# Distance Sensor Assistance to GPS Positioning

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***Abstract*—Global Positioning System (GPS) requires the signals from at least four observable satellites with good condition. However, in urban areas, the number of the observable satellites in line-of-sight decreases because urban areas have many buildings. In such case, the position estimation cannot be performed well. In order to estimate even if the good observable satellites are decreased, we use distance sensors. Many vehicles have the distance sensors that can measure traveling distances. In our proposal, the distance sensors assist in calculating own position. By the combination of GPS and the distance sensors, the proposed method can continue robust positioning even if we are in urban areas. Our experimental results will evaluate our effectiveness of the proposal.**

***Keywords–GPS; Positioning; Urban canyon; Lack of observable satellites.***

## I. Introduction

In recent years, location-based services have been increasing. These services require the information of user's position. Car navigation systems can be given as an example. In general, these services use the Global Positioning System (GPS). GPS can estimate user's position by using flying satellites around the Earth. We also focus on a novel GPS positioning in this paper [1].

The position estimation by GPS calculates the position, based on the measurement of the distances between the GPS receiver and the GPS satellites. In order to perform the position estimation, GPS receiver requires at least four satellites that can be received in line-of-sight [2]. However, in urban areas, the number of the observable satellites decreases because of buildings. Also, in urban canyon, GPS signals from the observable satellites tend to degrade because of multi-path propagation. The signals from satellites in non-line-of-sight should not be used because the errors are included in the propagated paths. In order to avoid degraded signals, the number of direct-path satellites that we prefer to use is more decreased. In such cases, the position estimation cannot be performed well. For example, the estimator cannot estimate the receiver's position if the number of observable satellites becomes less than three. Or, the performance becomes worse compared to the conditions in open sky where we can observe more than four satellites. Actually, conventional GPS systems rely on other information such as map information, base stations of assisted cellular networks, or Wi-Fi networks against the bad GPS measurement. In this paper, we will propose another assistance by a sensor data.

Many vehicles have wheel rotation sensors that can measure traveling distances. We proposed the novel positioning method that uses the information of the previous traveling distance [3][4][1]. Our proposed method uses the information of both the distance sensor and the previous position that is estimated under good condition. Higher accuracy in position estimation is expected by our proposed method, even when the number of observable good satellites is decreased. Also, it can be expected to prevent the impossibility of the position estimation when the number of the observable satellites becomes three. Moreover, better position estimation is expected by our proposed method, even when available satellites includes large error. By both the traveling distance and previous position, the proposal can add a quasi-satellite to GPS satellites. So, we can expect robust positioning by assistance of the sensor data.

In this paper, we will present the proposal's performance by field experiments. In the experiments, the position estimation is performed by traveling on a bicycle. From experimental results, we will evaluate the effectiveness of our proposed method.

This paper organized as follows. In Section II, we will introduce related works briefly. In Section III, we will show the case that the number of the observable satellites becomes three as the worst case. Under the worst case of three satellites, we will show our proposal in detail. In Section IV, we will evaluate our proposed method under three satellites. In Section V, we will show application of the proposal. We will apply our proposal to the case that more satellites exist. In Section VI, we will evaluate in case that adding the quasi-satellite by the proposal to the available all satellites. Finally, Section VII summarizes the paper.

## II. Related Work

In this section, we will show the related methods to improve the accuracy of the position estimation. Here, we are introducing the traditional and basic technologies.

### A. Differential GPS (DGPS)

The Differential GPS (DGPS) is the method for improving the position estimation accuracy [5]. The DGPS uses the GPS base station. The GPS base station transmits the information of the error amount in the GPS measurement to near GPS receivers. Measuring of the error information is performed accurately at the GPS base station.

Generally, the position estimation by GPS calculates the position by using the measurement of the distances between the GPS receiver and the GPS satellites. However, some errors are included in the distance measurement. The distance errors by clock difference, the ionospheric delay, and the troposphere delay can be given as examples. The estimation accuracy of user's position can be improved by correcting the error information that is generated at the GPS base station. However, the GPS estimation is used in various locations, such as urban areas, rural areas, sea, and mountains. In the urban areas, the DGPS cannot correct the propagation delay caused by the reflection at buildings. Therefore, in such case, the position estimation cannot be performed well.

### B. Dead Reckoning (DR)

The Dead Reckoning (DR) is the method of performing position estimation by the information of the relative movement [6]. In other words, DR uses the information how much we traveled from the previous position. Since the DR does not require any infrastructures, the DR is not limited to any area.

In vehicles, various sensors exist to detect the direction. Usually, the angular velocity is detected by the angular velocity sensor. The angular velocity can be calculated by integrating the traveling direction [7]. Also, it is possible to detect the direction by using a fiber optic gyroscope. The fiber optic gyroscope is a device for determining the direction by measuring the time difference of the light when the angular velocity is added to the fiber optic.

By using vehicle speed pulses, it is possible to detect the traveling distance. The vehicle speed pulse is the signal that is generated according to the rotational speed of the drive shaft of the vehicle. We can measure the traveling distance based on the circumference of the tire and the vehicle speed pulse.

In the DR, the relative position can be estimated by using the moved direction and the traveled distance. The DR is often used with a map matching technique, as described in the next section. The combination of both DR and map matching are often used in the car navigation systems.

### C. Map Matching

The map matching is the method for finding the appropriate position of the vehicle on the road by using the map information [8]. It is used in combination with the position estimation methods such as GPS and DR.

Currently, the map matching and the DR are commonly used in the car navigation systems [14]. The DR is a system for determining the relative position from the previous position. In DR, the error accumulates when the error occurs in the distance sensor and the direction sensor. This problem can be solved by using the map matching, but the map matching cannot be used in a place that does not have the map information. So, in order to estimate the absolute position, the DR with the map matching is often used with GPS. However, the sensor data from the DR does not improve the positioning accuracy of GPS directly.

As other works for multipath propagation in urban area, there are a lot of researches which can mitigate the multipath interference. For example, techniques which focus on signal tracking on a receiver or channel estimation by multiple correlators are shown[9], [10]. Moreover, the mitigation methods of Direction-of-Arrival (DoA) estimation by array antenna are also shown[11], [12]. The purpose of these methods is to prevent the degradation of the positioning performance in case of the multipath propagation with LOS satellites. Also, most of recent cellular phones use assisted-GPS (A-GPS), which is based on 3G/4G connection [13]. By using A-GPS, users can estimate own position rapidly under the environment of weak or bad satellite signals.

As a new method, we want to improve the position estimation accuracy by adding the sensor data, to the GPS position estimation directly. Here, we use a distance sensor. We proposed the positioning method that can estimate the absolute position by the combination of the distance sensor and GPS [3][4][1]. By our proposed method, the absolute position can be estimated even if only GPS cannot estimate own position exactly because of the bad environment. The bad environment for the conventional GPS is under lack of the observable satellites in line-of-sight. This case often happens in urban areas. The worst case is that the number of the observable satellites is three. The proposed method can estimate the own position but the conventional cannot estimate. We will treat this worst case in Sections III and IV. Of course, if the number of satellites is more than four, our proposed method may estimate well by assistance of the distance sensor. We will introduce this case in Sections V and VI. In our proposal method, we will keep estimating the user's absolute position by assistance of the distance sensor.

## III. Proposed Method Under Three Satellites

In this section, first, we will show the problems of the position estimation by GPS. As a worst case, we assume that the number of the observable good satellites is three. Thereafter, we will show our proposed method that uses the distance sensor and the previous position information.

### A. Problems of Position Estimation by GPS

The GPS positioning estimates the receiver's position based on the distances between the receiver and the satellites [15]. For estimating the 3D position of the receiver, the receiver needs three relations, that is, three satellites. Moreover, the receiver needs to estimate own clock error because the general receivers are equipped with inexpensive crystal clocks. Therefore, the GPS position

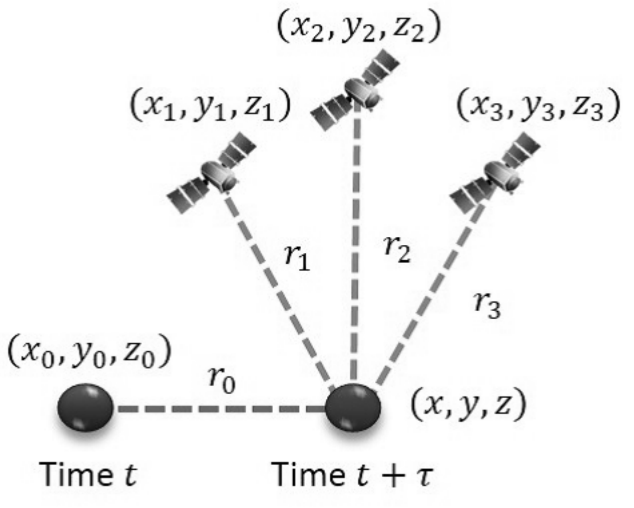


Figure 1. Assumed situation of our research.

estimation needs four relations, that is, four satellites. For carrying out the positioning calculation, the GPS position estimation needs at least four observable satellites. The observable satellite means the satellite that can receive its signal in line-of-sight. However, in urban areas, the number of the observable satellites are decreased because the receiver cannot observe the low elevation satellites due to the interference of buildings. Therefore, the receiver is not always possible to observe more than or equal to the four satellites in line-of-sight. So, the number of the observable satellites tends to decrease. Such decrement results in degradation of the position estimation accuracy. As a worst case, the receiver cannot estimate its own position if the number of the observable satellites becomes less than four. This is the most common problem in urban area.

To solve the above problem, we propose the novel GPS estimation that uses the distance sensor and the previous position information. The proposal method uses the previous receiver's position. We can measure the traveling distance from the previous position by the distance sensor. We assume the previous position as the quasi-satellite, which uses both the previous position and the traveled distance. By the proposed method, we expect that the position estimation is possible even if the number of the observable satellites are three. In addition, we also expect the improvement of position estimation accuracy when the number of the observable satellites are low. The detailed procedure is shown in the next subsection.

### *B. Proposed Position Estimation Algorithm*

The proposed method is assumed to be used in position estimation on vehicles such as cars or bikes. The assumed situation of our research is shown in Figure 1. We consider 2 observation times (the time $t$ and $t+\tau$). The position coordinate of the receiver at the time $t$ is $(x_0, y_0, z_0)$. And the position coordinate of the receiver at the time $t+\tau$ is $(x, y, z)$. We want to estimate the position $(x, y, z)$. We define that the position of the $i$-th satellite is $(x_i, y_i, z_i)$. By using the orbital information of the satellites that is contained in the satellite signals, the positions of the satellites can be defined. Also, the variables $r_i (i = 1, 2, 3)$ are the distances between the receiver and the satellites. On the other hand, the variables $r_0$ is the distance between the time $t$ position and the time $t+\tau$ position. Here, we assume that the position was correctly estimated by the adequate satellites at the time $t$. After traveling, we assume that the observable satellites are decreased at the time $t+\tau$. The number of the observable satellites at the time $t+\tau$ are assumed as three. In this case, the position estimation becomes impossible because of lack of observable satellites.

In this paper, for the purpose of simple explanation, we assume that the number of the observable satellites are four at the time $t$. The GPS positioning at the time $t$ uses the satellite positions and the distances between the receiver and the satellites. The true distance $\rho_i$ between the $i$-th satellite and the receiver can be expressed as follows.

$$\rho_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \tag{1}$$

The clock error may be included to the distance between the receiver and the satellite. Therefore, the distance $r_i$ can be expressed by (2).

$$r_i = \rho_i + s \tag{2}$$

Here, the clock error is represented as the parameter $s$. The unit of clock error $s$ is meter. This equation can be applied to all the observable satellites. As the number of the observable satellites are four at the time $t$, the following four equations can be obtained.

$$\begin{cases} r_1 = & \rho_1 + s \\ r_2 = & \rho_2 + s \\ r_3 = & \rho_3 + s \\ r_4 = & \rho_4 + s \end{cases} \tag{3}$$

By solving (3), it is possible to find out the position of the receiver $(x_0, y_0, z_0)$ and the clock error $s$ [16].

In case of the time $t+\tau$, the above method is not applicable for the position estimation because the number of the observable satellites are three. Therefore, we will estimate the position $(x, y, z)$ by adding the quasi-satellite. That is, we use the previous position $(x_0, y_0, z_0)$ and the distance between the current position and the previous position $r_0$. The distance $r_0$ can be represented by (4).

$$r_0 = \sqrt{(x_0 - x)^2 + (y_0 - y)^2 + (z_0 - z)^2} \tag{4}$$

At the time $t+\tau$, we can observe the three satellites. So, we can obtain the three equations (2). By using the three relations based on (2) and (4), we can derive the following equations.

$$\begin{cases} r_1 = & \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} + s \\ r_2 = & \sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2} + s \\ r_3 = & \sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2} + s \\ r_0 = & \sqrt{(x_0 - x)^2 + (y_0 - y)^2 + (z_0 - z)^2} \end{cases} \tag{5}$$

It is possible to estimate the position $(x, y, z)$ by (5).

The advantage of the proposed method is that we can increase the available satellites by adding (4). In addition, the proposed method only uses the GPS receiver and the distance sensor, no other infrastructure is needed. In our proposal, we need the distance between the current position and the previous position. In recent vehicles, it is easy to measure the traveling distance because most of the vehicles have distance sensors, such as speed pulses. Also, our proposal use the previous position. So, we note the proposal may accumulate the positioning errors that are based on the estimation errors of the previous position.

### C. Calculation Process of Position Estimation

Because the simultaneous equation (5) is nonlinear, the solution can be obtained by the sequential approximation that is performed the linearization around the initial value. Here, the procedure of the sequential approximation is shown below. As a notation, subscripts of the right shoulder of the following variables indicate the times of the sequential approximation.

1) we prepare the suitable initial values $x^0, y^0, z^0, s^0$ about $x, y, z, s$.
2) By using the receiver position $x^0, y^0, z^0$ and the clock error $s^0$, we calculate the distances between the receiver and the satellites.

$$\begin{cases} r_1^0 = \sqrt{(x_1 - x^0)^2 + (y_1 - y^0)^2 + (z_1 - z^0)^2} + s^0 \\ r_2^0 = \sqrt{(x_2 - x^0)^2 + (y_2 - y^0)^2 + (z_2 - z^0)^2} + s^0 \\ r_3^0 = \sqrt{(x_3 - x^0)^2 + (y_3 - y^0)^2 + (z_3 - z^0)^2} + s^0 \\ r_0^0 = \sqrt{(x_0 - x^0)^2 + (y_0 - y^0)^2 + (z_0 - z^0)^2} \end{cases} \quad (6)$$

3) The residual error $\Delta r_i = r_i - r_i^0$ can be determined by using the distance $r_i (i = 0, 1, 2, 3)$, which is actually measured.
4) Since it is possible to approach the correct solution by compensating same amount corresponding to the residual error for $x^0, y^0, z^0, s^0$, the compensation amount is determined using the partial derivative about $x, y, z, s$.

$$\begin{aligned} \frac{\partial r_i}{\partial x} &= -\frac{(x_i - x)}{r_i} \\ \frac{\partial r_i}{\partial y} &= -\frac{(y_i - y)}{r_i} \\ \frac{\partial r_i}{\partial z} &= -\frac{(z_i - z)}{r_i} \\ \frac{\partial r_i}{\partial s} &= \begin{cases} 1 (i = 1, 2, 3) \\ 0 (i = 0) \end{cases} \end{aligned} \quad (7)$$

From (7), the compensation amount $\Delta x, \Delta y, \Delta z, \Delta s$ to update $x^0, y^0, z^0, s^0$ can be represented as follows.

$$\begin{cases} \Delta r_1 = \frac{\partial r_1}{\partial x}\Delta x + \frac{\partial r_1}{\partial y}\Delta y + \frac{\partial r_1}{\partial z}\Delta z + \frac{\partial r_1}{\partial s}\Delta s \\ \Delta r_2 = \frac{\partial r_2}{\partial x}\Delta x + \frac{\partial r_2}{\partial y}\Delta y + \frac{\partial r_2}{\partial z}\Delta z + \frac{\partial r_2}{\partial s}\Delta s \\ \Delta r_3 = \frac{\partial r_3}{\partial x}\Delta x + \frac{\partial r_3}{\partial y}\Delta y + \frac{\partial r_3}{\partial z}\Delta z + \frac{\partial r_3}{\partial s}\Delta s \\ \Delta r_0 = \frac{\partial r_0}{\partial x}\Delta x + \frac{\partial r_0}{\partial y}\Delta y + \frac{\partial r_0}{\partial z}\Delta z \end{cases} \quad (8)$$

Here, the simultaneous equation (8) can be represented by the matrix form in order to simplify handling. We define the vectors $\Delta\vec{x} = [\Delta x, \Delta y, \Delta z, \Delta s]^{\mathrm{T}}$ and $\Delta\vec{r} = [\Delta r_1, \Delta r_2, \Delta r_3, \Delta r_0]^{\mathrm{T}}$ (the notation T expresses a transpose), the equation (8) can be expressed as follow.

$$G\Delta\vec{x} = \Delta\vec{r} \quad (9)$$

Here, the matrix $G$ is usually called as the observation matrix or the design matrix. The matrix $G$ can be expressed as follows.

$$G = \begin{bmatrix} \frac{\partial r_1}{\partial x} & \frac{\partial r_1}{\partial y} & \frac{\partial r_1}{\partial z} & \frac{\partial r_1}{\partial s} \\ \frac{\partial r_2}{\partial x} & \frac{\partial r_2}{\partial y} & \frac{\partial r_2}{\partial z} & \frac{\partial r_2}{\partial s} \\ \frac{\partial r_3}{\partial x} & \frac{\partial r_3}{\partial y} & \frac{\partial r_3}{\partial z} & \frac{\partial r_3}{\partial s} \\ \frac{\partial r_0}{\partial x} & \frac{\partial r_0}{\partial y} & \frac{\partial r_0}{\partial z} & \frac{\partial r_0}{\partial s} \end{bmatrix} = \begin{bmatrix} -\frac{(x_1 - x)}{r_1} & -\frac{(y_1 - y)}{r_1} & -\frac{(z_1 - z)}{r_1} & 1 \\ -\frac{(x_2 - x)}{r_2} & -\frac{(y_2 - y)}{r_2} & -\frac{(z_2 - z)}{r_2} & 1 \\ -\frac{(x_3 - x)}{r_3} & -\frac{(y_3 - y)}{r_3} & -\frac{(z_3 - z)}{r_3} & 1 \\ -\frac{(x_0 - x)}{r_0} & -\frac{(y_0 - y)}{r_0} & -\frac{(z_0 - z)}{r_0} & 0 \end{bmatrix} \quad (10)$$

The compensation amount $\Delta x, \Delta y, \Delta z, \Delta s$ in (8) can be derived by multiplying the inverse matrix of $G$ from the left of (9). Therefore, the compensation amount $\Delta x, \Delta y, \Delta z, \Delta s$ can be determined by solving (11).

$$\Delta\vec{x} = G^{-1}\Delta\vec{r} \quad (11)$$

5) The initial values $x^0, y^0, z^0, s^0$ are updated by $\Delta x, \Delta y, \Delta z, \Delta s$ as follows.

$$\begin{aligned} x^1 &= x^0 + \Delta x \\ y^1 &= y^0 + \Delta y \\ z^1 &= z^0 + \Delta z \\ s^1 &= s^0 + \Delta s \end{aligned} \quad (12)$$

6) After updating the initial value to $x^1, y^1, z^1, s^1$, we return to the Procedure 2. These procedures are repeated until $\Delta x, \Delta y, \Delta z, \Delta s$ becomes enough small.

By following the above procedure, our proposed method has the possibility to calculate the receiver's position $(x, y, z)$. In our experience, the solution can converge by repeating several times even if the initial values are $x^0 = y^0 = z^0 = s^0 = 0$.

## IV. Characterization by Field Experiment I

### A. Setup and Environment

The experiments were conducted in order to evaluate the ability and the effectiveness of the proposed method. In the proposed method, the position estimation is performed while updating the traveling distance and the previous position. The assumed environment is an urban area. There are many buildings in urban areas. The satellites with low elevations tend to be shaded by the buildings. As the worst case, the number of the observable satellites is only three. We use three satellites with high elevations.

In the experiment, we are using a bicycle as the moving vehicle. The bicycle is shown in Figure 2. As shown in Figure 2, a GPS antenna is attached to the loading platform. Also, the cycle computer is attached to the front wheel. The cycle computer is shown in Figure 3. The cycle computer is a device that senses a magnet mounted on the spokes of the tire to generate a pulse after each rotation. The bicycle also has a data logger system. Each generated pulse is saved in the data logger. The sampling interval of the data logger is 1 ms. The example of the saved pulses is shown in Figure 4. From Figure 4, the cycle computer outputs 0V when the magnet passes the front of the sensor. Except above, the cycle computer usually outputs 2V. We

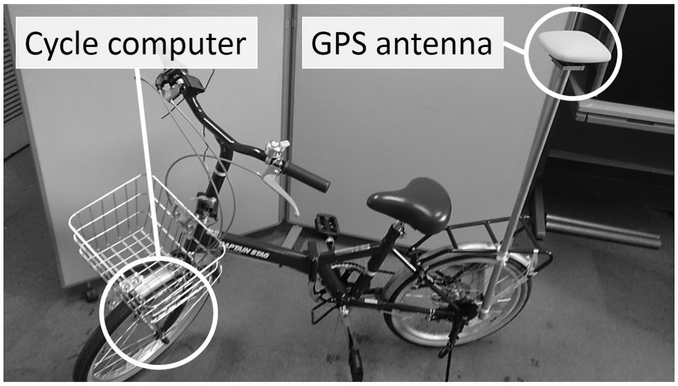


Figure 2. Experimental vehicle.

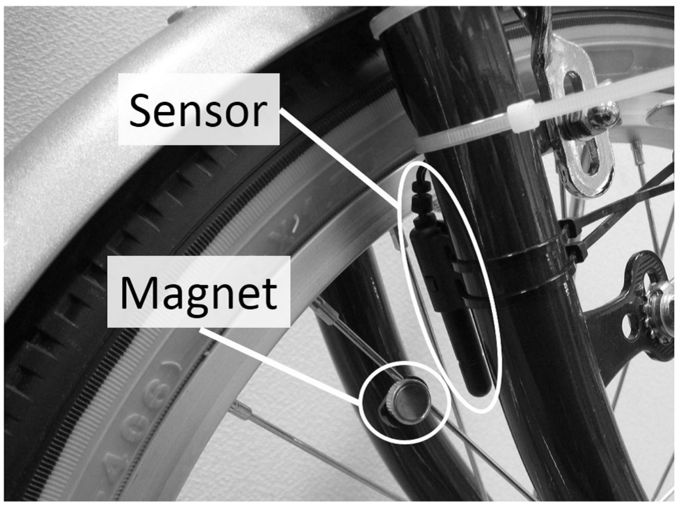


Figure 3. Cycle computer.

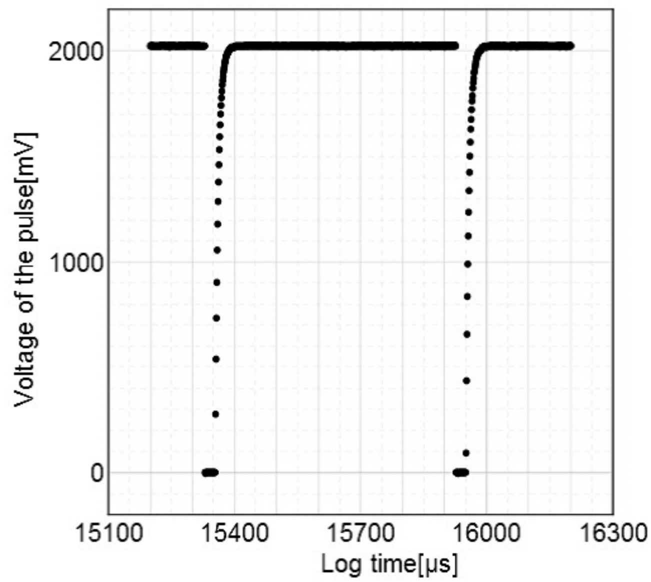


Figure 4. Waveform from the cycle computer.

TABLE I. SPECIFICATIONS OF THE EXPERIMENT ENVIRONMENT

| | |
|---|---|
| GPS receiver | JAVAD GNSS DELTA-G3T |
| Number of observable satellites | 8 satellites |
| Total traveling time | 50 s |
| Estimation interval of GPS receiver | 1 s |
| Data logger | EasySync DS1M12 |
| Cycle Computer | CATEYE CC-VL820 VELO 9 |
| Sampling interval of the data logger | 1 ms |
| Circumference of the tire of the bicycle | 1.515m |

can recognize the rotation of the tire by the pulses. Based on these pulses, the distance $r_0$ can be measured. The duration between an edge of a pulse and that of the next pulse is equal to the circumference of the tire. The traveling distance $r_0$ is calculated for each position estimation by using the circumference of the tire.

The experiment has been conducted under an open sky. The distance $r_0$ had measured while traveling by the bicycle. The position estimation of the proposed method is performed using the three satellites with high elevation and the previous position. For comparison, the positions are also estimated by the conventional method with all the observable satellites. As mentioned in Section II, there are the past researches. However, in this paper, we compare the stand-alone GPS estimation as standard.

A total distance of the experimental riding is 100 meters. In the first 20 meters, we rode toward east straightly. Then, we turned right. In the next 80 meters, we rode toward south straightly again. The cycle computer has a function that displays the speed according to the rotational speed of the tire. We kept the speed of the bicycle 10 km/h. From the start to the end, the time is 50 seconds. Table I summarizes the parameters of the experiment environment.

By using the data obtained in the experiment, the position estimation is performed per second. The GPS receiver can output the distance between the receiver and the satellites. The output distance includes some errors, such as the ionospheric delay error and the tropospheric delay error (so, the output distance is often called as the pseudo range). The ionospheric delay can be estimated by the transmitted messages from the satellites because the messages have coefficients of equations, which are modeled as the ionospheric delay. So, in this paper, we subtract the modeled ionospheric delay from the output distance. Similarly, we subtract the modeled tropospheric delay from the output distance. We use the remaining distance as the distance $r_i$ For comparison, the position estimation using all satellites was also calculated per second. In the proposed method, the position coordinates of the start is estimated by using the four satellites with high elevation.

## B. Position Estimation Results

Figure 5 shows the results of the proposed and conventional method of position estimation. The origin of Figure 5 is the starting point. The positioning results are plotted every second. According to Figure 5, by using the proposed

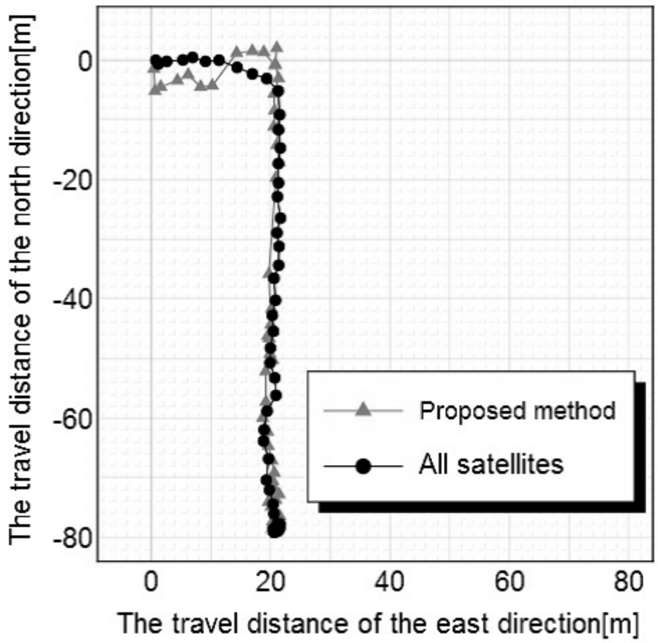


Figure 5. Position estimation results of all satellites and the proposed method.

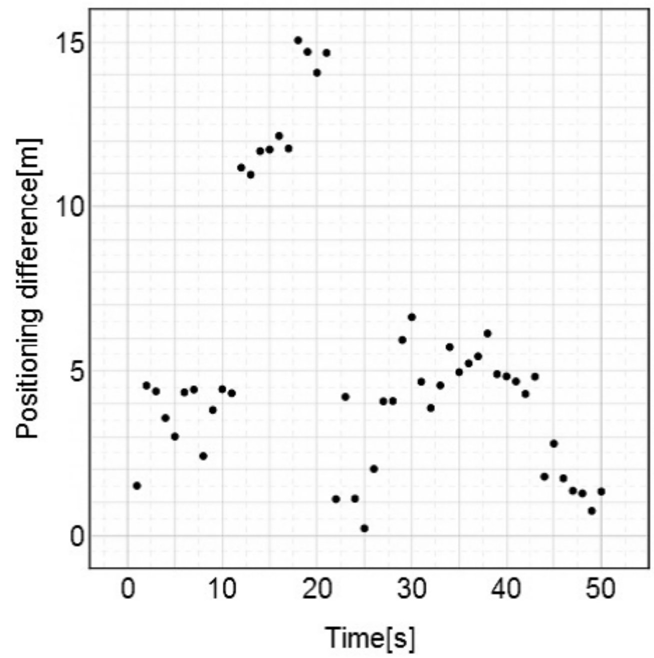


Figure 6. Positioning difference.

method, the position estimation can be performed even when the number of the observable satellites are three. First, the bicycle is moving towards east, from the start point. After going 20 meters straightly, the direction is changed to the south. Finally, the vehicle stops when the traveling distance becomes 80 meters from the turned point. As we can see, the proposal method can keep estimating the position when the direction of the traveling is changed while traveling.

In Figure 6, the positioning difference between the estimated position by the proposed method and the position by all the satellites is shown. The positioning difference is defined as Euclidean distance between both the positions. Figure 6 is plotted every second. The total traveling time is 50 seconds. The positioning difference is 5 meters or less until 11 seconds from the start time. In addition, from 22 seconds to 50 seconds, the positioning difference also under 5 meters. Also, a few samples are 7 meters or less. From 12 seconds to 21 seconds, the positioning difference is over 10 meters.

As another viewpoint of discussion, Figure 7 shows the cumulative probability distribution in order to check the distribution of the positioning difference. From Figure 7, 76 percent of the positioning differences are 5 meters or less. The rate of the positioning differences over 10 meters is about 17 percent.

By considering these results, the proposed method is able to estimate the receiver's position by using the previous position and the three satellites with high elevation. However, there are some cases when the positioning differences are more than 10 meters. These large differences are a problem that will have to be resolved. One of the above reasons is the satellites constellation. Generally, in case of the four satellites, the good satellites constellation

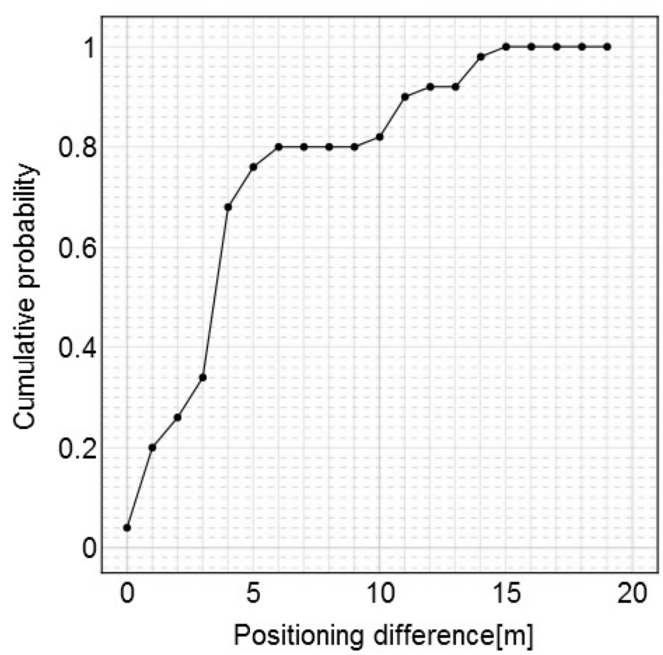


Figure 7. Cumulative probability distribution of the positioning difference.

can be presented as follows [17].

- One satellite is near zenith, that is, with high elevation.
- Other three satellites are distributed and surrounded uniformly with low elevations.

In this experiment, three satellites with high elevation have been selected. A better selection has to be consider for the above appropriate satellite constellation. By a better selection, we expect that the proposal can become better.

We now investigate other reasons why the large differences occur. In this paper, the properties were evaluated by comparing the position estimation results of using all satellites. However, the measurement error of the estimation results by all satellites may be included. In oder to investigate in more detail, it is necessary to evaluate the characteristics by comparing the true position with the proposed method.

The proposed process is not much different from the conventional process. In the proposed method, we just use the traveling distance instead of the range from a satellites. So, there is no big difference in calculation time compared to the conventional positioning. We hope that our proposal can be calculated in real-time.

## V. Application of proposal

The proposed method in Section III was used for three satellites. In this section, we will apply the assistance of the distance sensor by the proposal for estimating own position by available all satellites. By using both the distance sensor's data and the previous position, we can add the quasi-satellite to the available satellites. That is, we can increase the number of available satellites. For example, we can estimate own position by 11 satellites (10 satellites and a quasi-satellite) even when we can observe 10 satellites.

The calculation equations are as follows. These are similar to (8).

$$\begin{cases} \Delta r_1 = \frac{\partial r_1}{\partial x}\Delta x + \frac{\partial r_1}{\partial y}\Delta y + \frac{\partial r_1}{\partial z}\Delta z + \frac{\partial r_1}{\partial s}\Delta s \\ \Delta r_2 = \frac{\partial r_2}{\partial x}\Delta x + \frac{\partial r_2}{\partial y}\Delta y + \frac{\partial r_2}{\partial z}\Delta z + \frac{\partial r_2}{\partial s}\Delta s \\ \Delta r_3 = \frac{\partial r_3}{\partial x}\Delta x + \frac{\partial r_3}{\partial y}\Delta y + \frac{\partial r_3}{\partial z}\Delta z + \frac{\partial r_3}{\partial s}\Delta s \\ \vdots \\ \Delta r_N = \frac{\partial r_N}{\partial x}\Delta x + \frac{\partial r_N}{\partial y}\Delta y + \frac{\partial r_N}{\partial z}\Delta z + \frac{\partial r_N}{\partial s}\Delta s \\ \Delta r_0 = \frac{\partial r_0}{\partial x}\Delta x + \frac{\partial r_0}{\partial y}\Delta y + \frac{\partial r_0}{\partial z}\Delta z \end{cases} \tag{13}$$

The variable $r_n, (n = 1, 2, \cdots, N)$ means the distances between the receiver and the satellites. The variable $N$ is the number of the used satellites.

As mentioned before, the distance $r_n$ is often called as the pseudo range because the distance $r_n$ includes errors. The lower the elevation angle of the satellite is, the larger these errors become. Generally, the relation between the standard deviation of the errors in the pseudo range and the elevation angle can be approximated as:

$$\sigma(\theta) = \frac{0.8}{\sin\theta}, \tag{14}$$

where the variable $\theta$ means the elevation angle of the satellite [18]. In the positioning processes, we have to consider the bad influence by the above error. So, the calculation equations are expanded into the weighted equations as follows [18]:

$$\begin{cases} \frac{1}{\sigma_1}\Delta r_1 = \frac{1}{\sigma_1}\frac{\partial r_1}{\partial x}\Delta x + \frac{1}{\sigma_1}\frac{\partial r_1}{\partial y}\Delta y + \frac{1}{\sigma_1}\frac{\partial r_1}{\partial z}\Delta z + \frac{1}{\sigma_1}\frac{\partial r_1}{\partial s}\Delta s \\ \frac{1}{\sigma_2}\Delta r_2 = \frac{1}{\sigma_2}\frac{\partial r_2}{\partial x}\Delta x + \frac{1}{\sigma_2}\frac{\partial r_2}{\partial y}\Delta y + \frac{1}{\sigma_2}\frac{\partial r_2}{\partial z}\Delta z + \frac{1}{\sigma_2}\frac{\partial r_2}{\partial s}\Delta s \\ \frac{1}{\sigma_3}\Delta r_3 = \frac{1}{\sigma_3}\frac{\partial r_3}{\partial x}\Delta x + \frac{1}{\sigma_3}\frac{\partial r_3}{\partial y}\Delta y + \frac{1}{\sigma_3}\frac{\partial r_3}{\partial z}\Delta z + \frac{1}{\sigma_3}\frac{\partial r_3}{\partial s}\Delta s \\ \vdots \\ \frac{1}{\sigma_N}\Delta r_N = \frac{1}{\sigma_N}\frac{\partial r_N}{\partial x}\Delta x + \frac{1}{\sigma_N}\frac{\partial r_N}{\partial y}\Delta y + \frac{1}{\sigma_N}\frac{\partial r_N}{\partial z}\Delta z + \frac{1}{\sigma_N}\frac{\partial r_N}{\partial s}\Delta s \\ \frac{1}{\sigma_0}\Delta r_0 = \frac{1}{\sigma_0}\frac{\partial r_0}{\partial x}\Delta x + \frac{1}{\sigma_0}\frac{\partial r_0}{\partial y}\Delta y + \frac{1}{\sigma_0}\frac{\partial r_0}{\partial z}\Delta z \end{cases} \tag{15}$$

In order to calculate the compensation amount $\Delta\vec{x} = [\Delta x, \Delta y, \Delta z, \Delta s]^{\mathrm{T}}$, we used (11). In this section, we can determine the compensation amount as follows:

$$\Delta\vec{x} = \left(G^T W G\right)^{-1} G^T W \Delta\vec{r}, \tag{16}$$

where

$$W = \begin{pmatrix} \frac{1}{\sigma_1} & 0 & \cdots & 0 & 0 \\ 0 & \frac{1}{\sigma_2} & \cdots & 0 & 0 \\ \vdots & & \ddots & & \\ 0 & 0 & \cdots & \frac{1}{\sigma_N} & 0 \\ 0 & 0 & \cdots & 0 & \frac{1}{\sigma_0} \end{pmatrix}. \tag{17}$$

By using the above compensation, we can update the values $x, y, z, s$ such as (12).

## VI. Characterization by Field Experiment II

### A. Setup and Environment

In order to confirm and evaluate the characteristics of the extended method in Section V, we conduct other simulations in addition to Section IV. In Section IV, we simulated the case that the number of satellites are three. In this section, we will apply our proposal to the estimation under available all satellites. By using the distance sensor's data, we add the quasi-satellite to the available satellites.

In urban canyon, the following problems are famous.

1) There are many tall buildings. In case of satellites with high elevations, a receiver can receive direct path signals from the satellites in line-of-sight. However, in case of satellites with middle elevations, the signal tends to be shielded by the buildings. So, the receiver cannot receive direct-path-signal. In case of indirect-path signal, estimated positions have large errors.
2) Also, the signals from satellites with low elevations tend to be shielded by the buildings. So, the propagated path includes large errors. Then the estimated positions have large errors.

By using our proposal, we can prepare the quasi-satellite. So, we can use the quasi-satellites instead of the above bad satellites.

The following simulations use the real satellites information that is measured in Section VI. The satellites constellaton, which is recorded in the measurement, is shown in Figure 8. The number of available satellites are eight. In the simulations, the weight of the quasi-satellite by the wheel sensor is set as 1. In other words, the weight $W$ in (17) is set as 1.

In this section, we simulated the following two scenarios.

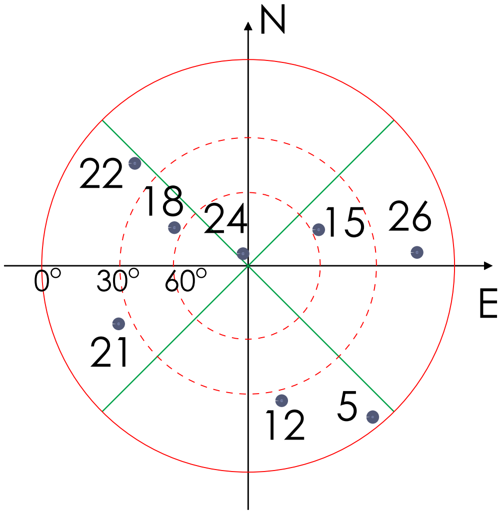


Figure 8. Satellites constellation.

Case A The satellite number PRN:18 includes large error. The elevation of the satellite PRN:18 is 58°, that is middle elevation. In order to simulate including the error, we add +10 meters to the measured range intentionally. In the proposal, the position can be estimated by using the quasi-satellite instead of PRN:18.

Case B The satellite number PRN:5 includes large error. The elevation of the satellite PRN:5 is 7°, that is very low elevation. In order to simulate including the error, we also add +10 meters to the measured range intentionally. In the proposal, the position can be estimated by using the quasi-satellite instead of PRN:5.

In both Case A and Case B, we treat the positioning result by all 8 satellites as true positions because the measurement is conducted under open-sky.

### *B. Position Estimation Results (Case A)*

In Case A, the simulated results are shown in Figures 9 - 12. Figure 9 shows the plots that are estimated positions by both all 8 satellites and 8 satellites (10 meters is added to the range of PRN:18). Figure 11 shows the plots that are estimated positions by both all 8 satellites and 7 satellites (proposal without PRN:18). Figures 10 and 12 show the positioning differences to all 8 satellites.

Figure 9 shows the plots that are estimated positions by both all 8 satellites and 8 satellites (10 meters is added to the range of PRN:18). The origin is the start point. The positioning results are plotted every second. In results of the all 8 satellites, the bicycle is moving towards east, from the origin. After going 20 meters straightly from the start point, the direction is changed to the south. The bicycle stops when the traveling distance becomes 80 meters from the turned point. In results of the 8 satellites (10 meters is added to the range of PRN:18), the start point is 5 meters away from the origin. The trajectory is almost similar to that of the all 8 satellites case. The bicycle stop point of

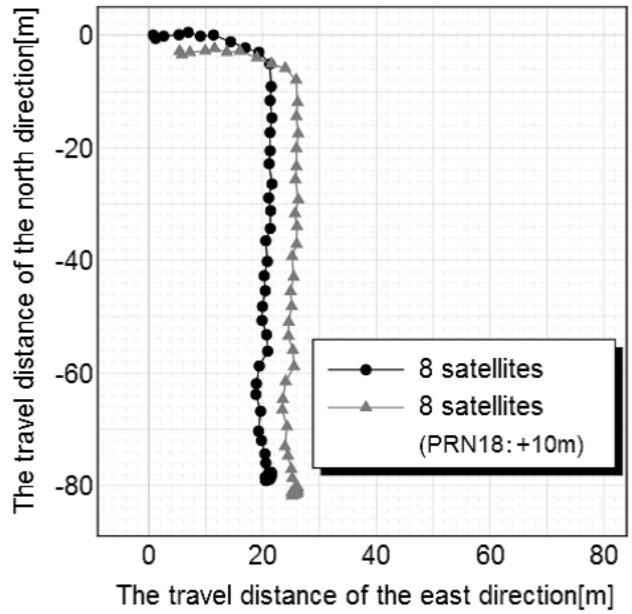


Figure 9. Positioning results (8 satellites vs 8 satellites(PRN 18: Indirect Path)).

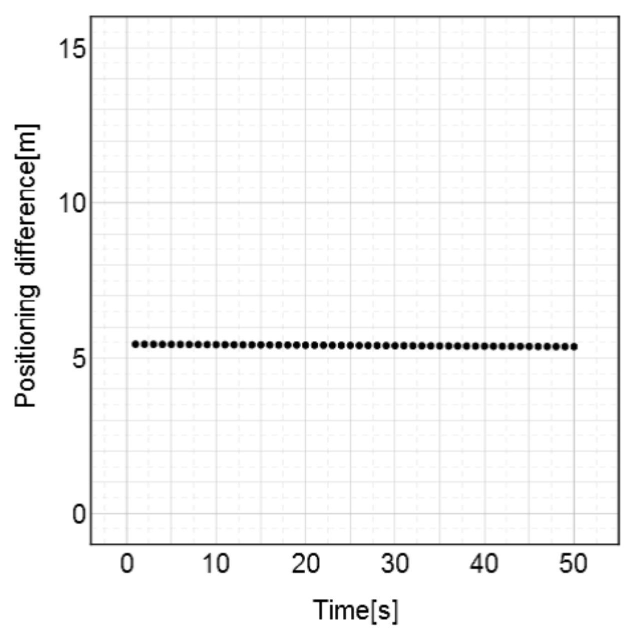


Figure 10. Positioning differences (8 satellites vs 8 satellites(PRN 18: Indirect Path)).

8 satellites (10 meters is added to the range of PRN:18) is 5 meters away from the stop point of all 8 satellites.

Figure 10 shows the positioning differences between the estimated position by the all 8 satellites and 8 satellites (10 meters is added to the range of PRN:18). Figure 10 is plotted every second. The total traveling time is 50 seconds. The positioning differences are around 6 meters or less from the start point to the stop point. The plots

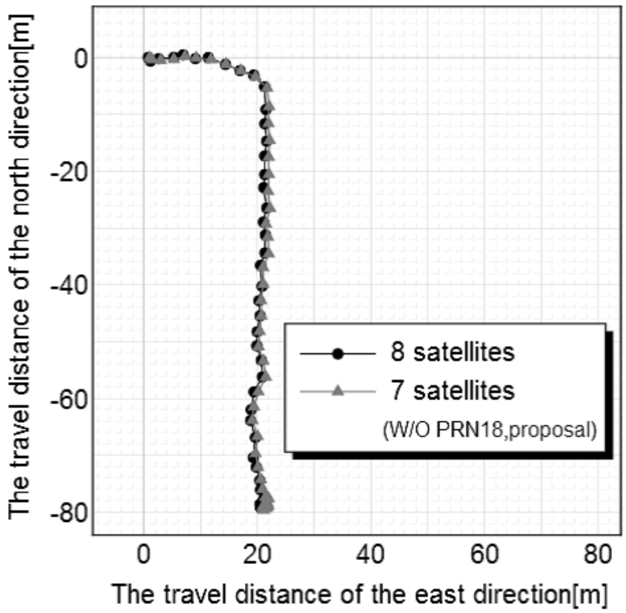


Figure 11. Positioning results (8 satellites vs 7 satellites(w/o PRN 18, proposal)).

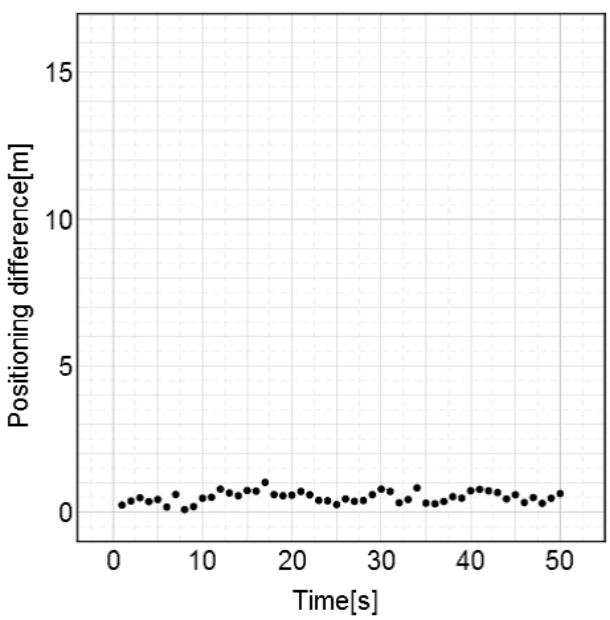


Figure 12. Positioning differences (8 satellites vs 8 satellites(w/o PRN 18, proposal)).

do not have large oscillation. The reason may be that the added error is a constant value. Average of the positioning difference is 5.40 meters. The positioning difference result must be influence by adding +10 meters.

Figure 11 shows the plots that are estimated positions by both all 8 satellites and 7 satellites (proposal without PRN:18). The origin is the start point. The positioning results are plotted every second. The result of the all 8 satellites case is same to that of all 8 satellites case in Figure 9. And, the results of the 7 satellites (proposal without PRN:18) are the result like the all 8 satellites.

In Figure 12, the positioning differences between the estimated position by the all 8 satellites and 7 satellites (proposal without PRN:18) are shown. Figure 12 is plotted every second and the total traveling time is 50 seconds. The positioning difference is around 1 meters or less from the starts to stop point. Average of the positioning difference is 0.51 meters.

By considering these results, the proposed method is effective in case A. Because, average of positioning difference in Figure 12 is less than that in Figure 10. The results suggest that the assistance of the distance sensor is effective instead of use of the satellite that includes large error. That is, the proposed method is effective.

### *C. Position Estimation Results (Case B)*

In Case B, the simulated results are shown in Figures 13 - 16. Figure 13 shows the plots that are estimated positions by both all 8 satellites and 8 satellites (10 meters is added to the range of PRN:5). Figure 15 shows the plots that are estimated positions by both all 8 satellites and 7 satellites (proposal without PRN:5). Figures 14 and 16 show the positioning differences to all 8 satellites in case of the above simulations respectively.

Figure 13 shows the plots that are estimated positions by both all 8 satellites and 8 satellites (10 meters is added to the range of PRN:5). The origin is the start point. The positioning results are plotted every second. The moving of the bicycle is same in case of Section IV-B.

Figure 14 shows the positioning differences between the estimated position by the all 8 satellites and 8 satellites (10 meters is added to the range of PRN:5). Figure 14 is plotted every second. The total traveling time is 50 seconds. The positioning differences are almost constant because the added error is constant. Average of the positioning difference is about 0.26 meters.

Figure 15 shows the plots that are estimated positions by both all 8 satellites and 7 satellites (proposal without PRN:5). The plots in case of all 8 satellites is same to that in Figures 9, 11, and 13. The positioning results are plotted every second. The results of the 7 satellites (proposal without PRN:5) are close to the results of the all 8 satellites.

In Figure 16, the positioning differences between the estimated position by the all 8 satellites and 7 satellites (proposal without PRN:5) are shown. Figure 16 is plotted every second. Average of the positioning differences are 0.29 meters.

Comparing Figures 14 and 16, the rate in case that the positioning differences of the proposal are lower than that of the conventional with PRN:8, which includes the ranging error is 48%. If the satellites that include errors are enough low angle, the calculation process decides that the weight in (17) is extremely low. So, without our proposal, the influence of the satellite with low angle is not big problem. That is, the effective of the proposal is low. We can find that the proposal is always not effective.

## VII. Conclusion and Future Work

In this paper, we proposed the novel positioning algorithm that used not only the GPS satellites but also the

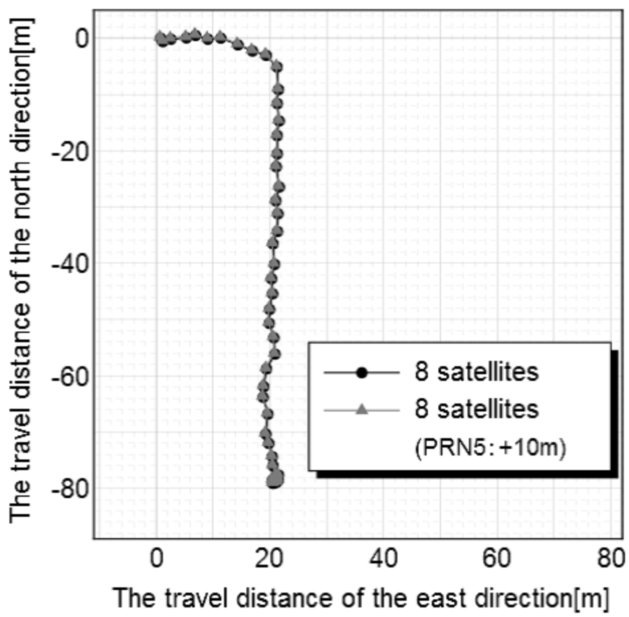


Figure 13. Positioning results (8 satellites vs 7 satellites(PRN 5: Indirect Path)).

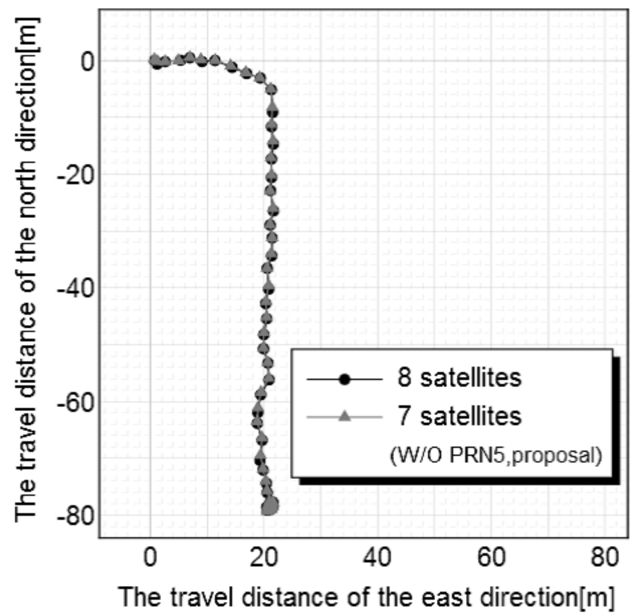


Figure 15. Positioning results (8 satellites vs 7 satellites(w/o PRN 5, proposal)).

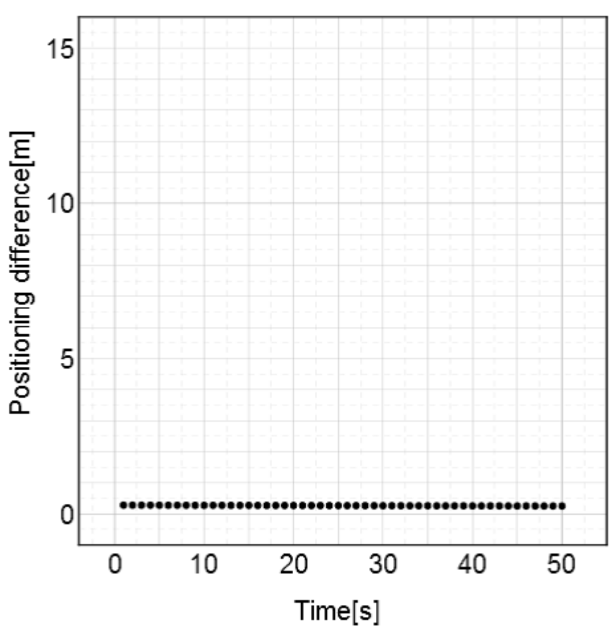


Figure 14. Positioning differences (8 satellites vs 8 satellites(PRN 5: Indirect Path)).

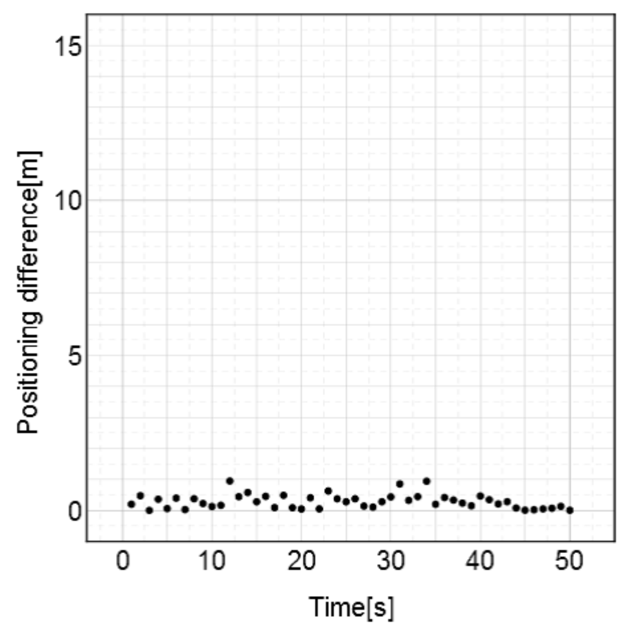


Figure 16. Positioning differences (8 satellites vs 8 satellites(w/o PRN 5, proposal)).

distance sensor information. In other words, the distance sensor assisted the GPS calculation process. By using our proposal, the problem that degrades GPS signals in urban canyon is improved. Because the proposal used the previous own position and the traveled distance that was measured by the distance sensor, the proposal can regard the sensor information as the quasi-satellite. The typical problem in urban area is the decrement of the observable satellites. The proposal can add the quasi-satellite. So, we can keep estimating own position even when the number of the observable satellites becomes low. Also, in case there are adequate number of the observable satellites, the quasi-satellite can help the conventional positioning. As a future work, we will try effective usage of the quasi-satellite. For example, we will find effective combination of the real satellites and quasi-satellite.

In order to evaluate the proposed method, we evaluated two cases of both insufficient number of satellites and adequate number of satellites. In the insufficient case, we confirmed that the proposal was able to keep estimating own position robustly. In the adequate case, we confirmed that the quasi-satellite instead of the bad satellite that includes errors can reduce the positioning errors but the proposal was always not effective.

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