Interference-aware Supermodular Game for Power Control in Cognitive Radio Networks

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***Abstract*—This Cognitive radio has proved to be a promising solution to improve the utilization of the radio spectrum. This new concept allows different wireless networks to operate on the same spectrum bandwidth. An efficient power control is thus crucial to make this coexistence possible and beneficial. In fact, unlicensed users (or secondary users, SUs) should communicate without harming the Primary users' (PUs) transmissions. In this paper, we propose a new power control mechanism for the SUs based on a powerful mathematical tool, the Game Theory. Our algorithm is based on a non-cooperative supermodular power control game in which we define a new utility function under total transmit power constraint, but also under interference constraint. We prove the existence of the Nash Equilibrium analytically and by means of simulations.**

Keywords- cognitive radio networks; game theory; power allocation; supermodular game.

I. INTRODUCTION

The wireless networking technologies are evolving rapidly in a very diverse manner (e.g., 3G+ [2] and 4G [3] cellular networks). This dramatic increase of the demand for spectral bandwidth and quality service is limited by the scarcity of spectrum resources. The CR (Cognitive radio) [4] is viewed as an effective approach for improving the utilization of the radio spectrum. The main idea of CR is to make secondary users (SUs), equipped with smart radios, able to sense the environment, detect the unused spectrum resources (spectrum holes) and decide when and how to access these holes. The CR also permits to the SUs to have underlay access to spectrums at the same time with the primary user, without causing harmful interference to the latter. Therefore, new challenges related to spectrum sharing appear especially how to design efficient power allocation and channel assignment schemes.

Many researches were conducted to resolve the topic of power control for Cognitive Radio Networks (CRNs) with the powerful mathematical tool the Game Theory [1]. In fact, Game Theory, having proved its efficiency in economics, was introduced lately to solve problems related to radio resource allocation in telecommunications [1]. This instrument helps study the complex interactions among independent players in order to optimize the setting of various elements of the network. More precisely, the players can be the SUs in a CRN, and the various elements are the SUs' transmit power. Game Theory will thus help us to design and to resolve the topic of power control for underlay SUs' transmission in presence of PUs under interference level constraint. It will also allow us to investigate the existence and convergence to a steady state operating point called Nash Equilibrium (NE), when the SUs perform independent distributed adaptations in terms of transmit power.

Some power control games are non-cooperative games, such [5] and [6]. In these games, selfish users choose their transmit power and attempt to maximize their individual utilities without being careful about the impact of their strategies on other users. A typical solution to a non-cooperative game is the Nash Equilibrium Point (NEP) [1], which is an equilibrium point where each player has no chance to increase its utility by unilaterally deviating from this equilibrium. Saraydar et al. [5] propose a non-cooperative power control game model in CDMA networks and proves the existence and uniqueness of Nash equilibrium. Zhou et al. [6] considered the problem of joint power and rate control for SUs in cognitive radio network by using non-cooperative game theory given a certain QoS requirement of SUs. Rasti et al. [7] proposed a non-cooperative power game with pricing that is linearly proportional to the signal-to-interference ratio. Del Re et al. [8] present a power resource allocation technique based on game theory, considering mainly Potential Games. Such power allocation is aimed to the up-link communication in a centralized CRN. Jing and Zheng [9] presented a game theoretic solution for uplink resource allocation in multi-cell OFDMA systems. Steady state and convergence are analyzed with potential game. The game can be modeled as a potential game to guarantee the convergence of NE. Del Re et al. [10] provided an S-Modular game in order to solve the resource sharing between the PU and the SU in a distributed and fair way. Elias et al. [11] use the Game Theory to address the spectrum access for the SUs taking into account the congestion level observed on the available spectrum bands. The same authors also proposed [12] a joint pricing and network selection scheme in CRNs based on a Stackelberg (leaderfollower) game.

In this paper, we are interested in developing a new algorithm based on Game Theory for a distributed power

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control in CRNs. This algorithm should allow the SUs to transmit without harming the PUs communications and while guaranteeing SU's Quality of Service requirements. We use a supermodular game because the latter guarantees the existence of at least one NE to reach, according to the different best SUs' responses.

This paper is organized as follows: Section II introduces the game model including the system model and formulates the optimization problem. In Section III, we describe our distributed power allocation algorithm based on our proposed supermodular game theoretic. Simulation results are given and discussed in Section IV. Finally, Section V concludes this paper.

II. GAME MODEL

Game theory analyzes the strategic interactions among rational decision makers. Three important components in a game model are the set of players, the strategy space of each player, and the payoff/utility function, which measures the outcome of the game for each player.

As the cognitive radios are smart terminals that can learn from their environment and dynamically modifying their transmission parameters in order to optimize their performance. Therefore, their interactions can be modeled using a non cooperative power control game. In this game model, $G=\left\{N,\{P_i\}_{i\in N},\{U_i\}_{i\in N}\right\}$, the N secondary users are the players and $P_i=\left\{p_i, 0\le p_i\le P^{\max}\right\}$ are their strategies, which represents the sets of power allocation that influence their own performance. P^{max} is the SU maximum power. And, U_i is the desired performance, designed as the payoff or utility function.

A. System Model

We consider a cognitive radio system in which a primary network is consisting of one PU base station and M active PUs coexists on one hand, and a secondary cognitive network made by N SUs equipped with CRs in a spectrum underlay manner on the other hand. SUs can simultaneously transmit with PUs but have to strictly control their transmit power to avoid harmful interference with PUs. We denote each transmitter and its intended receiver pair by a single index i (i = 1,…., M), referred to as a user. For simplicity, we neglect the interference from other adjacent cells. Thus, for each pair of SUs (v_i, v_j) located within mutual communication range, the signal-to-interference-and-noise ratio (SINR) received at user i can be written as:

$$\gamma_i=\frac{h_{ii}p_i}{\sum_{j=1,j\neq i}^{N}h_{ji}p_j+\sum_{m=1}^{M}h_{mi}p_m+\sigma^2} \tag{1}$$

where h_{ii} and p_i are respectively the channel gain, and the power level for the i^{th} player SU, in watts and is a parameter that we used in this paper for power control between [0, p^{max}]. h_{ji} is the cross channel gain from transmitter j to receiver i. The channel gain is determined by the log-normal shadowing path loss model. P_j is the transmission power of other SU different from SU i. and p_m is the transmission power of PUs and σ^2 is the additive white Gaussian Noise power (watts). Then, the transmission rate of the SU i at time t is:

$$R_i(t)=W\log_2\left[1+\eta\gamma_i\right] \tag{2}$$

where η is the SNR gap and it is related to the BER, bit error rate. It is given by $\eta = -1.5/\ln(5^*BER)$[12].

B. Utility Function Design

In this section, we seek to design a proper utility function that not only reflects the benefit of the player but also facilitates the implementation of power control algorithms in terms of convexity and global convergence. The key is to find utility expressions that are not only physically meaningful for a CRN but mathematically attractive for ensuring global convergence to the NEP [13] as well. In this paper, we adopt the $R(t)$ in eq.(2) as the QoS metric for SU players and accordingly construct the novel utility function U_i as an SINR-related form. The utility function represents the future benefit that a player will achieve when adopting a certain strategy, i.e., power allocation. However, the overall network optimum is usually not achieved at the NEP, since selfish users are only interested in the individual benefit. To improve the efficiency of the NE of non-cooperative games in CRNs, pricing can be introduced when designing the non-cooperative game, in order to guide the selfish users to a more efficient NE [14].

Each SU maximizes its own data rate at the cost of high power consumption, which causes interference to other SUs and brings down their data rate. In order to keep a SU from selfishly transmitting the highest transmit power, the system should first impose certain throughput fairness among the SUs, but also a pricing function. In our paper, we propose that this pricing function reflects constraints on the SU transmit power as well as constraints on the interference level caused on a PU. The utility function is therefore expressed by:

(P1): $$u_i\left(p_i, p_{-i}\right) = N\log\left(R_i\right) - \beta p_i - \alpha h_{im} p_i \quad (3)$$

$$s.t\ \gamma_i \geq \gamma_{\min} \quad (a)$$

$$\sum_{i=1}^{N} h_{im} p_i \leq I_{th}, \quad m = 1,....,M \quad (b)$$

$$0 \leq p_i \leq P^{\max}, \quad i = 1,....,N \quad (c)$$

where R_i is the achieved SU throughput, β is a positive constant, considered as the price of each SU transmit power p_i. The second part of (3), $\alpha\, h_{im} p_i$, considers the interference caused on the PU *m* by the user SU *i*. The constraint (a) reflects the minimum required quality of service for each SU; and the constraint (b) reflects that the aggregated interference caused at the PU should be below a predefined threshold I_{th}.

On the other hand, the power allocation can be formulated as an optimization power control problem given by:

$$\max_{p_i} u_i\left(p_i, p_{-i}\right) \quad (4)$$

$$s.t. \quad \gamma_i \geq \gamma_{\min}$$

$$\sum_{i=1}^{N} h_{mi} p_i \leq I_{th}, \quad p = 1,....,M$$

$$0 \leq p_i \leq P^{\max}, \quad i = 1,...,N$$

C. Existence of Nash Equilibrium

The NE gives the best strategy given that all the other players stick to their equilibrium strategy too. However, the question is how to find the Nash equilibrium, especially when the system is implemented in a distributed manner. One approach is to let players adjust their strategies iteratively based on accumulated observations as the game unfolds, and hopefully the process could converge to some equilibrium point.

The NE is the steady state in the game, in which no player can increase its utility function from unilaterally deviating its action. However, it does not follow that there is a NE in every game. Therefore, it becomes necessary to prove the existence of NE. For example, when the game can be modeled as a super modular game, convergence to the NE is guaranteed.

Theorem:

Our proposed game model can be shown as a supermodular game.

Proof: 1) Since [0, P^{max}] is a compact subset of R, 2) Also, for the range of 0≤ P_i ≤P^{max}, the utility function is continuous. 3) In addition, the utility function chosen has an attractive property: it is twice differentiable. The remaining condition that we should check is whether $\partial^2 U(Pi)/\partial Pi \partial Pj > 0$ or not.

Let $B = \sum_{j=1, j\neq i}^{N} h_{ij} p_j + N_0$ then the partial differential form of the above payoff function is:

$$\frac{\partial U(p_i)}{\partial p_i} = \frac{N h_{ii}}{\left(B + h_{ii} p_i\right)\log\left(1 + \frac{h_{ii} p_i}{B}\right)} - \beta - \alpha h_{im} \quad (5)$$

Let $C = \left(B + h_{ii} p_i\right)\log\left(1 + \frac{h_{ii} p_i}{B}\right)$, then:

$$\frac{\partial^2 U(p_i)}{\partial p_i \partial p_j} = \frac{-N h_{ii} h_{ij}\left(\log\left(1 + \frac{h_{ii} p_i}{B}\right) + \frac{h_{ii} p_i}{B}\right)}{B^2} \quad (6)$$

Since log(1+x) < x for all x>0, and ($h_{ii}p_i/B$) is positive; therefore, $\frac{\partial^2 U(p_i)}{\partial p_i \partial p_j} > 0$. According to the definition and property of game modes, this game is a supermodular game and therefore must be at least one NE in this supermodular game.

D. Solution of the game

Since the existence of NE was proved, we consider the problem of how to identify it. The optimal transmit power or NE can be obtained in such a way that each SU maximizes its own utility function iteratively. The problem can be expressed s follow:

$$P_i^* = \arg\max U_i\left(p_i, p_{-i}\right) \quad , i \in \mathrm{N} \quad (7)$$

where $P_i^* \in \left[0, P^{\max}\right]$

It should be noted that there is no sufficient guarantee in this game with regard to constraint (a), (b) and (c) of the problem (P1). First, the protection of PU should be assured by keeping the interference below a threshold, and the rigid SINR requirement of each SU must be respected especially if the SU experiences strong interference. In the next section, we will give details to solve this problem.

III. DISTRIBUTED POWER ALLOCATION GAME

In this section, an algorithm based on Lagrange techniques is developed to solve (4). This algorithm will have provable convergence and is suitable for distributed implementation. Because the model relates the optimum solution with 3 constraint conditions, let λ_i and μ_i denote

Lagrange multipliers corresponding to minimum SINR constraints (a) and the interference constraints (b) respectively.

The Lagrangian function of the convex equivalent of (3) is then:

$$L(p,\lambda_i,\mu_i)=U_i(p_i)+\sum_{i=1}^{N}\lambda_i(\gamma_i-\gamma_{\min})+\sum_{i=1}^{N}\mu_i\left(\sum_{i=1}^{N}h_{mi}p_i-I_{th}\right) \quad (8)$$

The problem (P1) is equivalent to:

$$\max_{p} L(p,\lambda_i*,\mu_i*)$$
$$s.t \quad 0\le p_i\le P^{\max},\, i=1,2,\ldots,N \quad (9)$$

The problem (P2) is solved via the following first-order algorithm that utilizes the gradient of L(p,λ,μ) to simultaneously update primal and dual variables with constant step size β and [x]+=max{0,x} :

$$p_i(k+1)=p_i(k)+\beta\frac{\partial L(p,\lambda,\mu)}{\partial p_i} \quad (10)$$

$$\lambda_i(k+1)=\left[\lambda_i(k)+\beta\gamma_i\right]^{+}$$
$$\mu_i(k+1)=\left[\mu_1(k)+\beta\sum_{i=1}^{N}h_{mi}p_i\right]^{+} \quad (11)$$

The gradient $\nabla L(p,\lambda,\mu_0,\mu)$ is used in (8) to find the maximum of $\nabla L(p,\lambda,\mu_0,\mu)$ with respect to *p*, and convergence will lead to the NE.

IV. PERFORMANCES EVALUATION

To evaluate the performances of the proposed algorithm, the simulations have been performed with a reduced number of users. Just one PU receiver has been placed in the scenario, while at most five SUs have been considered for the secondary system. The cell radius is R = 500m. The propagation model takes into consideration of path loss and frequency selective fading. The background noise δ2 is 5 × 10^{-15} Watts. The transmit power of PU is 10Watts .In such a scenario, the game converges quickly to the Nash equilibrium after 2-3 iterations.

First, we examine the convergence performance of the proposed game model in terms of SU transmit power. Fig. 1 illustrates the evolution of the SU transmit power for the five secondary users. It shows that the transmit power for each SU converges to the steady state. From this figure, we observe that there all the five SUs are transmitting with reasonable at the maximum power. This can not only enhance the power consumption for these SUs but also reduced the level of the interference to the PU. The limitation of the overall interference in the system is thus achieved.

Fig. 2 illustrates the achieved throughput by the different SUs versus their quality of service requirement in terms of BER. In fact, these SUs have not only satisfied the quality of service requirements but also realized a total throughput in the system of almost 14 Mbps.

TABLE I. THE LIST OF PARAMETERS FOR A SINGLE CELL COGNITIVE SYSTEM

Parameters	***Value***
W, the spectrum bandwidth	5 Mhz
Cell Radius	500 m
Number of users	5
Pmax, maximum power constraint	1 Watt

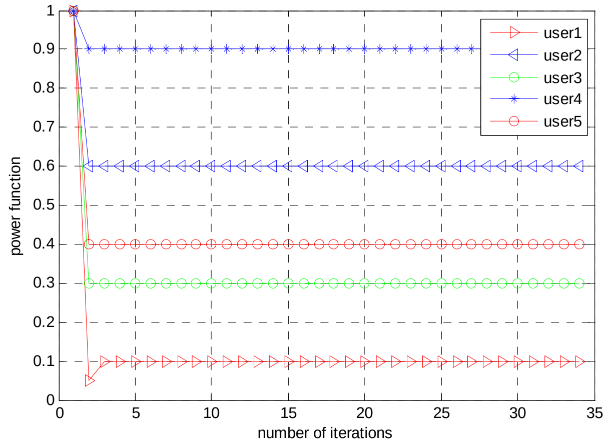

Figure 1. Convergence of SUs' transmit power.

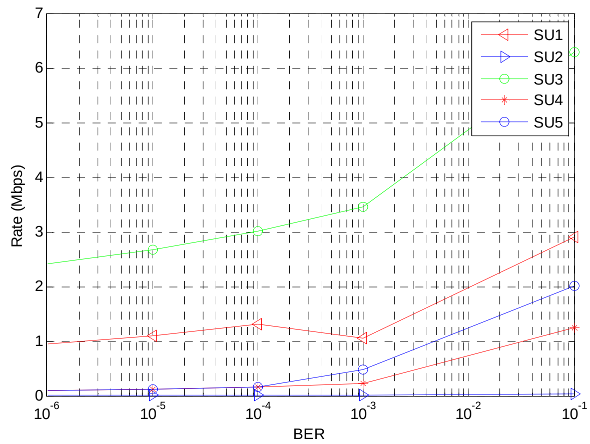

Figure 2. Achieved SUs' throughputs vs. SUs' target BERs

V. Conclusion and Future Work

In this paper, a non cooperative power control game is investigated for CR networks under quality of service and interference constraints. More precisely, we have introduced a new utility function in which the constraint on the interference caused by the SU to the PU is considered as well as the SU transmit power limitation. We have proved the NE for our game, and gave a distributed power control algorithm that converges to the NE.

The proposed algorithm used a pricing-based game to achieve the efficient power control which resulted in the maximum throughput for the cognitive network and respected the interference limitation as well. In the future, we intend to efficiently modify the price function so that we could maximize the throughput without altering the transmit power. Also, we intend to maximize the overall system throughput using cooperation between SUs.

References

[1] D. Fudenberg and D. K. Levine, "Game theory", MIT Press, Cambridge, MA, 1991.

[2] http://www.3gpp.org

[3] ITU-R Circular Letter 5/LCCE/2, "Invitation for submission of proposals for candidate radio interface technologies for the terrestrial components of the radio interface(s) for IMTAdvanced and invitation to participate in their subsequent evaluation," March 2008.

[4] J. Mitola III, "Cognitive Radio: An Integrated Agent Architecture for Software Defined Radio," Doctor of Technology Dissertation, Royal Institute of Technology (KTH), Sweden, May, 2000

[5] U. Cem Saraydar, B. NarayanMandayam, and J. David Goodman "Efficient power control via pricing in wireless data networks," IEEE Transactions on Communications, vol. 50 (2), pp. 291 – 303, 2002

[6] P. Zhou, W. Yuan, W. Liu, and W. Cheng, "Joint power and rate control in cognitive radio networks: A game-theoretical approach," in Proc. IEEE Int. Conf. Commun., pp. 3296–3301, 2008.

[7] M. Rasti, A. R. Sharafat, and B. Seyfe, "Pareto-efficient and goal-driven power control in wireless networks: A game-theoretic approach with a novel pricing scheme," IEEE/ACM Trans. Netw., vol. 17, no. 2, pp. 556– 569, Apr. 2009.

[8] E. Del Re, G. Gorni, L. Ronga, and R. Suffritti, "A power allocation strategy using Game Theory in Cognitive Radio networks," in Proceedings of the First ICST international conference on Game Theory for Networks, GameNets'09, pp. 117 - 123 , 13-15 May 2009.

[9] Q. Jing and Z. Zheng, "Distributed Resource Allocation Based on Game Theory in Multi-cell OFDMA Systems", International Journal of Wireless Information Networks, Vol. 16, pp.44-50. March 2009.

[10] E. Del Re, R. Pucci and L. S. Ronga, "Energy Efficient Non-Cooperative Methods for Resource Allocation in Cognitive Radio Networks" , communications and network journal, February 1, 2012

[11] J. Elias, F. Martignon, A. Capone, and E. Altman, "Non-Cooperative Spectrum Access in Cognitive Radio Networks: a Game Theoretical Model," Computer Networks, Elsevier , vol. 55, issue 17, pp. 3832-3846, December 2011.

[12] J. Elias, F. Martignon, and E. Altman, Joint Pricing and Cognitive RadioNetwork Selection: a Game Theoretical Approach, accepted forpublication in WiOpt 2012, Paderborn, Germany, May 2012.

[13] C. Yang, J.D. Li, and Zhi Tian, "Optimal Power Control for Cognitive Radio Networks Under Coupled Interference Constraints: A Cooperative Game-Theoretic Perspective", IEEE Transactions on Vehicular Technolgy, Vol. 59, No. 4, May 2010

[14] B. Wang , Y. Wu, and K.J. Ray Liu," Game theory for cognitive radio networks: An overview", Computer Networks , Volume: 54, Issue: 14, pp. 2537-2561 , 2010.