k$ parameters we need exactly p_j previous consecutive observations in order to calculate the first residual. Therefore, we need $i_{min} = \max(p_0, \ldots, p_k) + 1$ observations for the calculation of all residuals in the sample. For the calculation of the parameters of the XWAM control chart we use $n - i_{min} + 1$ weighted residuals calculated from the formula

$$z_i^\star = \sum_{j=0}^{k} w_j z_{i,j}, i = i_{min}, \ldots, n. \quad (13)$$

The central line of the chart is calculated as the mean of $z_i^\star$, and the control limits are equal to the mean plus/minus three standard deviations of $z_i^\star$, respectively. The operation of the XWAM control chart is a classical one. First decision is made after i_{min} observations. The weighted residual for the considered observation is calculated according to (13), and compared to the control limits. An alarm is generated when the weighted residual falls beyond the control limits.

The method for the construction of the XWAM chart was firstly proposed by Hryniewicz and Kaczmarek in [8] where they proposed an algorithm for the calculation of the weights of alternative models. This algorithm is based on the methods of computational intelligence, namely the DTW (Dynamic Time Warping) algorithm for comparison of time series. Unfortunately, this algorithm is computationally demanding, so in [13] they proposed its simplification, coined as the sXWAM (simplified XWAM). In this approach, Hryniewicz and Kaczmarek proposed not to compare original time series (observed and alternative), but their summarizations in terms of the autocorrelation functions of the pth order. Let $r_1, r_2, \ldots, r_p$ be the consecutive p values of the sample autocorrelation function, calculated using (5). Similarly, let $r_{1,i}, r_{2,i}, \ldots, r_{p,i}, i = 1, \ldots, J$ be the consecutive p values of the autocorrelation function of an alternative model. For given parameters of the alternative autoregression process $a_{1,i}, \ldots, a_{p,i}, i = 1, \ldots, J$ the values of $r_{1,i}, r_(2, i), \ldots, r_{p,i}, i = 1, \ldots, J$ can be found by solving the Yule - Walker equations (6). In general, the consecutive values of r_p can be computed using the following recursion equation

$$r_p = a_1 r_{p-1} + a_2 r_{p-2} + \cdots + a_p \quad (14)$$

Just like in [13], in this paper we consider only processes of the maximum fourth order. In such a case, explicit formulae for the first three autoregression coefficients are the following [13]:

$$r_1 = A_1, \tag{15}$$

$$r_2 = a_1 A_1 + a_2, \tag{16}$$

$$r_3 = \frac{a_1 B_1 + a_3 + (a_2 + a_4)(A_1 + A_2 B_1)}{1 - a_1 B_2 - (a_2 + a_4)(A_2 B_2 + A_3)}, \tag{17}$$

where

$$A_1 = \frac{a_1}{1 - a_2}, \tag{18}$$

$$A_2 = \frac{a_3}{1 - a_2}, \tag{19}$$

$$A_3 = \frac{a_4}{1 - a_2}, \tag{20}$$

$$B_1 = \frac{A_1(a_1 + a_3) + a_2}{1 - (a_1 + a_3)A_2 - a_4}, \tag{21}$$

$$B_2 = \frac{A_3(a_1 + a_3)}{1 - (a_1 + a_3)A_2 - a_4}. \tag{22}$$

Hence, the consecutive values of $r_4, r_5, \ldots$ can be directly computed from (14).

As the measure of distance between the estimated autocorrelations $r_1, r_2, \ldots, r_p$ and the correlations calculated for the ith alternative model $r_{1,i}, r_{2,i}, \ldots, r_{p,i}, i = 1, \ldots, J$ Hryniewicz and Kaczmarek-Majer [13] used a simple sum of absolute differences (called the Manhattan distance in the community of data mining)

$$dist_{i,MH} = \sum_{k=1}^{p} |r_k - r_{k,i}|, i = 1, \ldots, J. \tag{23}$$

In [1] Hryniewicz and Kaczmarek considered a slightly more general version of the sXWAM chart. As alternative models, they considered those autoregression models with k lowest values of $dist_{i,MH}$. Their weights, after some standardization, are inversely proportional to the distances of the closest models. The design of the sXWAM chart for residuals is thus much simpler than the original XWAM chart. The values of the autoregression functions for different alternative models can be computed in advance, and stored in an external file. This file can be read by a computer program, and used for choosing the model that fits to the observed sample (and its estimated autoregression function).

In this paper we try to improve the method used for finding alternative models. In order to do this we propose to use not only the autocorrelation functions ($ACF(p)$), but also the partial autocorrelation functions ($PACF(p)$). The kth value of the $PACF(p)$ function of the $AR(p)$ process, $\psi_p(k)$, is defined as the correlation between random variables X_t and X_{t+k} when the effect of the intermediate variables $X_{t+1} \ldots X_{t+k-1}$ that affect X_t and X_{t+h} has been removed. For the autoregression processes the $PACF(p)$ function has a very practical property. When the considered process is the AR process of the pth order, $AR(p)$, then all values of the $PACF(p)$ function are equal to zero for all $k > p$. The values of $\psi_p(k)$ can be found by solving the following system of equations

$$\begin{aligned} r_1 &= \psi_p(1) + \psi_p(2) r_1 + \ldots + \psi_p(p) r_{p-1} \\ r_2 &= \psi_p(1) r_1 + \psi_p(2) + \ldots + \psi_p(p) r_{p-2} \\ &\ldots \\ r_p &= \psi_p(1) r_{p-1} + \psi_p(2) r_{p-2} + \ldots + \psi_p(p) \end{aligned} \tag{24}$$

The system (24) is the so-called Toeplitz system of linear equations, and can be solved using the Durbin-Levinson recursion (see [14] for more information). The estimated values of the $PACF(p)$ functions can be calculated from (24) if we put estimated values of autocorrelations instead of their theoretical values. The distance between the estimated (from the sample) $PACF(p)$ function and the theoretical $PACF(p)$ of an alternative model is calculated using (23) with values of r_k and $r_{k,i}$ replaced, respectively, by ψ_k and $\psi_{k,i}$. Then, the sum of both distances, for the $ACF(p)$ and the $PACF(p)$ functions, is used for choosing the best alternative models and their weights.

V. Results and Discussion

In this section we present the results of computational experiments. In these experiments we use the original real-life data, and artificial data generated by adding some disturbances to the original data.

A. sXWAM control chart

The example of the sXWAM chart is presented in Figure 9 for the same original data that have been used for the construction of the control chart presented in Figure 8. For the design of this chart it was assumed that the weight for sample data is $w_0 = 0.7$. Five alternative process models have been found using the algorithm described above: $AR(-0.9, 0.5, 0.4, -0.3)$ with relative weight $w_1' = 0.201$, $AR(0.8, 0.7, -0.5, -0.3)$ with relative weight $w_2' = 0.201$, $AR(0.9, 0.5, -0.4, -0.3)$ with relative weight $w_3' = 0.200$, and $AR(-0.8, 0.7, 0.5, -0.3)$ with relative weight $w_4' = 0.199$, and $AR(0.8, 0.5, -0.3, 0.4)$ with relative weight $w_5' = 0.199$. We can see that in this case we have observed only one alarm generated at the same time point as one of the alarms generated on the control chart with non-weighted residuals. Experiments with artificially shifted process levels have shown that the detection ability of the proposed sXWAM chart are similar to that observed for the chart with non-weighted residuals.

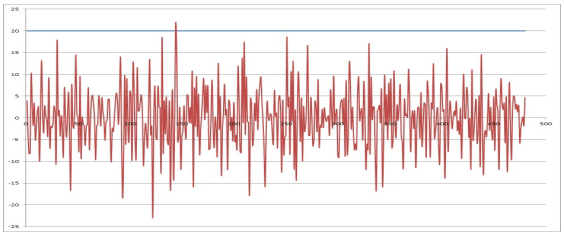

Figure 9. Control chart for weighted residuals with one false alarm.

B. Forecasting performance

Finally, we evaluate the forecasting performance of the proposed weighted model averaging approach with $ACF(p)$ and $PACF(p)$ (called $WAM*$, for short). We now present in detail the results obtained for the considered process of blood pressure measurements and the training database of AR processes of the first order. We assume that the model is estimated basing on a small sample ($n = 20$), and then, its verified for 6-step-ahead forecasts ($h = 1, 2, ..., 6$). The considered process has 480 observations that are used to create 454 samples. Each sample is divided into the first 20 observations that are used to estimate the predictive model, and the remaining 6 observations are used to verify its predictive performance. Alternative models and forecasts are calculated for each sample.

For example, the first sample is created for the first 26 observations of the process. The $WAM*$ approach delivers the following best alternative models and their weights: $AR(-0.4), w_1 = 0.34$, $AR(-0.5), w_2 = 0.33$, $AR(-0.3), w_3 = 0.33$. For the next sample (starting with n=2 element of the blood pressure measurements and ending with n=21 element of this process), exactly the same best alternative models are calculated. Slightly different models are calculated for the fourth sample (starting with n=4 element of the blood pressure measurements and ending with n=23 element of this process): $AR(-0.3), w_1 = 0.35$, $AR(-0.2), w_2 = 0.34$, $AR(-0.1), w_3 = 0.31$. In Table I, the frequency of the applied alternative models is presented.

TABLE I. Frequency of alternative models calculated with the $WAM*$ approach within all samples of the process with blood pressure measurements.

	M_1	M_2	M_3
AR(0.2)	-	0.00	0.00
AR(0.1)	0.00	0.00	0.01
AR(0.0)	0.01	0.02	0.05
AR(-0.1)	0.06	0.04	0.07
AR(-0.2)	0.07	0.14	**0.21**
AR(-0.3)	0.18	**0.27**	**0.24**
AR(-0.4)	**0.39**	**0.21**	0.15
AR(-0.5)	**0.19**	0.19	0.14
AR(-0.6)	0.07	0.06	0.11
AR(-0.7)	0.03	0.03	0.02
AR(-0.8)	0.01	0.03	0.00

As observed in Table I, the most frequent best alternative model is $AR(-0.4)$, selected in 39% of cases. As the second best model, $AR(-0.3)$ is selected in 27% of the cases. As the third best alternative model, again $AR(-0.3)$ is selected in 24% of the cases.

Finally, we summarize the forecasting results to evaluate the accuracy of the proposed approach. The following measures are applied for evaluation: the mean absolute error (MAE), the standard deviation of the MAE (stdD-MAE) the mean squared error (MSE) and the standard deviation of the MSE (stdD-MSE). As observed, we compute the commonly used scale-dependent measures (MAE, MSE, MDAE) because they are useful when comparing different methods applied to the same set of data, and this is our case.

Table II shows the accuracy measures depending on the forecast horizon. It is observed that in all cases, the proposed $WAM*$ method delivers forecasts that are more accurate than the forecasts calculated according to the AR process estimated with the Yule-Walker equations.

TABLE II. Forecast accuracy (the horizon up to 6-step-ahead) measured with MSE, stdD-MSE, MAE and stdD-MAE according to the proposed $WAM*$ approach and the respective estimated AR process.

Fcst. hor.	Method	MSE	stdD-MSE	MAE	stdD-MAE
1	$WAM*$	57.32	85.92	5.92	4.72
1	est AR	60.08	92.59	6.07	4.83
2	$WAM*$	64.82	76.00	6.30	4.05
2	est AR	68.91	81.64	6.51	4.12
3	$WAM*$	66.51	65.69	6.39	3.46
3	est AR	70.01	69.56	6.58	3.52
4	$WAM*$	67.21	58.56	6.43	3.06
4	est AR	69.96	61.46	6.57	3.12
5	$WAM*$	67.49	52.76	6.44	2.75
5	est AR	69.77	54.89	6.56	2.79
6	$WAM*$	**67.64**	**48.21**	**6.45**	**2.52**
6	est AR	**69.64**	**49.79**	**6.56**	**2.55**

For example, the average MSE of the 6 forecasts for 6-steps-ahead amounts to 67.64 according to the proposed approach and to 69.64 for the estimated process. For the comparative purposes, we calculate the errors obtained for the proposed model averaging approach in comparison to the errors of the forecasts calculated from the process estimated basing on the Yule-Walker equations (referenced as 'est AR' forecast). The relative errors for the aforementioned measures are presented in Table III.

TABLE III. Relative change of MSE, stdD-MSE, MAE and stdD-MAE for forecasts (up to 6-step horizon) of the proposed $WAM*$ approach compared to the respective estimated AR process.

Fcst. hor.	Method	R-MSE	R-stdD-MSE	R-MAE	R-stdD-MAE
1	$WAM*$	0.95	0.93	0.98	0.98
2	$WAM*$	0.94	0.93	0.97	0.98
3	$WAM*$	0.95	0.94	0.97	0.98
4	$WAM*$	0.96	0.95	0.98	0.98
5	$WAM*$	0.97	0.96	0.98	0.98
6	$WAM*$	**0.97**	**0.97**	**0.98**	**0.99**

As observed in Table III, the proposed model averaging $WAM*$ approach enables to improve all forecasts estimated with the traditional Yule-Walker equations. The improvement ranges from 0.93 to 0.99 depending on the considered accuracy measure.

C. Control chart for residuals of differences of the first order

Now, let us show how this new improved method of prediction can be utilized in the construction of a control chart. For calculation of residuals of first differences we will use predictions calculated according the autoregression model (4). The estimated process model minimizes the modified Akaike's BIC criterion, calculated according to (10). Suppose, that our control chart has to be put into operation after observing first 20 measurements of blood pressure. Hence, our sample of first differences consists of 19 observations. The optimal (with respect to the BIC criterion) model, estimated from this sample, is the following: $\mu_0 = 0.1642$ and $a_{0,1} = -0.559$. Thus, our residuals a described by the autoregression model of the first order. Let us assume now that our alternative models have to be chosen from among autoregression models of first and

second order. In this case, three best alternative models are the following: $(\mu_1 = 0.263, a_{1,1} = -0.9, a_{1,2} = -0.6)$ with the relative weight 0.371, $(\mu_2 = 0.232, a_{2,1} = -0.8, a_{2,2} = -0.5)$ with the relative weight 0.359, and $(\mu_3 = 0.253, a_{3,1} = -0.9, a_{3,2} = -0.5)$ with the relative weight 0.269. Because all our alternative models are of the second order, we can calculate only 18 prediction of first differences, and thus only 18 residuals. When we take into account only the estimated model, the mean value of calculated residuals is equal to 0.673, and sample standard deviation is equal to 10.393. Hence, the Shewhart two-sided control chart for the mean value, calculated using only estimated model, has the following control limits: $LCL = -30.50$, and $UCL = 31.85$. When we use the estimated model for the calculation of residuals for the remaining 460 measurements, the control chart will look like this presented in Figure 10. As we can see, both control lines are rather distant from the charted process values. Thus, this chart may not be effective in the detection of process instability.

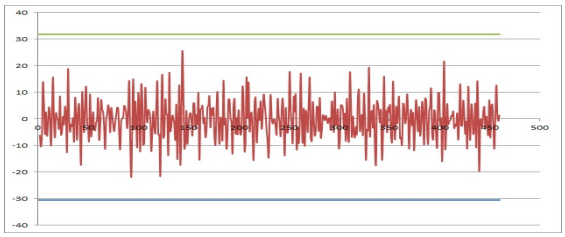

Figure 10. Control chart for residuals of differences of the first order related to measurements of blood pressure (estimated AR model used for prediction).

Now, let us consider a chart designed using only weighted residuals calculated according to the chosen alternative models. In this case, the mean value of calculated residuals is equal to 0.303, and sample standard deviation is equal to $6,628$. Hence, the Shewhart two-sided control chart for the mean value, calculated using only estimated model, has the following control limits: $LCL = -19.58$, and $UCL = 20.19$. Note, that in this case the standard deviation is significantly smaller, and the control lines a closer to the process observations. When we use the weighted alternative models for the calculation of residuals for the remaining 460 measurements, the control chart will look like this presented in Figure 11. We can see from Figure 11 that in this case four points on the chart fall beyond the control limits. Thus, we observe four alarms, and two of them (these showing significant increase of blood pressure) are definitely false. Moreover, insignificant (from a medical point of view) shifts of the monitored process may cause many false alarms.

Finally, let us use the $WAM*$ approach, and construct a chart for which we assign the weight w_0 to the estimated model, and the weight $1 - w_0$ to the alternative models. Suppose, that we take $w_0 = 0.5$. In this case, the mean value of calculated residuals is equal to 0.3488, and sample standard deviation is equal to $8,294$. Hence, the Shewhart two-sided control chart for the mean value, calculated using the $WAM*$ approach, has the following control limits: $LCL = -24.39$, and $UCL = 25.37$. When we use the $WAM*$ approach for the calculation of residuals for the remaining 460 measurements, the control chart will look like this presented in Figure 12.

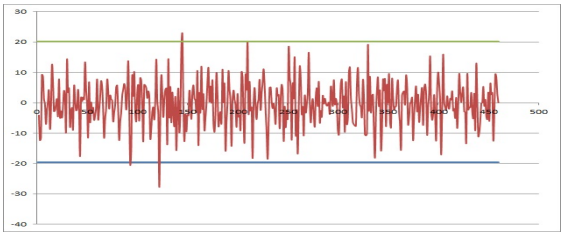

Figure 11. Control chart for residuals of differences of the first order related to measurements of blood pressure (weighted alternative AR models used for prediction).

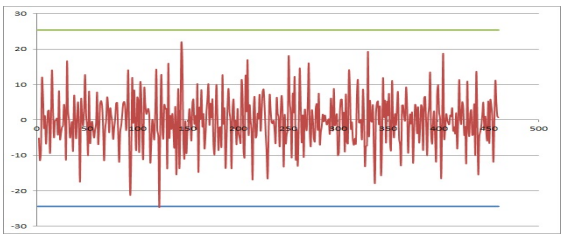

Figure 12. Control chart for residuals of differences of the first order related to measurements of blood pressure ($WAM*$ approach used for prediction).

The control chart displayed in Figure 12 represents a compromise between a chart designed using only an estimated model, and a chart designed using alternative models. On the one hand, in contrast to the chart presented in Figure 10, it should react to dangerous shifts of blood pressure, as the control limits are not so wide, as in the case presented in Figure 10. However, on the other hand, in contrast to the chart presented in Figure 11, where the control limits seem to be too narrow, no alarm signals are displayed for this seemingly stable process. It means that the control limits are not too narrow, and the chart should not generate too many false alarms in the case of insignificant variations of the monitored process. One has to stress here, that the behavior of a control chart presented in Figure 10 may lead to dangerous consequences (no necessary alarm signals). Observing too many false alarms is not so dangerous, but may decrease reliance on the used control chart. Therefore, in statistical process control we are always looking for compromise solutions, such as that presented in Figure 12.

In the design of our control chart we have used AR models of maximal second order. One could ask, however, about a similar design using models of higher order. In our calculations we also considered models of the fourth order. Predictions using such models are definitely more accurate. However, when the number of available measurements is, as in our case, strongly limited, then the effective sample size (the number of calculated residuals) becomes smaller. For example, when we use models of the fourth order, the sample size decreases to 15. Thus, the total variability of residuals may even increase, and the designed chart may be less effective.

VI. Conclusions

This paper has to be considered as a methodological one. As an illustrative example we have considered monitoring stability of a short and non-stationary processes using a simple tool such as a Shewhart control chart. We consider a monitored process stable, if its predicted future values are not very

different from the observed ones. Such processes are often encountered in practice, and in this paper are represented by a certain health-recovery process, where natural randomness of measured health-related characteristics is accompanied by random or deterministic trends. Statistical analysis of non-stationary processes is usually very difficult and costly for implementation, as it requires large amount of available data and sophisticated specialized software. It can be used at intensive-care hospital units or in cases when patient's life is endangered. However, in many cases, it is completely sufficient to monitor the state of health using personal measuring devices, and to alarm a patient (or his/hers physician) only in the case of unexpected events. We are of the opinion that this can be done using simple tools, like Shewhart control charts. Such procedures using simple software implemented, e.g., in personal measurement equipments. The chart proposed in this paper should definitely fulfill such requirements of simplicity.

In our research, we assume, as it is often done in statistical process control (SPC), that at its initial stage the monitored process is supervised (e.g., by a physician), and considered as stable. Data from this stable period, considered as our sample data, are used for the identification of the monitored process and construction of a control chart. Because simple methods for monitoring non-stationary processes do not exist, we propose to monitor differences of the first order (i.e,, differences between values of consecutive measurements). This approach is effective for linear or approximately linear trends. When we consider, as in this paper, short series of observations, this assumption seems to be rather realistic. However, it is possible to apply the proposed methodology for differences of a higher order. For example, in this paper, we also consider differences of the second order which can be used in the case of processes with alternating (e.g., morning and evening) process levels. In our investigations we have assumed that our series of observations are rather short, and the monitored process has to be identified using a small sample of measurements. This assumption reflects reality when health-recovery processes is evaluated by a physician for only short time, and the period in which the process has to be stable is also short (e.g., until a next treatment is applied). For this reason, we have proposed a novel statistical tool, sXWAM chart, developed recently by us, and the new chart, designed using the $WAM*$ approach, proposed by us in this paper.

The performance of the proposed method has been verified using real-life data obtained from a patient recovering from a hypertension episode, who measured his blood tension once a day for a period of 480 consecutive days. Statistical analysis of these data has shown that basic assumptions about statistical independence of consecutive measurements, which are used in the design of control charts applied in the monitoring of hypertension patients, and used in the procedures described in Solodky et al. [21], and Hebert and Neuhauser [22], have not been fulfilled. Despite the limited amount of data considered in this research, the presented results show that the proposed method is promising. The proposed methodology is general, and can be used for the analysis of any type of health-related measurements. One has to admit, however, that the particular prediction model - described by the equation (2) - is too simple for a proper description of continuous 24/7 measurements, either in clinical environment or taken from wearable sensors. In such applications much more complicated models, that take into account, e.g., periodical changes of a measured process level, must be used. A good presentation of this problem can be found in the analysis of the results of the 10th Annual PhysioNet/Computers in Cardiology Challenge 2009, which was devoted to predicting the Acute Hypotension Episodes (AHE), and described in the paper by Moody and Lehman [23]. Its participants provided various sophisticated solutions such as: neural networks, a rule-based approach, decision trees or support vector machines. A short review of other recent approaches for the AHE prediction can be also found in the paper by Jiang et al. [24]. Some of these methods may be used in our approach as replacements for our prediction model (2). Unfortunately, their computational complexity is currently too high for such personal devices like tablets or smartphones. Therefore, their usage in our approach is possible, but only in the case of external data processing by sufficiently powerful computers.

In our current research (with participation of clinical psychiatrists) we apply the proposed methodology for the analysis of self-assessment data provided by patients suffering from psychiatric bipolar disorder. It is known from the investigations described recently by Bonsall et al. [25], Maxhuni et al. [26], and Vasquez-Montes et al. [27] that classical control charts cannot be used for monitoring the behavior of such patients. Preliminary results of the application of the procedure described in this paper, described in Kaczmarek-Majer et al. [28], are considered by psychiatrists as very promising, and show great potential of the proposed methodology.

Finally, one has to note, that despite great progress in the application of statistical process control (SPC) in health care the proposed solutions, like the one presented in this paper, are still in an experimental stage. Therefore, it will take some time before their application in medical devices will be approved by regulatory bodies.

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