Time Dependent Lévy Flights Models for Internet Traffic

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Abstract

Measurements of local and wide-area network traffic in the 90's established the relation between burstiness and self-similarity of network traffic. Several papers demonstrated that the widely used Poisson based models could not be applied for the past decade's network traffic. Recent papers have questioned the direct applicability of these results in networks of the new century. Some authors of these papers demand the revision of previous assumptions on the Poisson traffic models. They argue that as newer and newer network technologies are implemented and the amount of Internet traffic grows exponentially, the burstiness of network traffic might cancel out due to the huge number of aggregated traffic flows. Some results are based on analyses of high-speed Internet backbone links and other traffic traces. We analyzed the same traffic traces and applied novel methods to characterize them in terms of packet interarrival time. We demonstrate that the series of interarrival times in the 2003 traces is still close to a self-similar process. Since then, new traffic traces have been made public, including ones captured from OC-192 links of the Internet backbone in 2008. We also compare the 2008 traffic traces with the ones captured in 2003 and apply our analytical methods to illustrate the tendency of Internet traffic burstiness in recent years. We found that the burstiness of the interarrival times decreased significantly compared to earlier traces.

***Index Terms*— Internet traffic; burstiness; Lévy Flights.**

1. Introduction

Network congestion can be caused by several factors. The most dangerous cause of congestion is the burstiness of the network traffic. Recent results make evident that high-speed network traffic is more bursty and its variability cannot be predicted as assumed previously. It has been shown that network traffic has similar statistical properties on many time scales. Traffic that is bursty on many or all time scales can be described statistically using the notion of self-similarity. Self-similar traffic has observable bursts on all time scales.

One of the consequences of burstiness is that combining the various flows of data, as it happens for example in the Internet, does not result in the smoothing of traffic. Measurements of local area network traffic [20] and wide-area network traffic have proved [21] that the widely used Markovian process models could not be applied for today's network traffic. If the traffic were Markovian process, the traffic's burst length would be smoothed by averaging over a long time scale contradicting with the observations of today's traffic characteristics. Combining bursty data streams will also produce bursty combined data flow. Various papers discuss the impact of the burstiness on network congestion [1], [3] and [8]. Their conclusions are that congested periods can be quite long with losses that are heavily concentrated.

The self-similarity of network traffic was observed in numerous papers, such as [3], [9], [22] and [25]. These and other papers showed that packet loss, buffer utilization, and response time were totally different when simulations used either real traffic data or synthetic data that included self-similarity [10], [11].

Papers, such as [16] and [31] challenge the direct applicability of these results for today's network traffic. They argue that traditional Poisson models can be used again to characterize the aggregate traffic flow of multiplexed large numbers of independent sources in the Internet backbone [17], [30]. Their explanation is that as the amount of Internet traffic grows dramatically mainly due to the implementation of fiber optic backbone links, the burstiness of network traffic might cancel out as a result of the large number of multiplexed packet flows. The paper describes the analyses of packet traces captured in the Internet backbone. The authors found that packet arrivals followed the Poisson distribution at sub-second time scales, appeared to be nonstationary at multi-second time scales, and the same packet trace showed evidence of long-range dependence at scales of seconds and above.

By the end of 90's, many previous works also analyzed the burstiness and the correlation structure of Internet traffic in various time scales according to the behavior of the Transmission Control Protocol (TCP) in terms of timeouts, congestion avoidance, self-clocking, etc. The paper [5] applied a wavelet-based multiresolution tool to analyze the scaling behavior of Internet traffic on short time scales. This paper was one of the first works showing evidence that Internet traffic could be analyzed by a multifractal model. Recent studies in [[7], [2], and [24]] have also proved that Internet traffic exhibits not only monofractal properties, but also a multifractal nature. Actual measurements have demonstrated that low-aggregate network traffic can have more complex properties than assumed previously.

The paper [12] illustrated that short time scale burstiness was independent of the TCP flow arrival process and showed that in networks with light traffic, correlations across different flows did not have an effect on the short scale burstiness. Internet traffic was classified in alpha and beta flows in the paper [28]. It was shown that large transfers over high-capacity links, called alpha flows, produced non-Gaussian traffic, while the beta flows, low-volume transmissions, produced Gaussian and long-range dependent traffic. Long sequence of back-to-back packets can cause significant correlations in short time scales. The reasons of sending long back-to-back packets in TCP or UDP sources were analyzed in [14], such as UDP message segmentation, TCP slow start, lost ACKs, etc. The same authors in [15] identified the actual protocol mechanisms that were responsible for creating bursty traffic in small time scales. It was shown that TCP self-clocking could shape the packet interarrivals of a TCP connection in a two-level ON-OFF pattern. The pattern causes burstiness in time scales up to the round-trip time of the TCP connection.

In our paper we analyzed the same traffic traces as in [16] and [35], and applied novel methods to characterize them. The network traffic traces are considered as a time series of the arrival times of the packets. Due to space limitation the analysis of the packet lengths is omitted. The arrival times form a monotone increasing series. The interarrival times are independent, identically distributed random variables. The classical modeling of the interarrival times goes back to Erlang, who successfully modeled the phone calls by a Poisson process with interarrival times distributed exponentially. We generalize his model by changing the distribution to a general family of infinitely divisible distributions and by the corresponding Lévy processes [29]. Since a subset of these distributions—called $\alpha-$stable distributions (asymmetrical in our case)— provides self-similar processes, we can analyze not just if the packet traces are self-similar, but we go beyond the results of previous papers and measure how close these packet traces are to being self-similar. The instrument of our analysis is the so called Truncated Lévy Flights [34]. The current paper is a continuation of our research project to investigate the tendency of the Internet's traffic burstiness. As part of the research project we have been comparing traffic traces captured from various years covering the last and current decade. The current paper relies on traces captured in 2003 and 2008 in terms of burstiness of the interarrival times. In another paper [36] we already analyzed the traces captured only in 2003 in terms of the burtiness of both the interarrival times and packet lengths. The current paper uses similar statistical methods as in [36] and investigates the traces captured in 2008 and compares it to the characteristics of the traces from 2003. In this paper we demonstrate that the 2003 traces are totally different from the 2008 traces in terms of burstiness and we conclude that based on the sample traces, the Internet is losing its self-similar nature that was so prevalent for years.

The second section describes the mathematical models applied for the analyses of the traces. The third section discusses the types of traces used in our work. The fourth section presents the results of the application of our model for the data, followed by the conclusion in section five.

2. Model: Smoothly Truncated Lévy Flights

In this section we introduce a model: The Smoothly Truncated Levy Flights (STLFs). It will be applied in section IV for describing the distribution of the interarrival times of the packet traces. The time series of the interarrival times under consideration is the sequence of the differences between consecutive arrivals of packets collected in the Internet backbone. The data collection details are described in section III.

The Truncated Lévy Flights were introduced by Mantegna and Stanley [23] as models for random phenomena, which exhibit properties at small time-scales similar to those of self-similar Lévy processes. The Truncated Lévy Flights have distributions with cutoffs at large time-scales, i.e., they have finite moments of any order. Building on Mantegna and Stanley's ideas Koponen [19] defined the Smoothly Truncated Lévy Flights (STLFs), which had the advantage of a nice analytic form. Independently, the same family of distributions was described earlier by Hougaard [13] in the context of a biological application. The concept of the more general distribution, called tempered stable distribution, is due to Rosiński [27] (see, e.g., [34] and [33] for a partial history of these works).

Since the interarrival times are positive, we consider STLF with a totally asymmetric distribution. It is given by the cumulant function (log of the characteristic function)

$$\psi_X(u) = a\Gamma(-\alpha)\left[(\lambda - iu)^{\alpha} - \lambda^{\alpha}\right], \qquad (1)$$

where $\alpha \in (0,1)$ and $\lambda, a > 0$. A more general discussion of STLF is given in Appendix C. This distribution depends on three parameters: the *index* α, the *truncation* parameter λ, and the *scale* parameter a. These parameters provide some information about the position of the distribution in the following manner:

Property 1. If α and a are fixed and λ tends to zero, then the limit distribution is a totally asymmetric α-stable distribution and the corresponding Lévy process is self-similar.

Property 2. If λ and a are fixed and α tends to zero, then the limit distribution is Gamma with parameters (a, λ). In particular, if a is 1, then the limit is exponential, therefore the Lévy process is Poisson.

Property 3. If λ and α are fixed, then for small a the distribution is close to the α- stable distribution and for large a the distribution is close to Gaussian. More precisely, moments of any positive order ϱ (including fractional) have the following asymptotics:

$$\log \mathbf{E}(|X|^{\varrho}) \sim \begin{cases} \min(\varrho/\alpha, 1)\log a + c_1, & \text{as} \quad a \to 0; \\ \varrho \log a + c_2, & \text{as} \quad a \to \infty. \end{cases}$$

For $m \geq 1$, the cumulants, derived from the cumulant function (1), are given in terms of the parameters α, λ, and a, namely,

$$\mathrm{cum}_m(X) = a\lambda^{\alpha - m}\Gamma(m - \alpha). \qquad (2)$$

3. Traffic traces

The traffic traces were captured from OC-48 (2.5 Gbps) connections of the Internet backbone collected by CAIDA [38][1] (The Cooperative Association for Internet Data Analysis). CAIDA's OC-48 traffic gathering devices compile packet headers at large peering points of several large Internet Service Providers (ISPs) in the United States. We used the traces collected in both directions of an OC-48 link at AMES Internet Exchange (AIX) on three different times. The traces have been divided into a collection of 5-minute files and another collection of 60-minute files to allow downloading the traces easier. These packet traces include the packet headers of packets with IP addresses anonymized with the prefix-preserving Crypto-PAn library. These traces include only IPv4 traffic. The precision of the traces is in the order of microseconds. Table 1 includes the details of the traces.

3.1. OC-192 traces

We also analyzed packet traces collected for four hours by CAIDA in May, 2008. The data sets contained anonymized traffic traces from an Internet data collection monitor on an OC-192 Internet backbone link (9953.28 Mbps). The Internet data collection monitor was located in Chicago, IL, and was connected to a Tier1 ISP between Chicago, IL and Seattle, WA. The traffic was captured by two network monitoring cards in both directions. A single card was connected to a single direction of the full-duplex backbone link. The directions were denoted by A (Seattle to Chicago) and B (Chicago to Seattle). The anonymized trace data contains layer 3 and layer 4 protocols: IP for layer 3, and TCP, UDP or ICMP for layer 4. These packets are originally encapsulated in layer 1 and layer 3 protocols. On the physical layer the protocol is PoS (Packets over SONET), on the data link layer the protocols are cHDLC (Cisco's version of HDLC), or PPPoHDLC (PPP over HDLC). Between layer 2 and 3 the service provider also inserts one or more MPLS headers [4].

The packets were captured more then an hour resulting in two traces direction A and B (Compressed size of direction A is 4.1 GB, compressed size of the trace in direction B is 14 GB).

The traces were captured by dedicated network measurement cards built by Endace Measurement Systems especially designed to provide very high quality packet time-stamps. Since GPS transmitters broadcast the current time based on atomic clocks, all Endace network measurement cards are equipped with ports allowing a GPS receiver to be connected providing clock synchronization.

Endace's DAG Universal Clock Kit (DUCK) provides per packet time-stamps that are both high resolution and capable of accurate synchronization to the Coordinated Universal Time (UTC). When a packet is captured, the DUCK time-stamps the beginning of the packet arrival in hardware unlike in NIC-based packet capture. NIC based time-stamping occurs in the host computer sometime after the packet has arrived estimating a time-stamp for the end of the packet arrival. The DUCK represents time in a single 64-bit fixed point number representing seconds since midnight on the first of January 1970. The high 32-bits store the integer number of seconds, the lower 32-bits contain the binary fraction of the second. This method provides a resolution of 232 seconds, or approximately 233 picoseconds.

Since the card's output file format was not supported by the majority of traffic analysis tools, CAIDA converted the original traces to a format with nanosecond

[1]Support for CAIDA's OC48 Traces is provided by the National Science Foundation, the US Department of Homeland Security, DARPA, Digital Envoy, and CAIDA Members.

Date	Duration	Length of the trace in bytes
Aug 14, 2002	3 hours, with 1 hour gap in one direction	108GB
Jan 15, 2003	1 hour, in both directions	30GB
Apr 24, 2003	1 hour, in both directions	13GB

Table 1. Details of the traces.

timestamp precision along with the packet lengths for both IPv4 and IPv6 packets separately. It is noticeable from the size of the traces that direction A had less traffic then direction B. A possible reason of the difference is that many content servers were located at one end of the link. Another interesting observation of the traffic was that based on a smaller sample, only a small portion (~8.6%) of IPv4 addresses was captured as both source and destination IP addresses in packets after merging both directions. This could be the indication that the network traffic in this area of the backbone may have been routed asymmetrically (Email communications with Emile Aben, Data Administrator, CAIDA/SDSC/UCSD).

4. Packet interarrival times

The series of interarrival times in the OC-48 traces are modeled as stochastic series. If the series correspond to a Poisson process, then the interarrival times have exponential distribution. In Figure 1 we fitted the Gamma distribution to the interarrival times of the OC-48 traces captured on April, 24, 2003 (20030424-001000-0-anon.pcap). (The Gamma distribution is more general than the exponential distribution. It reduces to the exponential distribution, if the shape parameter is 1.)

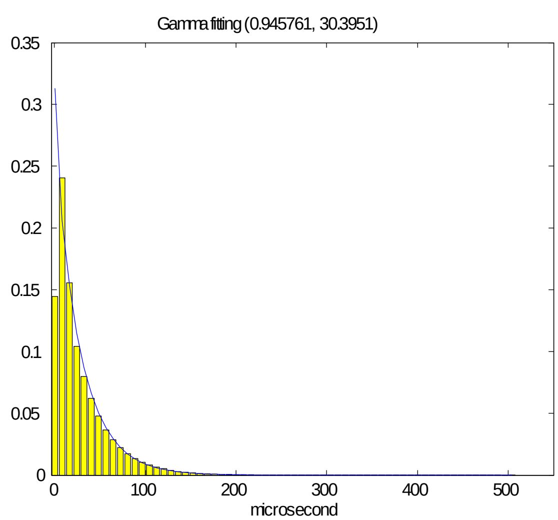

Fig. 1. **Gamma distribution of the interarrival times.**

Although the estimated parameters (0.945761, 30.3951) suggest that the distribution of the interarrival times is close to the exponential distribution, the Kolmogorov-Smirnov test strongly rejects the hypothesis that the series follows the Gamma distribution. Consequently, the corresponding process cannot be a Poisson process. Therefore we reject the hypothesis that the packet trace follows a Poisson process.

We continue the search for a distribution that would be suitable for characterizing the interarrival times by applying the family of Lévy processes.

The Poisson process is one of the simplest Lévy processes (see, e.g., [29]) with the main assumption that the increments—the interarrival times— are independent, homogeneous and exponential. Changing the distribution of the increments we obtain a wide variety of Lévy-stable processes as candidates for modeling the interarrival times [37]. Lévy-stable processes show heavy tail behavior making it impossible to apply them for the measured interarrival times: Figure 1 depicts that there are very few measurements after 200 micro second. The heavy tail of a distribution also implies that the moments do not exist, so these distributions are not appropriate for modeling purposes. Other members of the family of Lévy processes, the Smoothly Truncated Lévy Flights (STLF), have higher order moments. Since they have been successfully applied for finance, biological, and physical phenomena it is reasonable to apply it for traffic analysis as well. Some applications of the STLF are demonstrated in [23], [19], [13], and [34]. The following formula of the cumulants of STLF provides a means for estimating the parameters by the method of moments, i.e., calculating the empirical values from the traffic traces and compare them with the theoretical values above:

$$\mathrm{cum}_m\left(X\right) = a\lambda^{\alpha-m}\Gamma\left(m-\alpha\right),$$

More precisely, for a given trace we calculate the estimated cumulants $\widehat{\mathrm{cum}_m}$, $m = 1, 2, \ldots 8$, then we use the least squares method for finding the estimates $\widehat{a}$, $\widehat{\lambda}$, and $\widehat{\alpha}$ (for the details, please see the authors).

4.1 Analysis of OC-48 traces

We carried out these calculations for the OC48 trace captured on April 24, 2003 (20030424-002500-0-anon.pcap). Figure 2 shows the log of estimated cumulants $\widehat{\mathrm{cum}_m}$, and the log of cumulants $\widetilde{\mathrm{cum}_m}$, $m = 1, 2, \ldots 8$, of the Smoothly Truncated Lévy Flights when the parameters are estimated, i.e.,

$$\widetilde{\mathrm{cum}_m}\left(X\right) = \widehat{a}\widehat{\lambda}^{\widehat{\alpha}-m}\Gamma\left(m-\widehat{\alpha}\right),$$

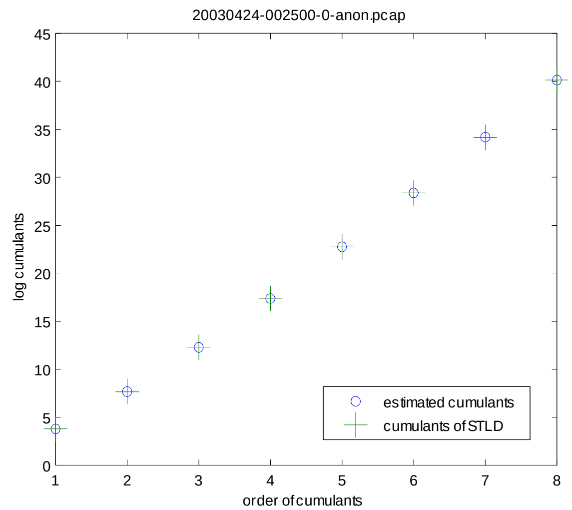

Fig. 2. **Comparison of the cumulants and estimated cumulants of the OC48 trace.**

$Trace$ $(Direction\,'0')$	α	λ	a
00_0	0.13106	0.02010	1.21465
05_0	0.17247	0.01871	1.37994
10_0	0.2162	0.0177	1.5266
15_0	0.13525	0.01828	1.23914
20_0	0.15424	0.01771	1.29506
25_0	0.15570	0.01768	1.30567
30_0	0.19040	0.01773	1.42341
35_0	0.22436	0.01802	1.56060
40_0	0.25456	0.01717	1.68368
45_0	0.19989	0.01760	1.44594
50_0	0.11637	0.01699	1.14885
55_0	0.14671	0.01818	1.26795

Table 2. The estimated parameters of the OC-48 5 minute traces in direction '0'.

where $\widehat{\alpha} = 0.15570$, $\widehat{\lambda} = 0.01768$, $\widehat{a} = 1.30567$. Since the fitting is good, it implies that this trace is close to the self-similar process because the value of λ is very small. At the same time the trace is not too far from the exponential distribution considering that the value of α is small and a is close to 1.

Table 2 and 3 show the estimated parameters of the OC-48 five minute traces.

In general, we can conclude that the distribution of these traces are close to α - stable distribution, since the estimations of λ are very small, hence the process is close to a self-similar process (see Property 1 in section II. A). It is also clear from the parameters that the traces in direction '1' are closer to the exponential distribution (see Property 2 in section II. A) than the ones in direction '0', since the parameter α is small and a is close to 1 at least in these traces: 05_1, 10_1, 25_1, 45_1, and 50_1. Therefore, the traces in direction '1' are closer to

$Trace$ $(Direction\,'1')$	α	λ	a
00_1	0.19944	0.02666	1.31111
05_1	0.06867	0.03078	0.99380
10_1	0.08212	0.02729	0.99712
15_1	0.17804	0.02431	1.29634
20_1	0.17372	0.02390	1.28337
25_1	0.07781	0.02525	0.98236
30_1	0.16226	0.02449	1.20388
35_1	0.11714	0.02452	1.07384
40_1	0.20719	0.02221	1.35552
45_1	0.07819	0.02307	1.00036
50_1	0.08903	0.02259	1.01705
55_1	0.16310	0.02214	1.21355

Table 3. The estimated parameters of the OC-48 5 minute traces in direction '1'.

a Poisson process then the traces in direction '0'. The reason for the different characteristics of the traffic traces in directions '0' and '1' is under investigation.

4.2 Analysis of OC-192 traces

Since the OC-192 datasets are significantly larger that the OC-48 datasets, we consider the parameters of the model being time dependent. We analyze the behavior of the model at every 0.1 second. The parameters of the model are estimated for the duration of 1 second interval. We assume that the traffic traces are locally stationary. The following figures depict the characteristics of the traffic flow in direction B and A. (We use the notations direction A and direction B to clearly distinguish the results related to the traces captured in 2003 and in 2008.) Figure 3 clearly demonstrates that $\alpha\,(t)$ is not close to zero, therefore the traffic flow in direction B does not exhibit the attributes of a self-similar or a Poisson process (see Property 1 and 2).

We obtained a similar figure for $\lambda\,(t)$ and $a\,(t)$ as well, see Figure 4-5.

In Direction A α is equal to zero in 43% of the samples, while a's values are close to 1. Figure 6 shows these values of a. The figure demonstrates that the traffic trace is approaching the Poisson process in 43% of the total samples.

5. Conclusion

We presented a novel model for analyzing the self-similarity of Internet traffic captured by CAIDA. The network traffic traces were considered as time series of the arrival times of the packets. We characterized the traffic traces with three parameters of Lévy Flights and placed a particular trace somewhere in the space generated by the Poisson and self-similar Lévy processes. Previous papers characterized the same traces as either self-similar or not self-similar traces. We were able to measure how close these packet traces were to being self-similar. We analyzed two sets of traces; one captured

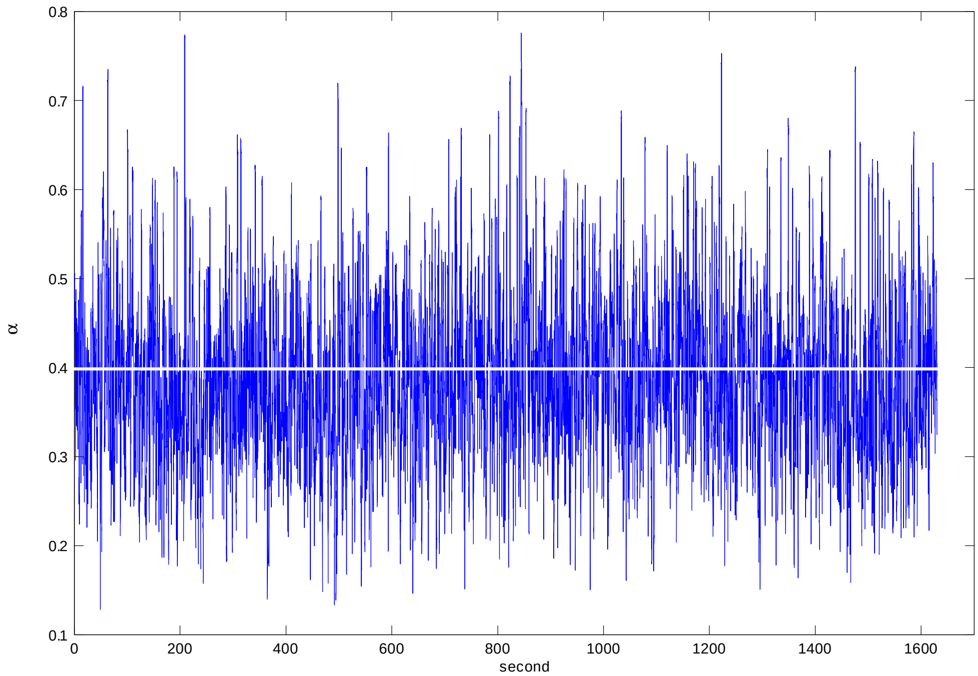

Fig. 3. $\alpha(t)$ **in Direction B.**

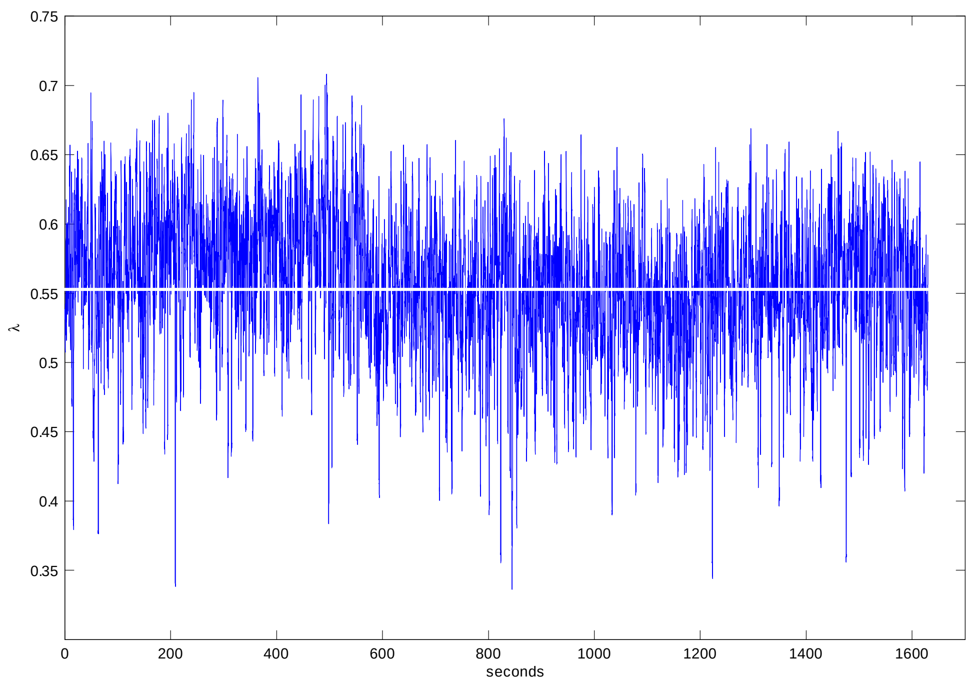

Fig. 4. $\lambda(t)$ **in Direction B.**

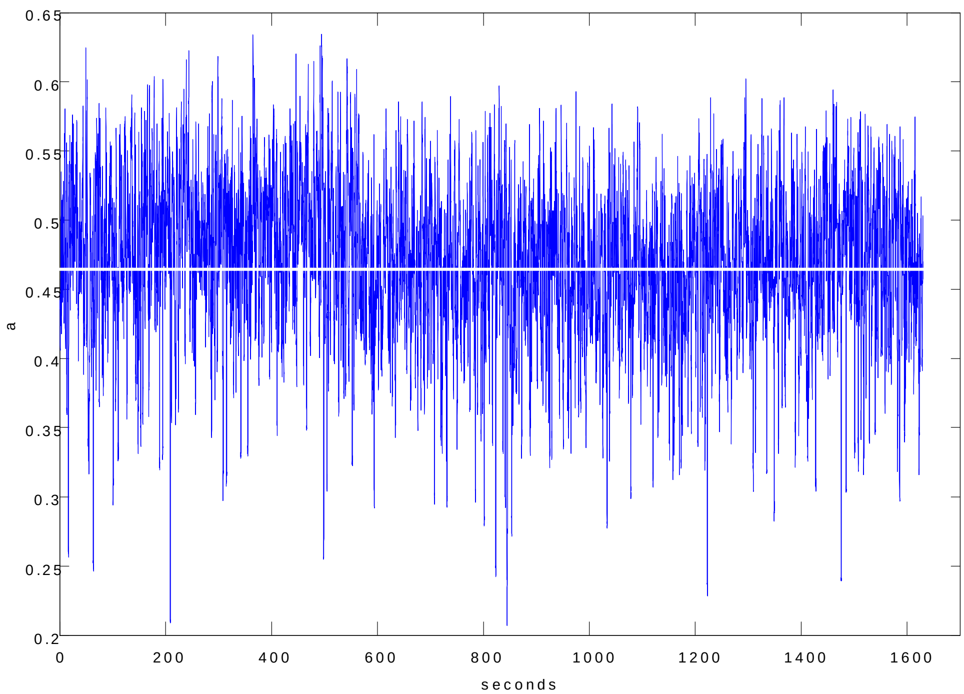

Fig. 5. $a(t)$ **in Direction B.**

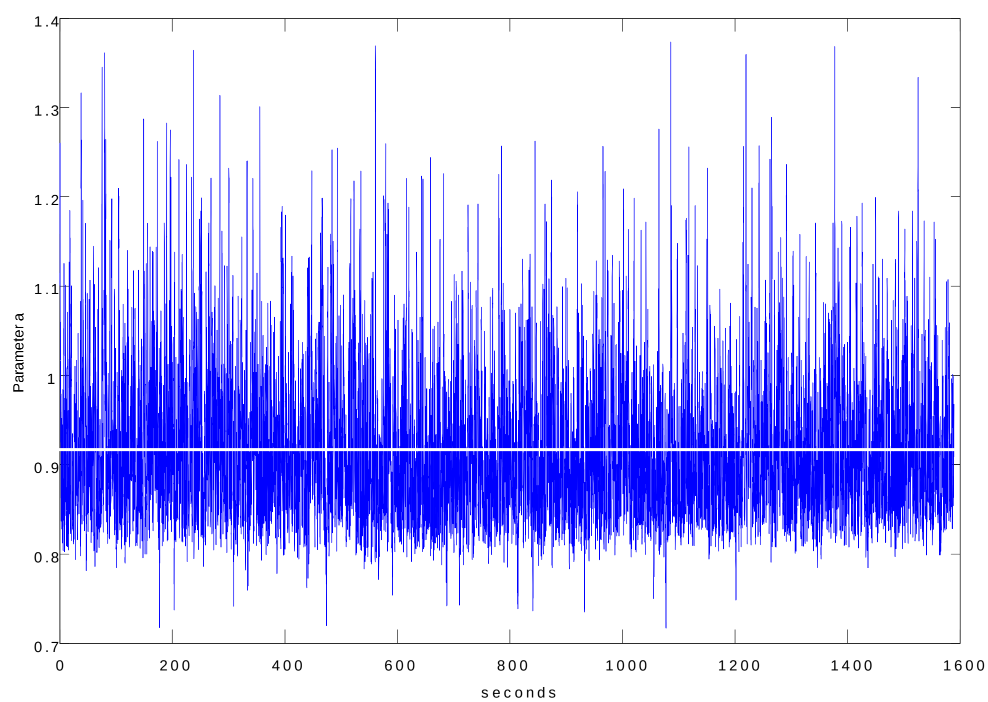

Fig. 6. Values of a **when** $\alpha(t) = 0$**, mean**$(a) = 0.91639$

from an OC-48 link in 2003 and another from an OC-192 trace in 2008. We concluded that the distribution of the 2003 traces was close to $\alpha-$stable distribution, since the estimations of λ were very small; hence the process was close to a self-similar process. It was also clear from the parameters that the traces in direction '1' were closer to the exponential distribution than the ones in direction '0'. Therefore, the traces in direction '1' were closer to a Poisson process then the traces in direction '0'. Regarding the traces from 2008 we concluded that in Direction B the traffic flow can be modeled by the Lévy Flights, but in Direction A, a large portion of the trace shows evidence of the properties of the traditional Poisson process.

6. Appendix

6.1. Cumulants

It is well known that there is a one to one correspondence between the moments and cumulants. The expected value is the cumulant of first order:

$$\mathrm{cum}_1(X) = \mathsf{E}X.$$

The cumulants of order 2 and 3 are equal to the central moments

$$\begin{aligned} \mathrm{cum}_2(X) &= \mathrm{Cov}(X,X) \qquad (3)\\ &= \mathsf{E}\left(X-\mathsf{E}X\right)^2, \\ \mathrm{cum}_3(X) &= \mathsf{E}\left(X-\mathsf{E}X\right)^3, \end{aligned}$$

but this is not true for higher order cumulants. One might easily check this for the case of cumulants of order four. Let us denote the central moment of k^{th} order by $m_k = \mathsf{E}\left(X-\mathsf{E}X\right)^k$, then we have

$$\begin{aligned} \mathrm{cum}_4(X) &= m_4 - 3m_2^2, \qquad (4)\\ \mathrm{cum}_5(X) &= m_5 - 10m_3m_2, \\ \mathrm{cum}_6(X) &= m_6 - 15m_4m_2 - 10m_3^2 + 30m_2^3, \\ \mathrm{cum}_7(X) &= m_7 - 21m_5m_2 - 35m_4m_3 + 210m_3m_2^2, \\ \mathrm{cum}_8(X) &= m_8 - 28m_6m_2 - 56m_5m_3 - 35m_4^2 \\ &\quad +420m_4m_2^2 + 560m_3^2m_2 - 630m_2^4, \end{aligned}$$

see [18] p.64, [32] p.10. If a sample $x_1, x_2, \ldots x_n$ is given, then the estimated expected value, i.e., first order cumulant is the mean $\overline{x}$, and the estimated k^{th} order central moment

$$\begin{aligned} \widehat{m}_k &= \overline{(x-\overline{x})^k} \\ &= \frac{1}{n}\sum_{j=1}^{n}\left(x_j-\overline{x}\right)^k. \end{aligned}$$

Now, the estimated cumulants are given in terms of estimated central moments (see formulae (4) above). For example, the 4^{th} order estimated cumulant $\widehat{\mathrm{cum}}_4$ is calculated by

$$\widehat{\mathrm{cum}}_4(X) = \widehat{m}_4 - 3\widehat{m}_2^2.$$

6.2. STLF

Let us recall that the STLF $X(t)$ is a Lévy process, i.e., a process with homogeneous and independent increments and $X(0)=0$. The probability distribution of $X = X(1)$ has characteristic function of the form

$$\varphi_X(u) = \exp\left(\psi_X(u)\right),$$

where the *cumulant function*

$$\psi_X(u) = a\lambda^\alpha\left[p\zeta_\alpha(-u/\lambda) + q\zeta_\alpha(u/\lambda)\right] + iub,$$

and $\lambda > 0,\ a,p,q \geq 0,\ p+q=1$, b is a real number, and

$$\zeta_\alpha(r) = \begin{cases} \Gamma(-\alpha)\left[(1-ir)^\alpha - 1\right], & \text{for } 0<\alpha<1; \\ (1-ir)\log(1-ir) + ir, & \text{for } \alpha = 1; \\ \Gamma(-\alpha)\left[(1-ir)^\alpha - 1 + i\alpha r\right], & \text{for } 1<\alpha<2. \end{cases}$$

(See [34] for details.)

Without loss of generality, we only consider the case when the shift parameter $b=0$. Parameters p and q describe the *skewness* of the probability distributions, and $p=q=1/2$ yields a symmetric distribution. Parameter λ will be referred to as the *truncation* parameter.

In the case of $0<\alpha<1$, the cumulant function is given by the formula

$$\psi_X(u) = a\lambda^\alpha\Gamma(-\alpha)\left[p\left(1+i\frac{u}{\lambda}\right)^\alpha + q\left(1-i\frac{u}{\lambda}\right)^\alpha - 1\right], \qquad (5)$$

and if $p=0$, the cumulant function

$$\begin{aligned} \psi_X(u) &= a\lambda^\alpha\Gamma(-\alpha)\left[\left(1-i\frac{u}{\lambda}\right)^\alpha - 1\right] \qquad (6)\\ &= a\Gamma(-\alpha)\left[(\lambda-iu)^\alpha - \lambda^\alpha\right], \end{aligned}$$

describes a distribution totally concentrated on the positive half-line. The distribution of X will be denoted by $STLF_\alpha(a,p,\lambda)$. The index α corresponds to the nontruncated limit when $\lambda=0$. In this case the distribution of X is the classical Lévy's $\alpha-$stable probability distribution. The scale parameter a tunes the time unit to a, hence the distribution of $X(t)$ is $STLF_\alpha(at,p,\lambda)$.

The role of the truncation parameter λ is obvious in the following particular case. For the one-sided $STLF_\alpha(a,0,\lambda)$ distribution with $0<\alpha<1$, the cumulant function has the form

$$\psi_X(u) = a\lambda^\alpha\Gamma(-\alpha)\left[\left(1-i\frac{u}{\lambda}\right)^\alpha - 1\right]. \qquad (7)$$

As $\lambda \to 0$, the distribution $STLF_\alpha(a,0,\lambda)$ converges to the $\alpha-$stable distribution $STLF_\alpha(a,0,0)$. The parameter λ looks appropriate for measuring the distance from the $\alpha-$stable distribution, but it can be noticed that scaling X will change the value of λ as well. More precisely, if X distributed as $STLF_\alpha(a,0,\lambda)$ then the distribution of cX is $STLF_\alpha(ac^\alpha,0,\lambda/c)$, where $c>0$. Therefore the distance from the $\alpha-$stable distribution can be measured by the parameter λ when the value a is fixed to 1.

For a fixed $\lambda, a>0$, as $\alpha\to 0$, the distribution $STLF_\alpha$ tends to the Gamma distribution $\Gamma(a,\lambda)$. Indeed, for $0<\alpha<1$, the Laplace transform ϕ_λ of $STLF_\alpha(a,0,\lambda)$ is

$$\phi_\lambda(u)=\exp\left(a\lambda^\alpha\Gamma(-\alpha)\left[(1+u/\lambda)^\alpha-1\right]\right),$$

and

$$\lim_{\alpha\to 0}\exp\left(-a\Gamma(1-\alpha)\frac{(\lambda+u)^\alpha-\lambda^\alpha}{\alpha}\right) = \exp\left(-a\log(1+u/\lambda)\right)=(1+u/\lambda)^{-a},$$

by the L'Hospital rule.

6.3. Estimating the parameters of $STLF_\alpha(a,0,\lambda)$

Take the logarithm of

$$\mathrm{cum}_m(X)=a\lambda^{\alpha-m}\Gamma(m-\alpha).$$

We obtain

$$\log\mathrm{cum}_m(X)=\log a+(\alpha-m)\log\lambda+\log\Gamma(m-\alpha). \quad (8)$$

Plug the estimated cumulants $\widehat{\mathrm{cum}_m}$ (see (4) above) into the left side of equation (8), then we have three unknowns a, λ, and α. In order to find the parameter values for the best fitting start with the system of equations when $m=2,3,4$, i.e.,

$$\begin{aligned}
\log\widehat{\mathrm{cum}_2}(X) &= \log a+(\alpha-2)\log\lambda && (9)\\
&\quad+\log\Gamma(2-\alpha),\\
\log\widehat{\mathrm{cum}_3}(X) &= \log a+(\alpha-3)\log\lambda && (10)\\
&\quad+\log\Gamma(3-\alpha)\\
&= \log a+(\alpha-3)\log\lambda\\
&\quad+\log(2-\alpha)+\log\Gamma(2-\alpha),\\
\log\widehat{\mathrm{cum}_4}(X) &= \log a+(\alpha-4)\log\lambda && (11)\\
&\quad+\log\Gamma(4-\alpha)\\
&= \log a+(\alpha-4)\log\lambda\\
&\quad+\log(3-\alpha)+\log(2-\alpha)\\
&\quad+\log\Gamma(2-\alpha).
\end{aligned}$$

The difference of the first two equations (9-10) gives

$$\begin{aligned}
\log\widehat{\mathrm{cum}_3}(X)-\log\widehat{\mathrm{cum}_2}(X) &= -\log\lambda+\log(2-\alpha)\\
&= \log\frac{2-\alpha}{\lambda},
\end{aligned}$$

hence

$$\alpha=2-\lambda\frac{\widehat{\mathrm{cum}_3}(X)}{\widehat{\mathrm{cum}_2}(X)}.$$

Similarly from the last two equations (10-11)

$$\alpha=3-\lambda\frac{\widehat{\mathrm{cum}_4}(X)}{\widehat{\mathrm{cum}_3}(X)},$$

therefore we obtain

$$\begin{aligned}
\widehat{\lambda} &= \frac{\widehat{\mathrm{cum}_3}(X)\widehat{\mathrm{cum}_2}(X)}{\widehat{\mathrm{cum}_4}(X)\widehat{\mathrm{cum}_2}(X)-\left[\widehat{\mathrm{cum}_3}(X)\right]^2},\\
\widehat{\alpha} &= 2-\frac{\left[\widehat{\mathrm{cum}_3}(X)\right]^2}{\widehat{\mathrm{cum}_4}(X)\widehat{\mathrm{cum}_2}(X)-\left[\widehat{\mathrm{cum}_3}(X)\right]^2},\\
\widehat{a} &= \frac{\widehat{\mathrm{cum}_2}(X)}{\widehat{\lambda}^{\widehat{\alpha}-2}\Gamma(2-\widehat{\alpha})}.
\end{aligned}$$

We obtain more precise estimations for the parameters, if we use these estimates as initial values and refine the estimates using nonlinear least squares, which minimizes

$$\sum_{m=1}^{8}\left[\mathrm{cum}_m(X)-a\lambda^{\alpha-m}\Gamma(m-\alpha)\right]^2.$$

6.4. Gamma distribution

The Gamma pdf is

$$f(x\,|a,b)=\frac{x^{a-1}}{b^a\Gamma(a)}\exp(-x/b),\ x>0,$$

where a and b are positive and called as shape and scale parameter respectively. If $a=1$, then it reduces to the exponential distribution.

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