Concepts for Computing Patterns in 15th Century Korean Music

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***Abstract*—Computational musicology denotes the use of computers for analyzing music. This paper proposes applying techniques from stringology for analyzing classical Korean music. Historically, Sejong, the fourth king of the Joseon Dynasty in Korea, intended to rule the country with courtesy and music following the teaching of Neo-Confucianism. For this, he invented a music score in which music could be written. He made the structure of the music score based on the meaning of Neo-Confucianism, and recorded contemporary music with two notation patterns. In this paper, we first study the patterns in the structure of the music score and then investigate the notation patterns by which contemporary music was recorded in the music score. Finally, we establish links between these musical patterns and computing pattern inference for music via the field of stringology. Future research directions are outlined.**

Keywords–Korean music; Lyndon word; Musicology; Notation pattern; Stringology; Structure of music score; V-word.

I. Introduction

Musicology is the study of music through scholarly analysis and research methodologies. ***Computational musicology*** is the use of computers in order to study music and integrates musicology, computer science and stringology (the study of strings). This interdisciplinary research area includes music information retrieval, pattern matching and music informatics. This paper introduces the computational study of patterns occurring in Korean music and proposes future research directions for this endeavour. We first overview the background for these musical patterns in the structure of the music score and the notation patterns devised for recording Korean music and then describe connections to the field of stringology.

King Sejong (reign 1418–1450) introduced 'yeack' ideology with the goal of ideal Confucian politics. Yeack ideology means ruling the subjects by courtesy and music rather than by punishments. For this he invented a form of music score that has been passed down to this day. This music score is a full score that notates lyric and various musical instruments. Moreover, it is a detailed music score that notates both one-third and one-fourth beats. The music score was improved one step further by his son, King Sejo, and then it has evolved with gradual changes up until now.

King Sejong made the structure of the music score based on the meaning of Neo-Confucianism, and recorded contemporary music with notation patterns fitting to each rhythm. Without understanding the notation patterns, therefore, the music recorded in the music score cannot be interpreted properly. Hence, we first study the structure of the music score and its meaning in Neo-Confucianism. Then, we investigate the notation patterns recorded in the music score and the corresponding rhythms of the 15th century Korean music.

The rest of the paper is arranged as follows. Section II explores the meaningful structure of the music scores of the Joseon Dynastry. Section III identifies the patterns of the music scores. Section IV discusses computational Korean musicology and applications of the stringology in musicology. Section V provides the summary and future directions.

II. The Structure of the Music Score and Its Meaning

A. Structure of the music score

The music score of the Joseon Dynasty (the version established by King Sejo) has the structure shown in Figure 1. Small squares form a vertical column, and vertical columns proceed (are read) from right to left. A small square is called a ***jeonggan*** and a vertical column is called a ***haeng***. A vertical column is divided into 6 ***daegangs*** by thick lines. A daegang consists of either three squares or two squares.

Figure 1 shows a full score, and thus five columns make one column set, in which the first column from right represents a string instrument (melody), the second column a wind instrument, the third column percussion instrument 1, the fourth column percussion instrument 2, and the leftmost column the lyric. A note of melody is notated by a pitch name, and percussion is notated by symbols of strokes. The black area at the beginning of the score means that this piece of music has an incomplete bar. This music score is called a ***jeongganbo***.

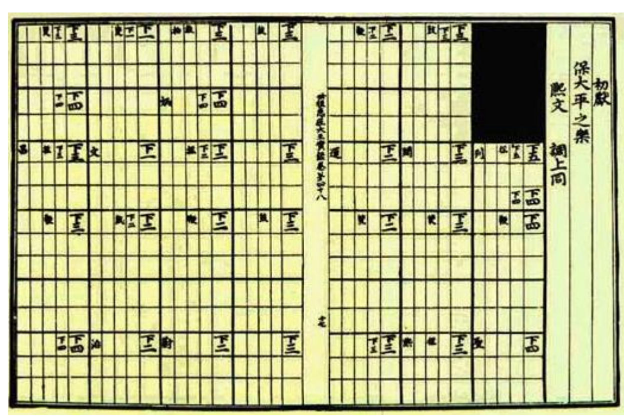

Figure 1. Structure of the music score

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If a vertical column in Figure 1 is drawn horizontally, it looks like Figure 2, i.e., a haeng consists of six daegangs, each of which consists of either 3 jeonggans or 2 jeonggans. Therefore, the music score in Figure 1 has the pattern of $(3\ 3\ 2\ 3)^5(3\ 2\ 3\ 3\ 2\ 3)^5(3\ 2\ 3\ 3\ 2\ 3)^5\cdots$, where (3 3 2 3) refers to the 3, 3, 2, and 3 squares in the rightmost column, and $(3\ 3\ 2\ 3)^5$ refers to the column set consisting of the rightmost five columns (under the black box).

Figure 2 shows the names in the structure of the music score.

haeng (column: 行)															
daegang			daegang		daegang			daegang			daegang		daegang		
jg	jg	jg	jg	jg	jg	jg	jg	jg	jg	jg	jg	jg	jg	jg	jg

Figure 2. The names in the structure of the music score

B. The meaning of the structure of the music score

The Neo-Confucian meaning of the music score is embedded in the number of squares. This music score consists of repetitions of 3+2+3=8 squares. In Figure 1, one haeng (i.e., column) consists of 16 squares, but in the original score made by King Sejong one haeng consisted of 32 squares.

Figure 3 shows the meaning of the structure of the music score.

year											
music						music					
8 diagrams spring			8 diagrams summer			8 diagrams fall			8 diagrams winter		
heaven	hum an	earth	heaven	hum an	earth	heaven	hum an	earth	heaven	hum an	earth
3	2	3	3	2	3	3	2	3	3	2	3

Figure 3. The meaning of the structure of the music score

The Neo-Confucian meanings of the numbers in the music score are shown in Figure 3. The numbers 3, 2, and 3 in a group of 8 squares mean heaven, human, and earth, respectively, and 8 means 8 diagrams from Neo-Confucianism, that is, one season. The number 16 means music, which is a fundamental doctrine of politics. One haeng (i.e., 32 squares) means one year consisting of spring, summer, fall, and winter, and it is repeated like a year [1], [2]. In this way, the structure of the music score was designed based on the meanings of Neo-Confucianism.

There has been research on the rhythm interpretation of this music score since the late 1950s. At first, there was a theory that interprets one square as the unit of beat [3], but it could not be used to play the music because the rhythm of music interpreted by the theory was strange. Later, this theory was slightly generalised to the theory that each square has the same length in rhythm [4], [5], but it had similar problems. Condit [6] and Hong [7] also proposed theories to interpret the rhythm of the music score, but these did not reflect the characteristics of Korean music.

III. Notation Patterns

There were two types of music in the 15th century Joseon: *hyangak* and *dangak*. Dangak is the music that came from China, and its lyric was written in Chinese characters. Hyangak is the indigenous music of Korea, and its lyric was written in the Korean language. The music of the 15th century Joseon was recorded in the music score with the following two notation patterns [8], [9]. Dangak was recorded with notation pattern 1, and hyangak was mostly recorded with notation pattern 2.

- Notation pattern 1: Melody, percussion, and lyric are notated in the unit of 8 squares.
- Notation pattern 2: Melody, percussion, and lyric are notated in the unit of 5 squares and 3 squares.

We will investigate each notation pattern by examining a representative music score of the notation pattern. The music score is a full score that has the column set consisting of 4 columns representing melody, percussion 1, percussion 2, and lyric.

A. Notation pattern 1: Rhythm with binary subdivision of a beat

Notation pattern 1 has the unit of 8 squares, which makes one beat, and one beat is subdivided into two half-beats (i.e., binary subdivision of one beat). But the binary subdivision of the beat is not easy to record in the structure of 3+2+3=8 squares. In notation pattern 1, therefore, 3+2=5 squares represent a half-beat, and the following 3 squares represent the second half-beat. In the first half-beat, the first 3 squares represent a quarter-beat, and the following 2 squares another quarter-beat. In the second half-beat, the first 2 squares represent a quarter-beat, and the following 1 square another quarter-beat. Hence, notation pattern 1 has the rhythm shown in Figure 4. That is, the pattern $(((32)(21))((32)(21)))^n$ in the music score is interpreted as the rhythm $(((aa)(aa))((aa)(aa)))^n$, where 'a' denotes a quarter-beat.

Figure 4 shows Notation Pattern 1.

Figure 4. Notation Pattern 1

Gimyeong (基命), an ancient song, which is a representative music passage in notation pattern 1, and its translation into Western stave notation are shown in Figure 5. In this music score, melody, percussions, and lyric are notated in the unit of 8 squares with occasional half-beats in melody. Each Chinese character in lyric lasts two beats. In dangak, each Chinese character has the same length in rhythm in most cases because Chinese characters have almost equal weights in meaning. All pieces of music in dangak were recorded in notation pattern 1, which fits well with Chinese traditional music that favours the binary subdivision of the beat.

B. Notation pattern 2: Rhythm with ternary subdivision of a beat

Notation pattern 2 has the unit of 5 squares and 3 squares, which makes one beat, and one beat is subdivided into 3 one-third beats (i.e., ternary subdivision of one beat). The

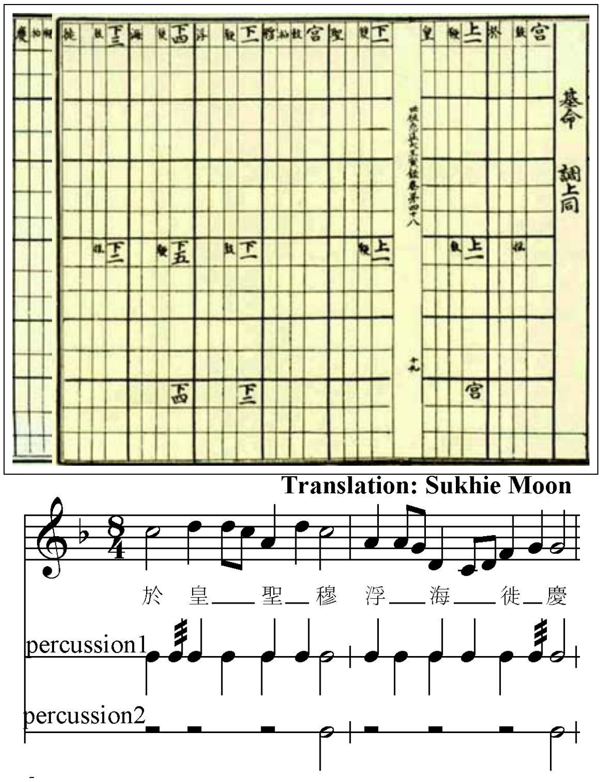

Figure 5. Gimyeong

ternary subdivision of the beat is also not easy to record in the structure of 3+2+3=8 squares. In notation pattern 2, 3+2=5 squares represent one beat, and the following 3 squares represent another beat. In the first beat of 5 squares, each of the first 2 squares, the following 1 square, and the last 2 squares represent a one-third beat. In the second beat of 3 squares, each square represents a one-third beat. Therefore, notation pattern 2 has the rhythm shown in Figure 6, where one beat is denoted by '♩.' for notational convenience. That is, the pattern $((212)(111)(212)(111))^n$ in the music score is interpreted as the rhythm $((bbb)(bbb)(bbb)(bbb))^n$, where 'b' denotes a one-third beat.

Figure 6 shows Notation Pattern 2.

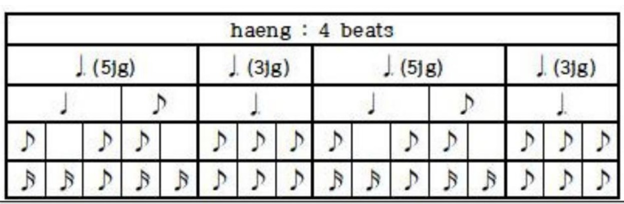

Figure 6. Notation Pattern 2

Cheongsanbyeolgok (青山別曲), a song of the Goryeo dynasty, which is a representative music passage in notation pattern 2, and its translation into Western stave notation are shown in Figure 7. In this music score, melody, percussions, and lyric are notated in the unit of 5 squares and 3 squares. It can be seen that Korean letters in lyric have different lengths in rhythm. This piece of music is hyangak, descended from the Goryeo dynasty, and was recorded in notation pattern 2, which fits well with indigenous music 'hyangak' that favours the ternary subdivision of the beat.

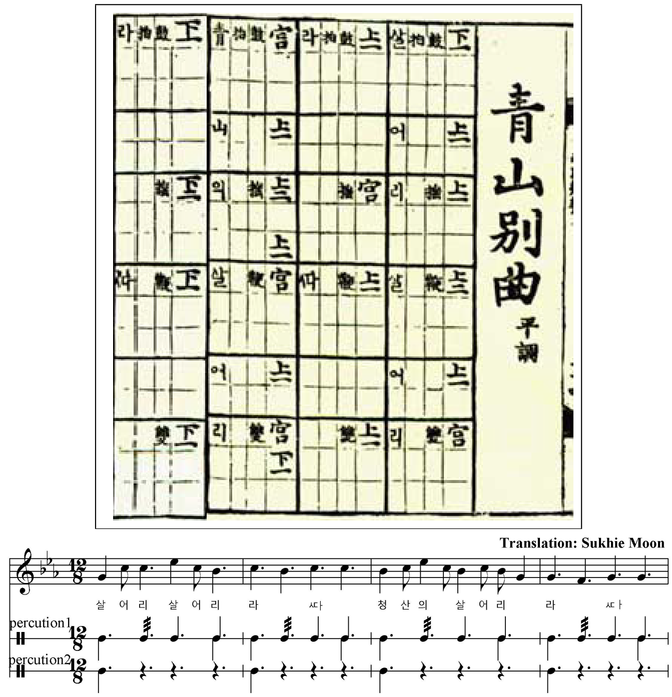

Figure 7. Cheongsanbyeolgok

Following this historical narrative on the creation, development and interpretation of Korean music scores, we proceed to establish a framework suitable for related computational music analysis using techniques from stringology.

IV. Overview of Computational Korean Musicology

A. Stringology notation

Stringology is the mathematical study of strings of data, that is sequences of symbols. Formally, given an integer $n \geq 1$ and a nonempty set of symbols Σ (bounded or unbounded), a ***string of length n***, equivalently ***word***, over Σ takes the form $\boldsymbol{x} = x_1 ... x_n$ with each $x_i \in \Sigma$. For brevity, we write $\boldsymbol{x} = \boldsymbol{x}[1..n]$ with $\boldsymbol{x}[i] = x_i$. We will assume that Σ is a totally ordered alphabet. The length n of a string $\boldsymbol{x}$ is denoted by $|\boldsymbol{x}|$. The set Σ is called an ***alphabet*** whose members are ***letters*** or ***characters***, and Σ^+ denotes the set of all nonempty finite strings over Σ. The ***empty string*** of length zero is denoted ε; we write $\Sigma^* = \Sigma^+ \cup \{\varepsilon\}$ and let $|\Sigma| = \sigma$. We use exponents to denote repetition, for instance if $\alpha \in \Sigma$ then α^3 means $\alpha\alpha\alpha$. If $\boldsymbol{x} = \boldsymbol{uwv}$ for strings $\boldsymbol{u}, \boldsymbol{w}, \boldsymbol{v} \in \Sigma^*$, then $\boldsymbol{u}$ is a ***prefix***, $\boldsymbol{w}$ is a ***substring*** or ***factor***, and $\boldsymbol{v}$ is a ***suffix*** of $\boldsymbol{x}$; we say $\boldsymbol{u} \neq \boldsymbol{x}$ is a ***proper prefix*** and similarly for the other terms. If $\boldsymbol{x} = \boldsymbol{uv}$, then $\boldsymbol{vu}$ is said to be a ***rotation*** (***cyclic shift*** or ***conjugate***) of $\boldsymbol{x}$. A string $\boldsymbol{x}$ is said to be a ***repetition*** if and only if it has a factorization $\boldsymbol{x} = \boldsymbol{u}^k$ for some integer $k > 1$; otherwise, $\boldsymbol{x}$ is said to be ***primitive***. For a string $\boldsymbol{x}$, the reversed string $\overline{\boldsymbol{x}}$ is defined as $\overline{\boldsymbol{x}} = \boldsymbol{x}[n]\boldsymbol{x}[n{-}1] \cdots \boldsymbol{x}[1]$. A string $\boldsymbol{x}$ is a ***palindrome*** if $\boldsymbol{x} = \overline{\boldsymbol{x}}$. A string which is both a proper prefix and a proper suffix of a string $\boldsymbol{x} \neq \varepsilon$ is called a ***border*** of $\boldsymbol{x}$; a string is ***border-free*** if the only border it has is the empty string ε.

We first illustrate this stringology notation for the patterns in music described in Section II. The pattern 3323 is a string over Σ, where Σ is the naturally ordered non-negative integers, and the string has the border 3 and proper prefixes 3, 33, and 332. The string 3323 is a proper suffix of the string 323323 and $(3323)^5$ is a repetition. The string 1 is a factor of each of the strings 111 and 212. The strings 111, 323323 and 212 are all palindromes.

Clearly a string, a sequence of symbols over an alphabet Σ, is a very fundamental and versatile representation of data. A collection of related or collated strings is often referred to as ***text***. A core stringology task is ***pattern matching***, the computation of patterns in strings, and arises in diverse areas of scientific information and text processing, for instance: retrieving information from a database, bioinformatics software utilies for molecular biology investigations, text editors for the Internet, data compression for Big Data, speech recognition, computer vision, computational geometry, and cryptography. Furthermore, the alphabet letters in Σ can be generalized to sets leading to ***indeterminate***, or equivalently ***degenerate***, strings which consist of sequences of nonempty subsets of letters over Σ. For example, $\boldsymbol{x} = 51877392921115$ is a string of integers whereas $\boldsymbol{x} = \{5,3\}\{1,1,1\}\{8,4\}\{3,9,0,9\}\{2,6,1,6\}\{7\}\{1,3,5,7\}$ is a degenerate string over the integers.

The application of stringology algorithms to measuring musical similarity was considered in [10] where it is also expressed that this similarity is a rather subjective measure. For instance, two otherwise identical musical packages might differ only by being played in a different key or some notes in one might have been recorded incorrectly – different definitions of musical similarity could potentially require distinct algorithmic solutions and both edit and hamming distances have been proposed for these kinds of analyses. Approximate matching methods were developed in [10] which measure the distance between musical passages by looking at the distance between the individual notes. String matching techniques such as approximate matching with gaps (allowing skipping of notes), polyphonic matching (searching for a pattern in a set of sequences of notes), and approximate repetitions have also been considered in computational musicology.

Ordering a set of strings is often used to enhance computational efficiency such as with indexing techniques. If Σ is a totally ordered alphabet then ***lexicographic ordering*** (***lexorder***) $\boldsymbol{u} < \boldsymbol{v}$ with $\boldsymbol{u}, \boldsymbol{v} \in \Sigma^+$ means that either $\boldsymbol{u}$ is a proper prefix of $\boldsymbol{v}$, or $\boldsymbol{u} = \boldsymbol{ras}$, $\boldsymbol{v} = \boldsymbol{rbt}$ for some $a, b \in \Sigma$ such that $a < b$ and for some $\boldsymbol{r}, \boldsymbol{s}, \boldsymbol{t} \in \Sigma^*$. We call the ordering $\ll$ based on lexorder of reversed strings ***co-lexicographic ordering*** (***co-lexorder***). Using the ordered Roman alphabet: $compute < computer < music < musicology < score$ while $music \ll score \ll compute \ll computer \ll musicology$. Ordering techniques can also capture patterns in strings.

B. Application of stringology in musicology

In combinatorics on words, ***Lyndon words*** are (generally) finite words (strings) which are lexicographically least amongst all their cyclic rotations.

Definition 1 (Lyndon word): A string $\boldsymbol{x}$ over an ordered alphabet Σ is said to be a ***Lyndon word*** if it is the unique minimum in lexorder $<$ in the conjugacy class of $\boldsymbol{x}$.

Lyndon words are primitive, have deep connections with algebra, and moreover, any string can be factored or decomposed efficiently into Lyndon words [11] including for degenerate strings [12]. Introduced by Lyndon in 1954 as standard lexicographic sequences, Lyndon words have been studied extensively and are finding an increasing range of applications: string combinatorics and algorithmics including specialized matching scenarios, computing the lexorder of substrings, digital geometry, and bioinformatics. Our interest here is applications of Lyndon words to musicology.

Consider the string 3323, which is part of the pattern of the Korean music score in Figure 1. We will show that 3323 is not a Lyndon word while the conjugate 2333 is a Lyndon word.

TABLE I. CONJUGATES OF 3323

3	3	2	3
3	3	3	2
2	3	3	3
3	2	3	3

TABLE II. THE LEXORDER OF THE CONJUGATES OF 3323

2	3	3	3
3	2	3	3
3	3	2	3
3	3	3	2

Although the string 3323 is not a Lyndon word it can be uniquely factored into Lyndon words as (3)(3)(23).

Interestingly, Lyndon words have been applied in the analysis of music repetition, or looping, which is a fundamental feature of music. When enumerating periodic musical structures (repetitions), the computation is done up to a cyclic shift. In this context, two strings, which are cyclic shifts of one another are considered the same, and primitive Lyndon words, provide a means to capture distinct representatives of the structure. In [13], two examples of traditional African repertoires have been analysed using Lyndon words: harp melodic canons played by Nzakara people from the Central African Republic and also asymmetric rhythmic patterns common to many cultures of Central Africa. This stimulates our interest in potential applications of Lyndon words to analysing Korean music.

We now define a non-lexorder called V-order [14]. Let $\boldsymbol{x} = x_1x_2 \cdots x_n$ be a string over Σ. Define $h \in \{1, \ldots, n\}$ by $h = 1$ if $x_1 \leq x_2 \leq \cdots \leq x_n$; otherwise, by the unique value such that $x_{h-1} > x_h \leq x_{h+1} \leq x_{h+2} \leq \cdots \leq x_n$. Let $\boldsymbol{x}^* = x_1x_2 \cdots x_{h-1}x_{h+1} \cdots x_n$, where the star * indicates deletion of x_h. Write $\boldsymbol{x}^{s*} = (...(\boldsymbol{x}^*)^*...)^*$ with $s \geq 0$ stars. Let $g = \max\{x_1, x_2, \ldots, x_n\}$, and let k be the number of occurrences of g in $\boldsymbol{x}$. Then the sequence $\boldsymbol{x}, \boldsymbol{x}^*, \boldsymbol{x}^{2*}, ...$ ends $g^k, ..., g^1, g^0 = \boldsymbol{\varepsilon}$. From all strings $\boldsymbol{x}$ over Σ we form the *star tree*, where each string $\boldsymbol{x}$ labels a vertex and there is a directed edge upward from $\boldsymbol{x}$ to $\boldsymbol{x}^*$, with the empty string $\boldsymbol{\varepsilon}$ as the root.

Definition 2 (V-order): We define V-order $\prec$ between distinct strings $\boldsymbol{x}$, $\boldsymbol{y}$. First $\boldsymbol{x} \prec \boldsymbol{y}$ if in the star tree $\boldsymbol{x}$ is in the path $\boldsymbol{y}, \boldsymbol{y}^*, \boldsymbol{y}^{2*}, \ldots, \boldsymbol{\varepsilon}$. If $\boldsymbol{x}, \boldsymbol{y}$ are not in a path, there exist

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smallest s, t such that $\boldsymbol{x}^{(s+1)*} = \boldsymbol{y}^{(t+1)*}$. Let $\boldsymbol{s} = \boldsymbol{x}^{s*}$ and $\boldsymbol{t} = \boldsymbol{y}^{t*}$; then $\boldsymbol{s} \neq \boldsymbol{t}$ but $|\boldsymbol{s}| = |\boldsymbol{t}| = m$ say. Let $j \in [1..m]$ be the greatest integer such that $\boldsymbol{s}[j] \neq \boldsymbol{t}[j]$. If $\boldsymbol{s}[j] < \boldsymbol{t}[j]$ in Σ then $\boldsymbol{x} \prec \boldsymbol{y}$; otherwise, $\boldsymbol{y} \prec \boldsymbol{x}$. Clearly $\prec$ is a total order on all strings in Σ^*.

To illustrate the star tree using ordered integers, if $\boldsymbol{x} = 53638$, then $\boldsymbol{x}^* = 5368$, $\boldsymbol{x}^{2*} = 568$ and $\boldsymbol{x}^{3*} = 68$; then since 68 is in the tree path we have $68 \prec 53638$.

We now introduce the V-order equivalent of the lexorder Lyndon word:

Definition 3 (V-Word): A string $\boldsymbol{x}$ over an ordered alphabet Σ is said to be a ***V-word*** if it is the unique minimum in V-order $\prec$ in the conjugacy class of $\boldsymbol{x}$.

TABLE III. THE V-ORDER OF THE CONJUGATES OF 3323

3	3	3	2
3	3	2	3
3	2	3	3
2	3	3	3

Hence, we see that although 3323 is not a V-word, and is in fact bordered, the conjugate 3332 of 3323 is a V-word. However, note that each of the Korean music score patterns 32, 21 and 3221 are V-words.

Interestingly, the rhythmic pattern 3 2 2 2 2 3 2 2 2 2 2 occurs in Aka Pygmies music [15], and this pattern forms a V-word.

V. Future Research Directions

This introductory paper has stimulated the flow of numerous research questions and directions regarding the application of stringology techniques to the analysis and processing of Korean music:

- apply the analysis of the musical structure to automated Korean music classification
- design and implement pattern matching techniques optimized for Korean music retrieval tasks
- apply indeterminate or degenerate strings to the pattern matching task of finding chords that match with single notes and analyzing chord progressions
- apply factoring techniques for indeterminate strings to music scores for aiding the identification of meaningful musical sequences
- investigate palindromes in the context of analyzing Korean music
- enumerate periodic Korean musical structures using Lyndon words
- apply V-words to music pattern inference and discovery

VI. Conclusion: From Ideology to Practice

King Sejong invented a form of a music score based on Neo-Confucian ideology. This music score has the structure, which is a repetition of 3+2+3=8 squares, and this structure embeds the meanings of heaven, human, earth, and four seasons in a year. He recorded contemporary music of the 15th century into the music score using two notation patterns. Notation pattern 1 records music with the binary subdivision of one beat, and notation pattern 2 records music with the ternary subdivision of one beat. In the late 16th century, these music scores, which were originally used only in the palaces, were handed down to aristocrats. From this time on the music score gradually lost Neo-Confucian meanings and has been transformed into notation patterns expressing the rhythm of music more directly. In the 20^{th} century the music score 'jeongganbo' finally abandoned Neo-Confucian meanings and became a music score that reflected the rhythm of music exactly.

This paper addresses computational musicology and has initiated the application of stringology techniques for analysing classical Korean musical patterns exhibited in both the structure and notation in the associated music scores. Lyndon words and V-words are mathematical structures with interesting combinatorial properties. By citing examples of patterns in African music which form Lyndon words we raise the question of using Lyndon words to analyse classical Korean music. Further, examples have been given of V-words arising in the structure of a Korean music score which indicate promising directions for further study. The proposed line of research impacts on the digital conservation of Korean culture and heritage.

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