

A Density Based Clustering Algorithm for Efficient Channel Allocation in Multi-radio Multi-channel Wireless Mesh Networks

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Abstract—Efficient channel selection is essential in 802.11 mesh deployments, for minimizing contention and interference among co-channel devices. The IEEE 802.11 standard provides at least three non-overlapping channels and thus its possible for a node equipped with more than one network interface card (NIC) to operate on different channels simultaneously. This may increase the aggregate bandwidth available. In this work, we propose a density based clustering algorithm for channel allocation (DCCA) for multi-radio multi-channel mesh networks. DCCA manages the network topology by partitioning the mesh network into balanced cluster and affects a fixed and static channel to each cluster to be used mainly for control traffic. DCCA reduces intra- and inter-clusters interference and can be used in conjunction with a dynamic channel assignment strategy. Simulation results show that our solution gives balanced clusters and reduces broadcast time.

Keywords-mesh networks; multi-channel; multi-rate; clustering; DCCA.

I. INTRODUCTION

Wireless Mesh Networks (WMNs) [1] have recently gained increasing attention and have emerged as a technology with great potential for a wide range of applications. WMN are composed of static wireless node which form the backbone. One or several nodes which belong to the mesh infrastructure can be configured as gateway to allow access to external network such as Internet. Moreover, some of mesh nodes can play the role of access point to allow mesh network access to client station. In mesh network, nodes are either stationary or minimally mobile and have ample energy supply. For wireless local area networks, an extension named IEEE 802.11s is being standardized. The essential motivation for the 802.11s standard is to provide a means by which a wireless backbone can be realised with minimal configuration effort useful in scenarios such as office buildings, home networking, and apartment blocks. The IEEE 802.11s working group specifies a new architecture defining three network components: Mesh Point (MP), Mesh Portal Point (MPP) and Mesh Access Point (MAP) [2]. The MP determines how to route packets through the mesh. The MPP is a particular MP connected to a wired gateway and it allows MP to access external network such as Internet. The MAP is a particular MP that allows client stations to access

to the mesh network.

Each node in WMN can be equipped with multiple radios and each node interface can be configured to a different channel. Allowing multiple channels use in the same network is often presented as a possible way to improve the network capacity. In fact, with proper design, leveraging multiple channels available today has several benefits, including increasing system throughput, decreasing end-to-end delay, and achieving better load balancing.

Several multi-channel researchers [3] use a dedicated control channel and one interface is statically tuned to the control channels. This interface will be used to ensure connectivity and to send broadcast and control messages. In these solutions, control channel can become a bottleneck under heavy loads. In addition, in IEEE 802.11, there are only three orthogonal channels. Thus, 33% of the channel resource is consumed exclusively for control purposes.

In this work, we propose a clustering algorithm based on a density metric to manage the mesh network topology. Our algorithm is called DCCA for Density based Clustering algorithm for Channel Allocation. DCCA constructs balanced clusters, ensures that neighbor clusters doesn't interfere with each other and provides connectivity between nodes. In fact, the intra-cluster connectivity is guaranteed by affecting a fixed channel FC to each cluster according to a coloration algorithm. In addition, the inter-cluster connectivity is ensured by an efficient inter-cluster channel assignment. DCCA allows configuring one interface on FC used to exchange control and data traffic and it can be used in conjunction with a dynamic channel assignment solution to assign channels to pending radio interfaces.

The remainder of the paper is structured as follows: in Section 2, we present multi-channel related works and challenges. Section 3 reviews and compares load balancing clustering algorithm. In Section 4, we detail our clustering algorithm and inter and intra-cluster channel assignment. A Simulation-based performance study is presented in Section 5. Section 6 concludes this paper.

II. MULTI-CHANNEL IN WMN

The main goal of channel assignment approaches is to allocate the available channels to network interfaces of nodes in a way that minimizes interference and maximizes the average throughput [3]. Several works were proposed in literature to achieve this goal. Before developing these works, we first present multichannel challenges.

A. Multichannel Challenges in WMN

1) *Ensure connectivity*: The wireless mesh network may be split due to channel assignment. This can happen if a node doesn't share a common channel with any of its neighbor. In this context, channel assignment must guarantee connectivity between nodes in wireless network.

2) *Support of broadcast*: There are two methods to broadcast messages in multichannel mesh network. The first one is the use of a dedicated control interface tuned statically to a fixed control channel to isolate control packets from data packets. The second method is to use different channels and broadcasted messages will be sent over all radio interfaces. Both of methods have drawbacks. In fact, in the first method the dedicated interface can be overloaded, whereas in the second method, a huge number of control messages is observed in the network.

3) *Balance load*: Utilizing multiple channels allows parallel transmissions on non-overlapping channels. However, without accounting for the channel load in terms of the contention group size (the number of nodes using the same channel in the vicinity) some channels may become overloaded which increases interference and degrades network capacity

4) *Minimize the overall interferences*: The interference generated by neighboring nodes to send control traffic should be minimized to decrease the packet loss probability and improve thereby the overall performance of the network. To reduce interference a node should minimize the number of neighbors who use with him a common channel.

B. Related work

Several researchers proposed to manage the network topology by using clustering in dynamic multi-channel solutions such as [4] and [5] or tree architecture such as [7]. Nevertheless, clustering algorithms used by these solutions are not adapted for multichannel. For example, Liu *et al.* [4] uses Max-Min D-cluster algorithm which clustering result depends mainly on the distribution of nodes identifier in the network. Max-Min D-cluster may lead to some small clusters (clusters with radius equal to one or clusters with small number of members). Small cluster may cause inter cluster interference. Thus, the clustering algorithm must be topology based. Makram *et al.* [5] requires a clustering at the beginning, wherein the MPs nodes are grouped into subsets of nearby nodes. It deploys the Highest Connectivity Cluster (HCC) [6] algorithm, where a node is elected as a

clusterhead if it is the most highly connected node (having the highest number of neighbor nodes). The HCC can construct unbalanced one hop clusters. Raniwala *et al.* [7] defines a multi-channel WMN tree architecture based. Each MPP is the root of a spanning tree and each node attempts to participate in one or multiple such spanning tree. The solution of Raniwala *et al.* [7] has a drawback of providing path to wired network only. Besides, it uses a dedicated control interface to broadcast control messages. This interface can become overloaded which increases collisions and interference.

III. CLUSTERING ALGORITHMS FOR LOAD BALANCING

Our first objective when forming clusters is to limit the number of mobile nodes in each cluster to reduce intra- and inter-cluster interference. As regard clustering schemes, they can be classified according to their objectives into six categories [8]. Dominating-Set based clustering, low-maintenance clustering schemes, mobility-aware clustering, energy-efficient clustering, load balancing clustering schemes and combined-metrics based clustering. In our work, we opted for load balancing clustering. In fact, this category attempts to limit the number of mobile nodes in each cluster to a specified range so clusters are of similar size. Obtained cluster sizes have significant importance. Indeed, a too-large cluster may have several nodes using the same FC channel causing to heavy of interference in intra-cluster and reducing system throughput. A too-small cluster however may produce a large number of clusters and thus increases the inter-cluster interference.

A. Load-balancing clustering schemes

1) *AMC (Adaptive Multi-hop Clustering)*: AMC [9] maintains a multihop cluster structure based on load-balancing clustering. For cluster maintenance each mobile node periodically broadcasts its information, including its ID, CID (Clusterhead ID), and status (clusterhead/member/gateway) to others within the same cluster. By such message exchange, each mobile node obtains the topology information of its cluster. Each gateway also periodically exchanges information with neighboring gateways in different clusters and reports to its clusterhead. Thus, a clusterhead can recognize the number of mobile nodes of each neighboring cluster. AMC sets upper and lower bounds (U and L) on the number of cluster members that a clusterhead can handle. When the number of cluster members in a cluster is less than the lower bound, the cluster needs to merge with one of the neighboring clusters. On the contrary, if the number of cluster members in a cluster is greater than the upper bound, the cluster is divided into two clusters.

2) *DLBC (Degree-Load-Balancing Clustering)*: Periodically, DLBC [10] runs the clustering scheme in order to keep the number of mobile nodes in each cluster around

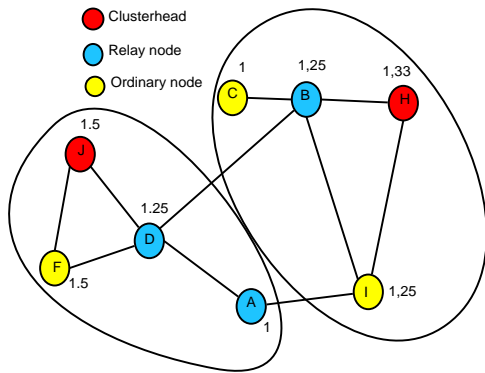


Figure 1. An example of DBC algorithm

a system parameter, ED (Elected Degree), which indicates the optimum number of mobile nodes that a clusterhead can handle. A clusterhead degrades to an ordinary member node if the difference between ED and the number of mobile nodes that it currently serves exceeds some value, MaxDelta.

3) *ACLB (Adaptive Cluster Load Balance method)*: A new approach in [11] is given. In hello message format, an "Option" item exists. If a sender node is a clusterhead, it will set the number of its dominated member nodes as "Option" value. When a sender node is not a clusterhead or it is undecided (H or non-CH), "Option" item will be reset to 0. When a clusterhead's Hello message shows its dominated nodes' number exceeds a threshold (the maximum number one clusterhead can manage), no new node will participate in this cluster.

4) *DBC (Density Based Clustering)*: DBC [12] adopts an approach based on a density metric to build clusters and to elect clusterhead. The density aims to characterize the node's importance inside the wireless network and in its neighborhood. The metric of density is the ratio between the number of edges between a node U and its neighbors, the number of edges between U 's neighbors and the number of nodes inside U 's neighborhood. Each node broadcast its density and the clusterhead will be the node having the highest density value. To elect a clusterhead, each node computes its density value and broadcasts it locally to all its neighbors. By receiving this value, each node is able to know which node will be the clusterhead. Once a cluster head is elected, the cluster head MAC Address and its density are locally broadcast by all nodes that decided to join that cluster. A cluster can then extend itself until it reaches a cluster frontier of another clusterhead. Let's consider an example of DBC algorithm in Fig. 1. Before partitioning the network, it is necessary to evaluate the density for each node. The density of node H is 1.33 ($1.33 = (2 + 1)/2$) because H has 2 edges with its neighbors (links (H, B) and (H, I)), 1 edge between its neighbors (link (B, I)) and 2 neighbors (B and I).

B. Comparison of Load-balancing Clustering

Table I
COMPARISON OF LOAD BALANCING CLUSTERING ALGORITHMS

	AMC	DLBC	ACLB	DBC
Clustering Multi-hops	2-hop	2-hop	1-hop	2-hop
Metric	NA	Degree	Degree	Density
Nono-verlapping Clusters	Yes	No	No	Yes
Balanced Clusters	Depend on U and L	Yes	Yes	In dense Network
Clusterhead in the middle	No	Yes	Yes	Yes
Distance between Clusterheads	More then three hops	More then three hops	Less then three hops	More then three hops
Direct connectivity to clusterhead	No	Yes	Yes	Yes

Table. I presents a summary of the load-balancing clustering schemes addressed before. We have fixed criterions related to multichannel to compare these clustering algorithms. The aim of this comparison is the selection of the most suitable clustering algorithm to manage network topology. The first criterion that we have considered is clustering multi-hop. In fact, multi-hop clustering is preferred than one-hop clustering and especially 2-hop clustering. In fact, if the radius of clusters is set to 1 then two non neighbor clusters may still interfere with each other. In contrast, if the radius of the clusters is 2 then non-neighbor cluster interference can only occur when the intermediate cluster is small enough, which reduces the possibility of non-neighbor cluster interference to a very small extent. If the radius of clusters is larger than 2, the efficiency of clustering algorithm will be reduced sharply and the intra-cluster interference on the Fc channel will increase. AMC, DLBC and DBC keep a multi-hop cluster structure while ACLB is 1-hop clustering algorithm. To have balanced clusters, a metric that consider topology is required. DLBC and ACLB use the metric of degree while DBC uses the metric of density. The density metric permits to obtain balanced clusters as it consider neighbor and links between neighbors. To reduce interference, clusters must be non-overlapping. AMC and DBC gives nonoverlapping clusters. The third criterion is the size of obtained clusters. In fact, The resulting clusters obtained from AMC are balanced if U (Upper bound) is closer to L (Lower bound). DLBC and ACLB give balanced cluster and DBC gives balanced cluster in dense network. The next criterion is the position of the clusterhead in the cluster. The ideal position is in the middle of the cluster to have a direct connectivity with major node members to minimize the number of exchanged message. In DLBC, ACLB and DBC clusterheads are in the middle while in AMC, clusterheads can be in the periphery of clusters. Finally, the last criterion is the distance between cluster-

heads. A distance of 3 hops and more is suitable. In fact, clusterheads will disseminate information on channel FC. Therefore, clusterheads must be distant to avoid interference between their transmissions. DLBC, ACLB and DBC require that neighboring clusterheads should be at least three hops away.

IV. DCCA: DENSITY BASED CLUSTERING ALGORITHM FOR CHANNEL ALLOCATION

We present environment features and constraints before describing our solution DCCA.

A. Assumptions

- 1) Initially, every node set one of its interfaces to a Default Channel.
- 2) The available non-overlapping channels are limited (3 at least for IEEE 802.11 standard).
- 3) Every node has at least two network interfaces but we don't require the same number of interfaces by node.
- 4) We consider that the mesh network is composed of static MP.
- 5) We assume that our network includes one MPP. However, if multiple MPP exist only one of them will be designated to unroll DCCA protocol. This MPP could be chosen based on the MAC address.

From Table I, we observe that the density algorithm is the more appropriate clustering algorithm to manage the network topology in a multichannel solution. We opted for DBC because it provides balancing and non-overlapping clusters and each cluster has at least a diameter of 2 which reduces the possibility of non-neighbor clusters interference. Moreover, the distance between neighboring clusterheads in DBC is 3 hops to avoid contention between clusterheads transmissions. Finally, clusterheads are closer to the middle of the cluster which implies less control exchanges into a cluster. However, DBC may construct unbalanced clusters in low density networks [12]. For this reason, we propose an improvement of DBC named DCCA (Density based Clustering algorithm for Channel Allocation) in order to obtain balanced clusters.

B. Clustering mechanism

DCCA is based on DBC. DBC computed the density of each node then it partitions network topology into clusters (see Sec. III-A4). In order to balance clusters sizes, each clusterhead in DCCA broadcast its MAC address and its cluster size. A relay node which is the node on the periphery of cluster such as node *A* receives messages from adjacent clusterheads and then knows the total number of nodes in each cluster. If this node finds that there is unbalance between the size of its cluster and one of its neighbor clusters (difference between node numbers in each cluster greater than 2), and if this node is 2 hops away from its clusterhead, it tries to migrate. Therefore, it sends an ATTACH message

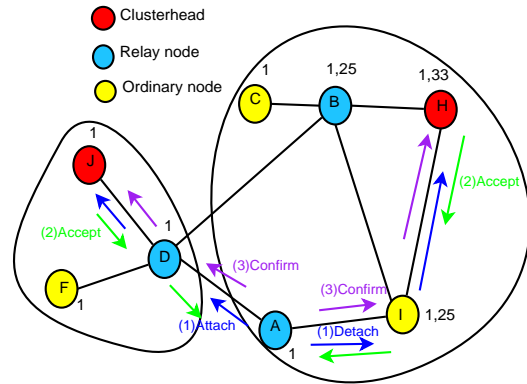


Figure 2. DCCA Algorithm

to the neighbor clusterhead (*J*) and DETACH to its appropriate clusterhead (*H*). The ATTACH message contains the relay MAC address, the neighbor clusterhead MAC address and the number of nodes in its cluster. The DETACH message contains the relay MAC address, its clusterhead MAC address and the neighbor cluster size. *J* accepts this migration only if the number of nodes in the ATTACH request is greater than the number of *J*'s cluster. In this case, it sends an ACCEPT message. Moreover, *H* accepts this migration only if the number of nodes in the DETACH request is lower than the number of *H*'s cluster. In this case, it also sends an ACCEPT message. If the relay node receives ACCEPT messages from *J* and *H*, it sends a CONFIRM message to *H* and *J*. Finally, this relay node will update its information about its new clusterhead. As a result of DCCA, the obtained clusters have approximately the same size (4 in this example).

C. Inter- and intra-cluster channel assignment

The purpose of the inter-cluster channel assignment is to distribute the available channels between clusters in a way that two neighboring clusters get different channels. In order to reach this objective, a modified DSATUR coloration algorithm [13] is deployed. In modified DSATUR, the MPP sends a declaration message PDEC (Portal Declaration) in the mesh network (we can use the PANN (Portal ANNouncement) message defined in IEEE 802.11s draft [2]). Upon reception of PDEC, each clusterhead sends a unicast PREG (Portal REGistration) to the MPP via the MP from which it received the PDEC. The registration message contains several information such as neighbor clusterhead addresses and a hop count field which calculate the number of hops between the MPP and the clusterhead. The MPP establishes a clusterheads table and a cluster connectivity matrix. The Clusterheads Table gives the level of each clusterhead which is obtained by dividing the distance between the clusterhead and the MPP by 3 (3 is the minimum distance between clusterheads)(example in (Fig. 3) the Clusterheads Table is given by Table. IV-C).

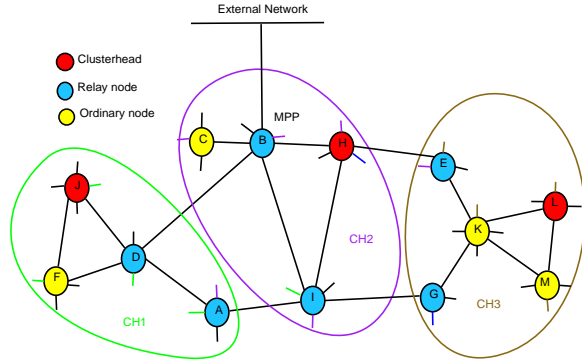


Figure 3. Inter-cluster channel assignment

In addition, the Connectivity Matrix permits to obtain the degree of a clusterhead which is the number of neighbor clusters (example in (Fig. 3) the Connectivity Matrix is given by (Table. IV-C)). For example in Fig. 3, the degree of clusterhead *H* is 2 as its cluster has two neighbor clusters.

Table II
CLUSTERHEADS TABLE

Id Clusterhead	Level
H	0
J	0
L	1

Table III
CONNECTIVITY MATRIX

	H	J	L
H	0	1	1
J	1	0	0
L	1	0	0

After a fixed timer (the necessary delay to receive all PREG messages from clusterheads), the MPP node affects to each clusterhead a fixed channel FC according to a modified DSATUR coloration algorithm. Indeed, this problem can be considered as a classical graph *k*-coloring problem: each clusterhead is a vertex in the graph. Two clusterheads are neighbor means that there are relay nodes between the two clusters. The Inter-cluster channel assignment algorithm (Algorithm 2) colors all vertices with *k* colors (*k* is the number of channels) so that neighbor vertices have different colors or the number of conflicts is as small as possible.

The MPP broadcasts a channel list message containing all clusters and affected fixed channels. Each node that receives this message configures one of its interfaces on the corresponding FC. A relay node has already an interface on FC of its cluster and affects its pending interfaces according to the algorithm (Algorithm 3).

In this way, intra- and inter-cluster connectivity are assured. Moreover, broadcast can be done efficiently. In fact,

Algorithm 1 DSAT

- 1: **if** no neighbor of *v* is colored **Then then**
 - 2: DSAT (*v*) = degree (*v*)
 - 3: **else**
 - 4: DSAT (*v*) = the number of different colors used in the first neighborhood of *v*
 - 5: **end if**
-

Algorithm 2 Inter-cluster Channel Assignment

- 1: Order the vertices in descending order of degree
 - 2: Color the vertex having the maximum degree with color 1
 - 3: Choose an uncolored vertex *x* having the maximum value of DSAT (Algo. 1)
 - 4: **if** conflict **then**
 - 5: Choose the vertex with the minimum level
// vertices having same DSAT
 - 6: **else**
 - 7: **if** conflict **then**
 - 8: Choose the vertex with the minimum ID
// vertices having same Levels
 - 9: **end if**
 - 10: **end if**
 - 11: Let FREE(*x*) a set of colors unused by the neighbors of *x*
 - 12: **if** FREE(*x*) $\subset \{1, 2, \dots, k\}$ is not empty **then**
 - 13: Color the vertex *x* with the smallest color in FREE(*x*)
 - 14: **else**
 - 15: Choose a color unused by the cluster of the MPP among the least used colors
 - 16: **end if**
 - 17: **if** all vertices are colored **then**
 - 18: Stop
 - 19: **else**
 - 20: Go to 3
 - 21: **end if**
-

relay nodes disseminate messages on all interfaces whereas other nodes disseminate messages on the interface configured on FC. Therefore, our proposal reduces interference involved by broadcast messages.

V. PERFORMANCE EVALUATION

We studied the performance gains of the proposed multichannel WMN architecture based on clustering through extensive Qualnet simulations. The following are the default settings for the simulations. We consider only the mesh network infrastructure. Nodes are uniformly distributed in $1500m \times 1500m$ simulation network. Each node is equipped with 2 NICs and the number of channels is set to 3. The ratio between the communication range and the interference range is set to 2. One of nodes is designated as the MPP node and is connected to the wired network. A random

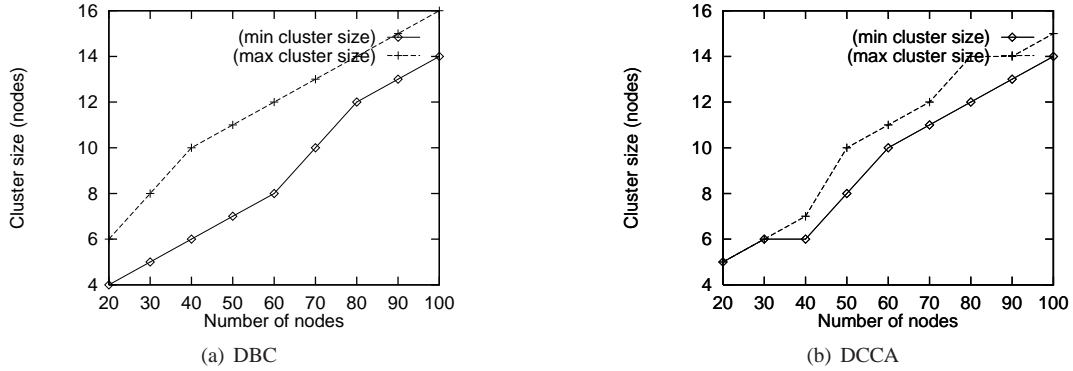


Figure 4. Obtained clusters size with: (a) DBC algorithm, (b) DCCA algorithm

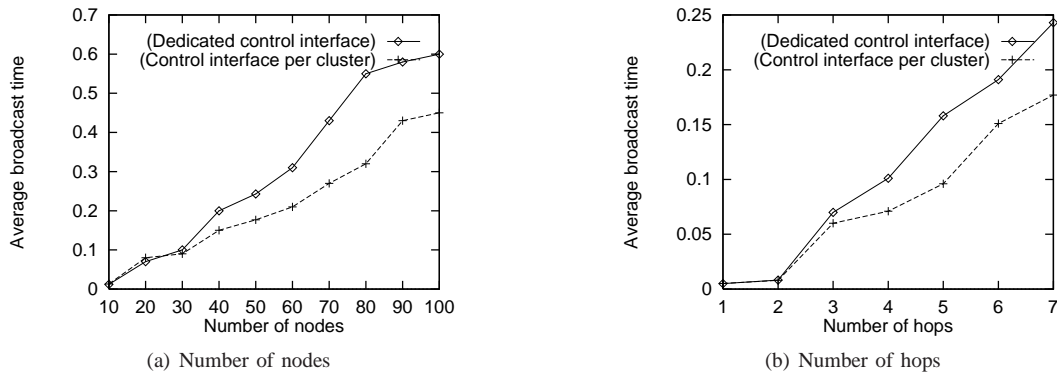


Figure 5. Average broadcast time as a fuction of: (a) the number of nodes, (b) the number of n hops

Algorithm 3 Relay Interfaces Configuration

- 1: **if** it is a relay to a single cluster (case of node B) **then**
- 2: It sets up one of its interfaces on the channel of the FC of the neighbor cluster
- 3: **else**
- 4: **if** the number of neighbor clusters exceeds the number of its interfaces **then**
- 5: It lefts one of its interfaces on the Default Channel.
- 6: **else**
- 7: It configures its interfaces on the channels of neighboring clusters
- 8: **end if**
- 9: **end if**

selected node sends a broadcast message on n hops away (n is a simulation parameter). Figures 4(a) and 4(b) show the maximum and minimum cluster sizes in terms of node members using the DBC and DCCA clustering algorithms respectively. DCCA gives more balanced clusters than DBC thanks to relay nodes migration if there is an unbalance between neighbor cluster sizes. That's way the difference between the maximum and minimum cluster sizes given by DCCA is smaller than the difference expressed by DBC. In

dense network (about 100 nodes), there is a constant and low difference between obtained cluster sizes using DCCA and DBC.

Figures 5(a) and 5(b) show the average broadcast time using a dedicated control interface and a control interface per cluster by varying the number of nodes and the number of broadcast hops (n). We observe that DCCA minimize the average broadcast time. In fact, DCCA uses different channels for neighbor clusters. Therefore, nodes broadcast messages using non overlapping channels which minimize interference and collision. The channel used by the dedicated control interface becomes overloaded which induces more time to propagate the broadcast message.

VI. CONCLUSION AND FUTURE WORKS

In this paper, we proposed a Density based Clustering algorithm for Channel Allocation (DCCA) in multi-radio and multi-channel wireless mesh network. DCCA uses a density metric and partitions the mesh network infrastructure into balanced clusters. Each cluster is affected a fixed channel FC using a coloration algorithm executed by the MPP. Obtained clusters have different fixed channels to minimize intra- and inter-cluster interference. Performance evaluation has shown that our proposal gives balanced clusters. DCCA also

minimize the average time of broadcast messages.

DCCA can be considered as a first step in a dynamic load aware channel assignment mechanism as it assigns a fixed channel per cluster; nodes can configure their pending interfaces according to a dynamic channel allocation. In future, we will use the obtained network topology from DCCA in a joint routing and channel allocation protocol.

VII. ACKNOWLEDGMENTS

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