Modular-Based Maintenance for Load-Sharing System with Random Repair Time and Non-Identical Components

Nasser Fard
Department of Mechanical and Industrial Engineering
Northeastern University
Boston, USA
e-mail: n.fard@neu.edu

Yuanchen Fang
Department of Mechanical and Industrial Engineering
Northeastern University
Boston, USA
e-mail: fang.yua@husky.neu.edu

Huyang Xu
Department of Mechanical and Industrial Engineering
Northeastern University
Boston, USA
e-mail: xu.hu@husky.neu.edu

***Abstract*—In this paper, decision-making of repairable load-sharing k-out-of-n is discussed. Decision variables are related to system degradation and restoration. By exploiting these decision variables combinations, the optimal design solution is selected by utilizing weighted principal component analysis based multi-response optimization. The mathematical modeling of the decision-making process is based on the statistical flowgraph model. The statistical flowgraph model is used to describe degradation and restoration with the advantage of computation over the traditional Markovian model. Based on the statistical flowgraphs of different factorial decision variables combinations, the reliability-related measurements of load-sharing system can be evaluated, which correspond to the responses in the multi-response optimization problem.**

Keywords- system reliability; modular design; multi-response optimization.

I. INTRODUCTION

To improve system reliability, the well-known approaches are based on the determination of component reliabilities and their system configuration. That is, the system reliability can be improved by reducing the system complexity, using the highly reliable components and structural redundancy. Additionally, if the system is repairable, then a planned maintenance, repair schedule and repair policy can be used to increase the system availability.

If the computation of system reliability is based on the critical assumption of independent failures among components, the system reliability is determined by applying an appropriate reliability for each component of the system and the rules of probability according to the system configuration. However, when component failures are dependent, more powerful methods, such as Markov analysis, may be needed [1].

Concentrating on a parallel configuration, which is used for including redundant components in the system, if independence is assumed across the components in the system, a failure of any component does not affect the failure rates of surviving components in a parallel configuration. On the other hand, in any multicomponent system, the failure of one component can affect the performance of the remaining components [2]. That is, many systems are structured to share loads among components, which is known as load-sharing. For a load-sharing system, the assumption of independence is unreasonable. The system reliability can be estimated from dependencies among components, the knowledge of components and their system configuration.

With the purpose of analyzing the reliability of load-sharing systems, the relationship between the load and the failure behavior of a component, described by the failure rate of the component, is considered. For example, Tierney [3] proposed two load-sharing settings for fibers in the parallel arrangement. Assume that there is little or no cohesion between fibers. Once a fiber fails, the surviving fibers share the steady and tensile load equally and uniformly. In the second setting, the load of a failed fiber is transferred to an adjacent fiber based on the shape of the set of adjacent failed fibers. As a generalization of the two previous settings, a load on any individual component monotonically increases as other components fail.

For a repairable system, when components fail, the system can be restored. Due to the variety of failure causes, the repair times are random. Therefore, studies of probabilistic repair times and repair performance levels are necessary. Since both time-to-failure and repair time are stochastic processes, as mentioned earlier, Markov process is the most common mathematical methodology for the reliability design of the repairable system.

Once the system is repairable, the maintenance of the system can be classified into three categories. The first one is the corrective maintenance. In this case, the system is repaired based on the system failure only. Once the system failure is significant due to a series of losses, preventive maintenance should be used. There are two policies for preventive maintenance. Given the failure distributions of components, one can plan a repair treatment before the failure of components occurs. On the other hand, if the analysis of component failures concentrates on the physical evaluations, conditional maintenance can be performed based on continuous records of specific measurements, which have thresholds to indicate the component failure events. The third maintenance strategy is reliability-centered maintenance. It is a corporate level maintenance strategy based on analysis and testing of factors, which affect the reliability of components systematically.

In the design phase, maintenance is a functional design problem. System modular design is beneficial to the competition since the system is reconfigurable based on the functional combination of modules in the system. By exploring the relationship between system configuration and separation of functional requirements, the functional combination is implemented. Proposed by [4], the modularity of a system depends on two characteristics of the design: 1) similarity between the physical and functional architectures of the design and 2) minimization of incidental interactions among physical components. In this paper, modular design concentrates on maintenance. The design decision includes the number of components in each maintenance module and the selection of component type for each module. It is assumed that each module has the same type of components.

For the load-sharing k-out-of-n systems, the repair time of each component is arbitrary distributed. This assumption is more valuable than constant repair time assumption in realistic applications. On the other hand, for the repairable load-sharing k-out-of-N configuration, Markov chain in which the states represent the number of failed components in the system has the strictest assumption. Comparing with the Markovian model, the flowgraph model is a graphical representation of a stochastic system in which possible outcomes are connected by directed line segments. This model provides a new computational way to the reliability evaluation of load-sharing k-out-of-N system based on Moment Generating Functions (MGFs). The use of MGFs simplifies the computational multiplication of different distribution functions. By linking covariates into branch transition, the MGFs of the system failure are evaluated under different covariates levels so that presence of external events can be described in quantitative way. In the proposed model, during the repair process of failed components, the operating component can fail.

The Weighted Principal Component Analysis (WPCA) based multi-response optimization [5] is used for determining components, system configuration and maintenance policy. This methodology is beneficial to the optimization problem, in which both network parameters and network structures (nodes and edges) of potential designs vary.

The paper provides a new computational framework based on statistical flowgraph model for repairable load-sharing k-out-of-n problem. Integrating the maintenance-based modular design concept into maintenance task, WPCA-based multi-response optimization is applied to determine the optimal design factorial combination. The remaining sections are organized as follows. Section 2 reviews the development of models for analyzing repairable load-sharing system. Section 3 proposes the flowgraph model and the methodology for computing system failure time MGF with different combinations of covariate levels. Section 4 presents the multi-response optimization for the module system and repair policy design, and Section 5 shows the detailed procedures of the proposed framework by a numerical example.

II. STATE OF THE ART

Most of the load-sharing k-out-of-n system models assume constant failure rate for every component, which can be either analytically solved [6] or represented by the Markov transition diagram [7]. On the other hand, the assumption of time-varying failure rates is proposed as well [8]. Another attempt was proposed by Liu [9], who modeled the component failure time distribution by proportional hazards model and the load changes by piecewise constant function. However, the generalizations of the models in these studies are limited by their computation complexity. Similarly, Liu and Mohammad et al. [10] presented a model in which the load-dependent time-varying failure rate of each component is expressed by Cox's proportional hazards model and provided a closed form expression for the system reliability when all components are identical. To reduce the computation complexity induced by multiple integrations for failure dependency, Suprasad et al. [11] proposed a series of models which can solve large systems in a short time. They considered two classes of models accounting for the effects of load history: tampered failure rate (TFR) model and cumulative exposure (CE) model. They converted the TFR load-shared model with general failure distributions and used the concept of supplementary variables in semi-Markov processes to model the effects of load history on system life for CE model [12]. A slightly different perspective of modeling load-sharing k-out-of-n system is based on task allocation and queueing, in which the load is considered as the tasks assignment on each component. Huang and Xu [13] studied such models and introduced the concept of queueing system and cumulative time in each state to generate a closed-form expression for reliability of load-sharing k-out-of-n system with arbitrary failure distributions.

Despite a wide range of applications for load-sharing redundant systems, the methods for lifetime-related performance evaluation and design of repairable load-sharing k-out-of-n systems are limited. The failure dependency as well as maintenance process complicate the states and transitions between states, so that it is of great challenge to model the lifetime reliability of such system. Shao and Lamberson [14] studied a Markov model for analyzing a shared-load repairable k-out-of-n system with imperfect switching. It assumed that all the components are identical with constant failure rates and constant repair rates. Although the repair rule declared in their paper considered more than one component at a time in each repair, the model used considered only one repair transition from each state to its one-step backward state, which means that only one component can be repaired at a time. Hasset et al. [8] extended Shao's model by considering time-varying failure rates and time-varying repair rates within states transitions and solved a 2- component failure-dependent parallel system. As stated by the authors, the computation of non-constant failure rate or repair rate models is rather challenging because the general solution for such model is intractable. A response to such intractability is to assume identical Weibull failure time distributions and identical constant repair rates. Even with these simple assumptions, the expression for system reliability and availability is extremely complicated and tedious to evaluate. Therefore, Amari et al. [15] proposed an efficient algorithm based on symmetric switching functions and iterative implementation to approximate the reliability, availability and failure distribution of a repairable k-out-of-n system with identical/non-identical components, which has O(kn) computational complexity. However, the cases with non-identical components still assume constant failure rates, and load-sharing was not considered in the paper. Different from the Markov models proposed in the previous papers, where

 ISBN: 978-1-61208-698-9

the states represent the number of failed components in the system, Mandziy et al. [16] modeled a detailed Markov chain where the states representing the failure of components at specific locations caused by specific fault mode for a simple 3-component system. In this study, the component failure time was distributed by Weibull, and load sharing effect was set by the scale functions depending on the status of all survival components in the system. A failed component could be repaired as long as it does not cause system failure, and the repair time was assumed exponentially distributed and identical for all components.

As a generalization of Markov process, semi-Markov process creates flexibility in modeling system degradation and recovery. In semi-Markov model, the states being successively visited are governed by a Markov chain, and the transition time distribution can be arbitrarily specified. Hellmich [17] modeled the repairable load-sharing k-out-of-n: G system with identical components by nonhomogeneous semi-Markov process, in which the failure time distribution of each component was arbitrary and repairable. But the repair time distribution was restricted to be exponential, because when the system was in the state that q components fail and another component failed, the process transited to the next state that q + 1 components fail and the repair of the previous failed component had to be forgotten which forced the repair process to be memoryless.

Although a semi-Markov multistate model provides a way allowing the transition time to a future state to depend on the duration of time spent in the current state, it is quite difficult to analyze data for semi-Markov models in practice. Flowgraphs model semi-Markov processes and allow a variety of distributions used within the multistate model. The “states” in the flowgraphs are all the possible outcomes of a stochastic system. The waiting time distributions of the change of states are formulated by MGFs. Moreover, flowgraphs can easily handle reversibility [18]. These give flowgraphs the natural advantage in analyzing time-to-event data and modeling system reliability performance. Jenab and Dhillon [19] used flowgraph to model the k-out-of-n system in which every component in the system had a failure detection – isolation – repair loop and all components were assumed to be identical and operating independently. Jenab and Dhillon [20] then extended this model to adapt the reversible multi-state case, where each unit in the system could transit from better states to worse states due to aging effect, or from worse states to better states due to repair, and the degradation and recovery process was a semi-Markov model and was represented by the flowgraph model. The load sharing was simply represented by changing the states from the level of degradation to the level of load carried by that unit in the system function, assuming all the units in the system are identical. In this paper, the flowgraph model is embedded in a novel framework for modular based design and extended for more general system in which the components are not necessarily identical and the failure and repair time distributions are not necessarily exponential.

From the previous reviews, it can be seen that Markov process is restricted to memoryless property. That is, the transition time from one state to another is exponential distributed. Although the semi-Markov process relaxes this restriction, the corresponding computational task is a challenge. Based on the property of MGFs, statistical flowgraph model is advantageous in regards to the challenge. Based on statistical flowgraph model, the repairable load-sharing k-out-of-n system is studied. By introducing the modular design concept, the intermediate layer between the top level and bottom level (component-level) is introduced, and the decision variables can be discussed for optimizing the corresponding repairable system. Specially, in the loading-sharing k-out-of-n system, the number of components in the system becomes a decision variable, which is denoted by N. Therefore, in this paper, the computational process encompasses the system degradation (step-by-step failures) and restoration (maintenance tasks) into a decision-making perspective. By using multi-response optimization methodology, the optimal factorial combination is obtained at last.

III. PROBLEM DESCRIPTION

A parallel model consists of n components in active redundancy, of which k ($1 \le k < N$) are necessary to perform the required function [21]. For many practical applications, load sharing is a suitable design to explain the dependence among components. Consider a k-out-of-N: G system with independent components, the system is put into operation at time zero, all components are functioning, and they are equally sharing a constant load that the system is supposed to carry [22]. When the system experiences component failures, the surviving components must carry the same load on the system. Considering the target performance levels prescribed in the design phase, the redundancy level is a decision variable, denoted by N, should be determined so that there is a suitable redundancy level in the system.

System design with prefabricated modules encompasses the production and use of preplanned modules as a solution to build with higher quality and more efficiency [23]. In order to manufacture systems in a manageable and economic way, prefabricated modules and adaptable module frames are selected, customized, and assembled [24]. For a k-out-of-N: G system, prefabricated modules are configured in the parallel structure to build redundancy of the system. Take a redundancy system with five components as example, which is illustrated in Figure 1. There are two types of modules: module 1 and module 2, in a shared-load k-out-of-5: G system, k = 1, 2, 3, 4, 5. Without loss of generality, the failure distributions of these components are not necessary to be identical.

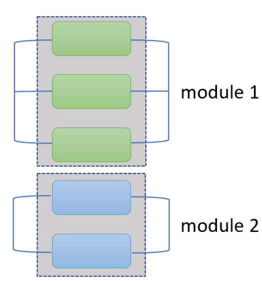


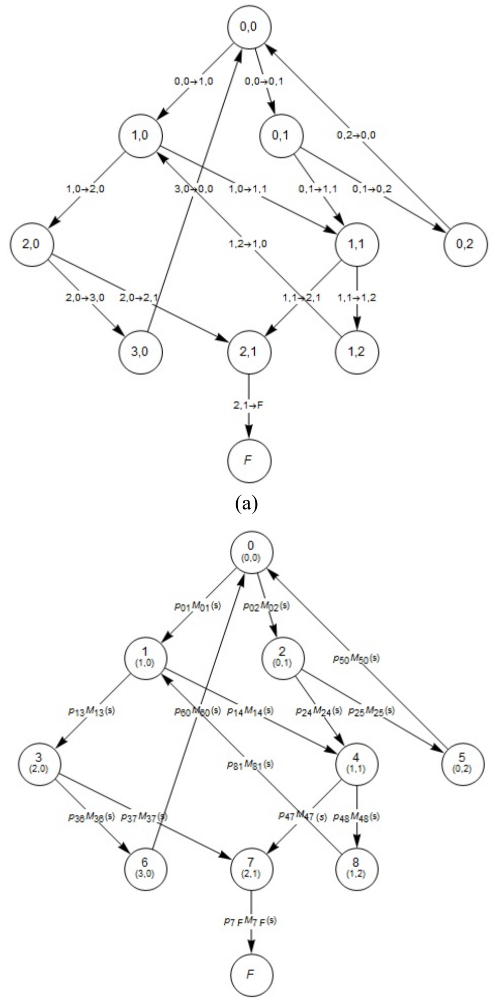
Figure 1. Shared-load k-out-of-5: G system with two types of module

On the other hand, considering the stability of manufacturing processes, the types of prefabricated modules should be limited at an affordable level. Suppose that the manufacturer produces M types of prefabricated modules, and M is a small integer. The numbers of components in each module are denoted by $\{m_1, \ldots, m_M\}$. Denote the numbers of each module in the system by $\{n_1, \ldots, n_M\}$. Then the total number of components in the system is $\sum_{j=1}^{M} m_j n_j = N$ which is equal to the redundancy level of the system.

For a k-out-of-N: G system, two types of cost, interface cost and encapsulate cost, are assumed. The interface cost depends on the number of modules in the system, and the encapsulate cost depends on the number of components in the module. Therefore, the total design cost of the system is $c_1 \sum_{j=1}^{M} n_j + c_2 N$, where c_1 and c_2 are cost coefficients of the interface cost and the encapsulate cost respectively, $\sum_{j=1}^{M} n_j$ is the total number of modules in the system.

Additionally, for a k-out-of-N: G system, a maintenance policy determines how and when the maintenance should be performed in order to avoid the system failures. Basic maintenance policies, such as age repair, periodic repair, and block repair polices are usually suggested for non-modularization systems. In particular, a maintenance policy, described by [25], is that the failed components are replaced if and only if the failed components are contained within the critical component set. Inspired by this policy, in this paper, we assume that a module can be replaced if and only if all components in the module are failed based on continuous monitoring.

The analysis of system performance presented in this paper is based on the system reliability analysis upon the flow-graph concept. The reliability of modularized shared-load k-out-of-N: G system is evaluated by using the concept of the flowgraph and MGF. A flowgraph is a graphical representation of a stochastic system in which possible outcomes are connected by directed line segments. Possible outcomes for the system reliability analysis are determined by the components failures in each module. Define a M-tuple, $\mathbf{O} = (O_1, \ldots, O_j, \ldots, O_M)$ in order to describe the possible outcomes, where O_j, j = 1, …, M, is a variable representing the number of failed components in the module j. If all components in the system are operating, then $\mathbf{O} = (0, \ldots, 0, \ldots, 0)$ and the system is in state 0. When one component in the system fails, assume that the failed component belongs to the module j, $j \in \{1, \ldots, M\}$, then $\mathbf{O} = (0, \ldots, 1, \ldots, 0)$. The number of states used to represent one component failure case is equal to the number of modules in the system. When the second failure occurs, it is assumed that the second failed component belongs to module j, for $j \in \{1, \ldots, M\}$. It is possible that i = j, that the two failures occur in the same module. If i = j, $\mathbf{O} = (0, \ldots, 2, \ldots, 0)$. If $i \neq j$, $\mathbf{O} = (0, \ldots, 1, \ldots, 1, \ldots, 0)$. Similarly, all the possible outcomes (states) are determined. Once the total number of failed components is greater than k, $\sum_{j=1}^{M} O_j > k$, the system fails and the system state enters to failure state, called F. For maintenance policy, once all components of a module fail, $O_j = m_j$, $j \in \{1, \ldots, M\}$, the module j is replaced immediately with measurable probabilistic repair time. For the shared-load k-out-of-5: G system in the Figure 1, Figure 2 (a) gives the flowgraph for k = 2 where the transitions of states are indicated on the branch.



(a) (b)

Figure 2. Flow graph for the example in Figure 1: (a) branches are labeled with transition between states; (b) branches are labeled with transmittances

In the flowgraph, each branch has a transition probability, p_{xy}, and a waiting time distribution associated with the transition from its beginning and ending nodes, $M_{xy}(s)$, where x and y denote the nodes in the flowgraph. Figure 2 (b) shows the flowgraph of the example in Figure 1 with k = 2, where the branches are labeled with transition probability and MGF of waiting time. p_{xy} is determined by the number of survival components in each module and the module in which the next failure occurs. Let $\mathbf{O_x} = (O_{x1}, \ldots, O_{xj}, \ldots, O_{xM})$ denote the component failure state of node x, x = 0, 1, …, F, where O_{xj} is the number of failed components in module j when the system is in state x. The transition probability from state x to state y, is

$$p_{xy} = \frac{m_h - o_{xh}}{N - \sum_{j=1}^{M} o_{xj}}, \text{for } \sum_{j=1}^{M} O_{xj} + 1 = \sum_{j=1}^{M} O_{yj},$$

where N is the total number of components in the system, and h represents the module in which the new failure occurs, $h \in \{1, \ldots, M\}$. For example, in Figure 2, state 4, (1, 1), represents that there is one failed component in module 1 and one failed component in module 2. Correspondingly, there are 5 – (1 + 1) = 3 survival components, in which two of them are in module 1 and the other one is in module 2. State 7, (2, 1), represents that the next failure component is in module 1. Thus, the probability of transformation from node 4 to 7 is

$$p_{47} = 1/[5 - (1 + 1)]\times 2 = 2/3$$

It is assumed that a module will be replaced when all components in that module fail. Thus, when the system reaches node Ox = (Ox1, …, Ox(j-1), mj, Ox(j+1), …, OxM), j = 1, …, M, module j is replaced with a new one, and the system will be transferred to Oz = (Ox1, …, Ox(j-1), 0, Ox(j+1), …, OxM), because the number of failed component in module j is restored to 0. In this case, the transition probability is

$$p_{xz} = 1, \text{ from } \mathbf{O_x} = (O_{x1}, \ldots, O_{x(j-1)}, m_j, O_{x(j+1)}, \ldots, O_{xM}) \text{ to } \mathbf{O_z} = (O_{x1}, \ldots, O_{x(j-1)}, 0, O_{x(j+1)}, \ldots, O_{xM})$$

Let T_{xy} be the random waiting time in state x until the transition to y occurs, x, y = 0, 1, …, F, x ≠ y, and $M_{xy}(s)$ be the MGF of T_{xy}

$$M_{xy}(s) = E(e^{sT_{xy}})$$

provided that the expectation exists for s in an open neighborhood of 0 [18]. T_{xy} on branches from state x to y, x < y, are the component failure times, and T_{xy} on branches from state x to y, x > y are the times to repair the failed component. T_{xy} can follow any arbitrary distributions. This paper assumes that the failure time of each component follows exponential distribution,

$$M_{xy}(s) = \frac{\lambda_{xy}}{\lambda_{xy} - s}, \text{for } \sum_{j=1}^{M} O_{xj} + 1 = \sum_{j=1}^{M} O_{yj},$$

where λ_{xy} is the failure rate of the failed component causing transition from node x to y and a function of various covariates and the shared load on that component. We assume that the repair time is normally distributed with mean μ and standard variation σ for all types of modules,

$$M_{xz}(s) = e^{\mu s + \frac{\sigma^2 s^2}{2}}, \text{for } \boldsymbol{O_x} = (O_{x1}, \cdots, m_j, \cdots, O_{xM}) \text{ and } \boldsymbol{O_z} = (O_{x1}, \cdots, 0, \cdots, O_{xM}),$$

The first step in this problem is to compute the overall transmittance of the entire flowgraph from the initial state 0 to the end state F, M(s). After identifying all paths, loops, and loops not connecting the path between nodes of state 0 and state F, M(s) is computed by Mason's rule. For details of computing M(s), refer to [18]. M(s) determines the distribution of the system life time. The flowgraph model concerns modeling the probabilities of the outcomes, the failure/repair time distributions of the outcomes, and manipulating the flowgraph to access overall failure time distribution. Once the system failure time MGF M(s) is computed, the system reliability measurements, such as mean time to failure, and average number of repairs at specified covariate levels can be determined. Therefore, for each combination of covariate levels, system life time, total design cost and performance deviation are considered, to obtain a criterion for design's quality.

IV. MODULE DESIGN USING MULTI-RESPONSE OPTIMIZATION

Suppose there are S different operating conditions. For the i^{th} operating condition, a system $\mathbb{S}^{(i)}$ consists of modules needs to be designed, i = 1, 2, …, S. Suppose there are M types of modules to be allocated to each system. Let m_j denote the number of components in the j^{th} type of module, j = 1, 2, …, M. Let $n_j^{(i)}$ denote the number of type j modules in system $\mathbb{S}^{(i)}$ designed for operating condition i, i = 1, 2, …, S, j = 1, 2, …, M. Therefore, the factors to be determined are m_j and $n_j^{(i)}$, i = 1, 2, …, S, j = 1, 2, …, M, and the possible values for them are the factor levels.

In this paper, system mean time to failure (MTTF), system failure time standard deviation (SD), average number of module repair before system failure (MRep), and total design cost (Cost) are selected as four response variables. The proposed method aims to obtain optimal values of m_j and n_j, while minimizing a function of four response variables through WPCA. Once the system MGF, $M_{\mathbb{S}^{(i)}}(s)$ is calculated following the method stated in Section 3, MTTF and SD can be obtained by

$$\text{MTTF}^{(i)} = \left.\frac{dM_{\mathbb{S}^{(i)}}(s)}{ds}\right|_{s=0} \quad (1)$$

$$\text{SD}^{(i)} = \left.\frac{d^2M_{\mathbb{S}^{(i)}}(s)}{ds^2}\right|_{s=0} - \text{MTTF}^2 \quad (2)$$

To compute the distribution of repair occurrence, an auxiliary constant 1 is created and its MGF, e^u, is attached to the branch of repair. For example, in Figure 2, branch 5 → 0, 6 → 0, and 8 → 1 are the repair transitions, and the transmittance about these branches are changed to $p_{50}e^{u}M_{50}(s)$, $p_{60}e^{u}M_{60}(s)$, and $p_{81}e^{u}M_{81}(s)$. The overall system life MGF is computed as described in Section 3. The joint MGF of the distribution of system life time and number of repairment is $M_{\mathbb{S}^{(i)}}(s,u)$. Then, MRep can be calculated by taking the first derivative of $M_{\mathbb{S}^{(i)}}(s,u)\big|_{s=0}$ over u and letting u = 0,

$$\text{MRep}^{(i)} = \left.\frac{dM_{\mathbb{S}^{(i)}}(s,u)\big|_{s=0}}{du}\right|_{u=0} \quad (3)$$

The total design cost for system $\mathbb{S}^{(i)}$ is determined by the number of components and number of modules in the system. Let pe_j denote the cost of individual component in the j^{th} type of module, j = 1, 2, …, M, and pi is the cost of interface of each module. Therefore, for a system $\mathbb{S}^{(i)}$ consisting of $n_j^{(i)}$ type j modules, the total design cost is

$$\text{Cost}^{(i)} = \sum_{j=1}^{M} pe_j \times n_j^{(i)} \times m_j + pi \times \sum_{j=1}^{M} n_j^{(i)} \quad (4)$$

The objective for this optimization is to find the best combination of m_j and $n_j^{(i)}$, i = 1, 2, …, S, j = 1, 2, …, M, such that $\text{MTTF}^{(i)}$ is as close as possible to a target value $\text{MTTF}_0^{(i)}$, and $\text{SD}^{(i)}$, $\text{MRep}^{(i)}$, $\text{Cost}^{(i)}$ are minimized simultaneously among all operating conditions.

 ISBN: 978-1-61208-698-9

WPCA based Multi-Response Optimization method is applied to the experimental design, and the unique optimal combination of m_j and $n_j^{(i)}$, i = 1, 2, …, S, j = 1, 2, …, M is determined. WPCA based multi-response optimization utilizes PCA to map the original data to a new vector of component scores and transforms the original response variables into uncorrelated principal components. Each component is multiplied by a weight to emphasize the contribution of components based on their corresponding variation. All the weighted components are combined into one multi-response performance index (MPI), and the optimal result is the factor level combination with the largest MPI. The detailed procedure of WPCA multi-response optimization with unique solution can be found in [5]. The optimal value of m_j, j = 1, 2, …, M gives the modular design which has high manufacturing performance and accommodates to various demands under different operating conditions.

V. EXAMPLE

Suppose a manufacturer is producing three redundancy systems of electric motors as in Table I, which are designed to supply power under three operating conditions:

TABLE I. OPERATING CONDITIONS

Operating condition	Type of service application	Operating temperature, °F	Operating altitude, ft
1	Heavy shock load	5	500
2	Light shock load	75	3600
3	Uniform and steady load	140	40

Two types of electric motor are considered in the design, which are shown in Table II:

TABLE II. TYPES OF ELECTRIC MOTORS

Electric motor type	Shaft Material	Shaft surface manufactured finish	Viscosity of lubricant used in	
			bearing system	gear system
A	Alloy steel	Polished	1.0	1.0
B	Cast aluminum	Ground	0.8	1.2

The objective is to design the two types of module $\mathcal{M}^{(A)}$ and $\mathcal{M}^{(B)}$, where $\mathcal{M}^{(A)}$ is a parallel structure of type A electric motors, and $\mathcal{M}^{(B)}$ is a parallel structure of type B electric motors, such that $\mathcal{M}^{(A)}$ and $\mathcal{M}^{(B)}$ have high resilience to accommodate the redundancy product design for three different operating conditions $\mathbb{N}^{(1)}$, $\mathbb{N}^{(2)}$, and $\mathbb{N}^{(3)}$. $\mathbb{N}^{(1)}$, $\mathbb{N}^{(2)}$, and $\mathbb{N}^{(3)}$ consist of different allocations of $\mathcal{M}^{(A)}$ and $\mathcal{M}^{(B)}$. Therefore, the objective is to determine the optimal number of components in the two modules $\mathcal{M}^{(A)}$ and $\mathcal{M}^{(B)}$,

$$m_A, m_B$$

and the number of modules $\mathcal{M}^{(A)}$ and $\mathcal{M}^{(B)}$ in the three redundancy systems $\mathbb{N}^{(1)}$, $\mathbb{N}^{(2)}$ and $\mathbb{N}^{(3)}$, respectively,

$$n_A^{(1)}, n_B^{(1)}, n_A^{(2)}, n_B^{(2)}, n_A^{(3)}, n_B^{(3)}$$

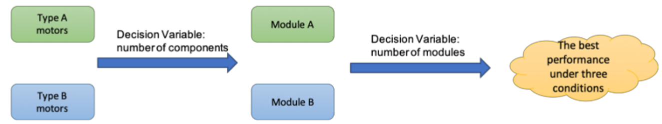


Figure 3. Illustrated Optimal Decision Procedure

for simultaneously meeting the requirements of mean time to failure (MTTF) for each operating condition, while minimizing the standard deviation of time to failure, average number of replaced modules before system failure, and the interface and encapsulate cost. Let $N^{(i)}$ denote the total number of electric motors in redundancy system i,

$$N^{(i)} = n_A^{(i)} m_A + n_B^{(i)} m_B, i = 1,2,3.$$

The failure time of an individual electric motor is exponentially distributed. Each module is considered as a tampered failure rate (TFR) load-sharing k-out-of-n: G system with identical electric motors where all surviving motors equally share the load. The system is consisted of different modules, which have non-identical types of electric motors. In this example, it is assumed that k = 3, and the module is replaced with a new one when all motors in that module fail, and the repair time is normally distributed with mean of 0.00048 million hour and standard deviation of 0.00024 million hour.

The failure rate model of an electric motor is based upon the failure rate of its parts, which includes windings, stator housing, armature shaft, bearings, and gears [26]. Failure mechanisms resulting in part degradation and failure rate distributions are independent in each failure mode. The total electric motor failure rate is the sum of the failure rates of each part in the motor, which are functions of covariates.

The target mean time to failure for each operating condition is assumed to be 8.76 thousand hours (1 year) for all operating conditions. The cost of individual electric motor is \$120 per type A motor and \$ 100 per type B motor, and the cost of module interface is \$100 per module. The possible choices of m_A and m_B are 1, 2, 3, or 4. The possible choices of $n_A^{(1)}, n_B^{(1)}, n_A^{(2)}, n_B^{(2)}, n_A^{(3)}, n_B^{(3)}$ are 0, 1, 2, or 3.

By introducing the covariate and load-sharing failure rate model to the system, we calculate the MGF of failure time for each combination of decision variables, m_A, m_B , $n_A^{(1)}$, $n_B^{(1)}$, $n_A^{(2)}$, $n_B^{(2)}$, $n_A^{(3)}$ and $n_B^{(3)}$. From the failure time MGF, MTTF and standard deviation of failure time are calculated. Factors $(m_A, m_B), (n_A^{(1)}, n_B^{(1)}), (n_A^{(2)}, n_B^{(2)}), (n_A^{(3)}, n_B^{(3)})$ are considered as four pairs of factors, and each factor pair consists of $4 \times 4 = 16$ levels. The 16^4 full experimental design is used. With cost of interface and encapsulate, the multi-response optimization experimental design is shown as Table III. Based on the Figure 3, we want to select the best system performance, described by the four responses. The objective is to determine the optimal number of components in the two module types and the number of modules in the redundancy systems. Therefore, in the Table III, the first eight columns of each row indicate sets of candidate designs, which are combinations of the number of motors in each module

 ISBN: 978-1-61208-698-9

associated with the three operating conditions. Based on these controllable design factors, we can evaluate the four responses (MTTF, SD, MRep, Cost) to describe the system performance. These four responses are computed based on the statistical flowgraph model described in Section 3.

For each operating condition, all these four responses are monitored so that responses in this example can be modeled in a three-level hierarchical structure. The top layer is about the system performance, the intermediate layer is about the three operating conditions, and the bottom layer is about the four responses. PCA-based multi-response optimization, as described earlier, can relax the response correlation problem, particularly for this hierarchical structure. On the other hand, there are two ways to solve the multi-response optimization problem: feature selection and dimensional reduction. For this hierarchical structure, feature selection is challenged since the intermediate levels (operating condition in this example) are equally important. This is another advantage of PCA-based multi-response optimization. In the experimental conduction perspective, the experiments were done by testing all the possible combinations of $(m_A, m_B), (n_A^{(1)}, n_B^{(1)}), (n_A^{(2)}, n_B^{(2)}), (n_A^{(3)}, n_B^{(3)})$. For easy visualization, the experiments with MTTF significantly deviating from the target value were eliminated from the table III.

Following the procedures proposed in [5] and [26], the unique solution WPCA based multi-response optimization method is applied on the data in Table III. Table IV summarizes the resulting MPIs. It can be seen that the optimal design for module $\mathcal{M}^{(A)}$ and $\mathcal{M}^{(B)}$, accommodating the three operating conditions, is to allocate two type A motors in module $\mathcal{M}^{(A)}$ and allocate three type B motors in module $\mathcal{M}^{(B)}$. The MTTF of system under operation condition 1, 2, and 3 are 8.2171, 8.1227, and 9.6936 thousand hours, respectively.

One of the most commonly used strategies of system design and repair rule is to consider every single component as an individual module which is subject to repair upon failure. To compare it with the proposed framework, the flowgraph is modified where the repair branch is added to every state with component failure. Let the system operates for 8.2171, 8.1227, and 9.6936 thousand hours under three operation conditions respectively. Following the procedures stated in Section 4 and the formulation of (3) and (4), the expected number of repairs and system configuration cost for different combinations of components are computed and summarized in Table V.

Compared the design options obtained in Table III and V, it is obvious that the design that immediately repairs every failed component leads to much greater times of repair action. For example, the expected number of repairs for the optimal solution under operating condition 1 (two modules with three type B motors in each module) is 0.1111. However, without considering modular repair rule, the system with six individual type B motors leads to an average of 1.6579 times of repair for operating the same length of time. Moreover, the system configuration cost is much higher under non-modular design (e.g., \$800 for the system with two modules with three type B motors in each module and \$1200 for the system with six individual type B motors), since the increased number of interfaces increases the total cost of the system.

VI. CONCLUSION AND FUTURE WORK

In this paper, incorporating the reliability concept into design for repairable systems is discussed. In the operating stage, for a disruptive event, the proposed maintenance strategy based on modular design provides a way to recover the system in the most appropriate way. In order to quantify the reliability-related performance in the design phase, a flowgraph model is introduced. The usage of flowgraph relaxes exponentially distributed assumptions for the state transition time, so that the proposed framework can model the problem with arbitrary distributions of failure time and repair time. Meanwhile, by linking the flowgraph with covariates, the model can be used when considering various external variates, such as different environmental conditions in which the system is operating. By applying the WPCA based multi-response optimization, the best design of modules and system can be obtained. The application of flowgraph is restricted by the complexity of the graph, because all the computations are based on functions [27]. The function-based operation limits the computation speed, and the higher graph complexity increases the number of functions involved, thus reducing the computation speed. Therefore, a novel algorithm for high efficiency flowgraph computation is needed and will extend the application of the proposed methodology in the future.

REFERENCES

[1] C. E. Ebeling, An Introduction to Reliability and Maintainability Engineering, 2nd ed.. Long Grove, IL: Waveland Press, Inc., 2010.

[2] I. Dewan and U. V. Naik-Nimbalkar, Load-Sharing Systems. Wiley Encyclopedia of Operations Research and Managment Science, 2010.

[3] L. Tierney, “Asymptotic bounds on the time to fatigue failure of bundles of fibers under local load sharing,” Advances in Applied Probability, vol. 14, no. 1, pp. 95-121, March 1982.

[4] K. Ulrich, “Fundamentals of product modularity,” in Management of Design. Dordrecht: Springer, pp. 219-231, 1994.

[5] N. Fard, H. Xu, and Y. Fang, “A unique solution for principal component analysis-based multi-response optimization problems,” The International Journal of Advanced Manufacturing Technology, vol. 82, no. 1-4, pp. 697-709, January 2016.

[6] E. M. Scheuer, “Reliability of an m-out-of-n system when component failure induces higher failure rates in survivors,” IEEE Transactions on Reliability, vol. 37, no. 1, pp. 73-74, 1988.

[7] H. Lin, K. Chen and R. Wang, "A multivariate exponential shared-load model," IEEE Transactions on Reliability, vol. 41, no. 1, pp. 165-171, 1993.

[8] T. F. Hassett, D. L. Dietrich, and F. Szidarovszky, "Time-varying failure rates in the availability & reliability analysis of repairable systems," IEEE Transactions on Reliability, vol. 44, no. 1, pp. 155-160, 1995.

[9] H. Liu, "Reliability of a load-sharing k-out-of-n: G system: non-iid components with arbitrary distributions," IEEE Transactions on Reliability, vol. 47, no. 3, pp. 279-284, 1998.

[10] R. Mohammad, A. Kalam, and S. V. Amari, "Reliability of load-sharing systems subject to proportional hazards model," Proceedings Annual Reliability and Maintainability Symposium (RAMS), pp. 1-5, 2013.

[11] A. V. Suprasad, M. B. Krishna, and P. Hoang, "Tampered failure rate load-sharing systems: status and perspectives," in Handbook of performability engineering, pp. 291-308, 2008.

[12] S. V. Amari and R. Bergman, "Reliability analysis of k-out-of-n load-sharing systems," Reliability and Maintainability Symposium (RAMS), pp. 440-445, 2008.

[13] L. Huang and Q. Xu, "Lifetime reliability for load-sharing redundant systems with arbitrary failure distributions," IEEE Transactions on Reliability, vol. 59, no. 22, pp. 319-330, 2010.

[14] J. Shao and L. R. Lamberson, "Modeling a shared-load k-out-of-n: G system," IEEE Transactions on Reliability, vol. 40, no. 2, pp. 205-209, 1991.

[15] S. V. Amari, M. J. Zuo and G. Dill, "O (kn) algorithms for analyzing repairable and non- repairable k-out-of-n: G systems," in Handbook of performability engineering, pp. 309-320, 2008.

[16] B. Mandziy, O. Lozynsky, and S. Shcherbovskykh, "Mathematical model for failure cause analysis of electrical systems with load-sharing redundancy of component," Przeglad Elektrotechniczny, vol. 89, no. 11, pp. 244-247, 2013.

[17] M. Hellmich, "Semi-Markov Embeddable Reliability Structures and Applications to Load- Sharing k-Out-of-n Systems," International Journal of Reliability, Quality and Safety Engineering, vol. 20, no. 2, 1350007, 2013.

[18] A. V. Huzurbazar, Flowgraph Models for Multistate Time-to-Event Data. Hoboken, NJ: John Wiley & Sons, Inc., 2005.

[19] K. Jenab and B. S. Dhillon, "K-out-of-n system with self-loop units. International Journal of Reliability," Quality and Safety Engineering, vol. 12, no. 1, pp. 61-73, 2005.

[20] K. Jenab and B. S. Dhillon, "Assessment of reversible multi-state k-out-of-n: G/F/Load- Sharing systems with flow-graph models," Reliability engineering & System safety, vol. 91, no. 7, pp. 765-771, 2006.

[21] A. Birolini, Reliability Engineering: Theory and Practice, 8th ed.. Springer, 2017.

[22] W. Kuo and M. J. Zuo, Optimal Reliability Modeling: Principles and Applications. Hoboken, NJ: John Wiley & Sons, Inc., 2003.

[23] U. Knaack, S. Chung-Klatte, and R. Kasselbach, Prefabricated Systems: Principles of Construction. Birkhäuser Architectur, 2010.

[24] H. Mayr, "GEM—A generic engineering framework for mechanical engineering based upon meta models," International Conference on Computer Aided Systems Theory (EUROCAST 1997), Springer Berlin Heidelberg, 1997, pp. 83-91, doi: 10.1007/BFb0025036

[25] M. Ram and J. P. Davim, Advances in Reliability and System Engineering. Springer, 2017.

[26] T. L. Jones, Handbook of Reliability Prediction Procedures for Mechanical Equipment. Naval Surface Warfare Center, Carderock Division, West Bethesda, Maryland, 2011.

[27] N. Fard, H. Xu, and Y. Fang, "Reliability assessment of load-sharing systems by flowgraph for non-identical components with time-varying repair rates," Proceedings of International Conference on Computers and Industrial Engineering (CIE), pp. 1-8, 2017.

TABLE III. MULTI-RESPONSE OPTIMIZATION EXPERIMENTAL DESIGN LAYOUT

Factor								**Response**											
Module Type		**Operating Condition**						**Operating Condition**											
A	B	1		2		3		1				2				3			
m_A	m_B	$n_A^{(1)}$	$n_B^{(1)}$	$n_A^{(2)}$	$n_B^{(2)}$	$n_A^{(3)}$	$n_B^{(3)}$	$MTTF^{(1)}$	$SD^{(1)}$	$MRep^{(1)}$	$Cost^{(1)}$	$MTTF^{(2)}$	$SD^{(2)}$	$MRep^{(2)}$	$Cost^{(2)}$	$MTTF^{(3)}$	$SD^{(3)}$	$MRep^{(3)}$	$Cost^{(3)}$
1	2	0	3	0	3	0	3	9.159	9.0651	0.7395	900	9.0652	8.9654	0.7395	900	6.5112	6.3642	0.7395	900
1	3	0	2	0	2	1	2	8.2171	5.8736	0.1111	800	8.1227	5.7918	0.1111	800	12.1395	9.8169	1.3793	1020
1	3	0	2	0	2	2	1	8.2171	5.8736	0.1111	800	8.1227	5.7918	0.1111	800	5.0004	3.6295	1.6667	840
3	1	2	0	2	0	1	2	9.0182	6.4383	0.1111	920	8.9048	6.3401	0.1111	920	5.0332	3.5329	1.6667	860
3	1	2	0	2	0	2	1	9.0182	6.4383	0.1111	920	8.9048	6.3401	0.1111	920	12.5481	10.0771	1.3793	1120
2	2	0	3	0	3	3	0	9.159	9.0651	0.7395	900	9.0652	8.9654	0.7395	900	6.9375	6.7645	0.7395	1020
2	2	0	3	1	2	3	0	9.159	9.0651	0.7395	900	9.3472	9.2459	0.7395	940	6.9375	6.7645	0.7395	1020
2	2	1	2	0	3	3	0	9.4473	9.3526	0.7395	940	9.0652	8.9654	0.7395	900	6.9375	6.7645	0.7395	1020
2	2	1	2	1	2	3	0	9.4473	9.3526	0.7395	940	9.3472	9.2459	0.7395	940	6.9375	6.7645	0.7395	1020
2	3	0	2	0	2	1	2	8.2171	5.8736	0.1111	800	8.1227	5.7918	0.1111	800	9.6936	11.4477	0.3668	1140
2	3	0	2	0	2	2	1	8.2171	5.8736	0.1111	800	8.1227	5.7918	0.1111	800	8.5759	9.1844	0.5541	1080
3	2	2	0	2	0	1	2	9.0182	6.4383	0.1111	920	8.9048	6.3401	0.1111	920	8.5049	9.0895	0.5541	1060
3	2	2	0	2	0	2	1	9.0182	6.4383	0.1111	920	8.9048	6.3401	0.1111	920	9.9104	11.6823	0.3668	1220
3	2	2	0	0	3	1	2	9.0182	6.4383	0.1111	920	9.0652	8.9654	0.7395	900	8.5049	9.0895	0.5541	1060
3	2	2	0	0	3	2	1	9.0182	6.4383	0.1111	920	9.0652	8.9654	0.7395	900	9.9104	11.6823	0.3668	1220
3	2	0	3	2	0	1	2	9.159	9.0651	0.7395	900	8.9048	6.3401	0.1111	920	8.5049	9.0895	0.5541	1060
3	2	0	3	2	0	2	1	9.159	9.0651	0.7395	900	8.9048	6.3401	0.1111	920	9.9104	11.6823	0.3668	1220
3	2	0	3	0	3	1	2	9.159	9.0651	0.7395	900	9.0652	8.9654	0.7395	900	8.5049	9.0895	0.5541	1060
3	2	0	3	0	3	2	1	9.159	9.0651	0.7395	900	9.0652	8.9654	0.7395	900	9.9104	11.6823	0.3668	1220
3	3	1	1	1	1	2	0	8.6172	6.1652	0.1111	860	8.5133	6.0747	0.1111	860	6.0477	4.0822	0.1111	920
1	4	2	1	2	1	2	1	13.1177	8.5674	2.1667	940	12.98	8.4503	2.1667	940	9.3589	5.7286	2.1667	940
2	4	1	1	1	1	1	1	8.0723	5.9972	0.1738	840	7.9792	5.9132	0.1738	840	5.5548	3.9151	0.1738	840
2	4	1	1	1	1	2	1	8.0723	5.9972	0.1738	840	7.9792	5.9132	0.1738	840	10.1503	11.9017	0.4611	1180
4	2	1	1	1	1	1	1	8.3358	6.1035	0.1738	880	8.2359	6.0151	0.1738	880	5.6737	3.9467	0.1738	880
4	2	1	1	1	1	1	2	8.3358	6.1035	0.1738	880	8.2359	6.0151	0.1738	880	10.141	11.863	0.4611	1180

TABLE IV. MULTI-RESPONSE OPTIMIZATION RESUTLS

Factor								**MPI**
Module Type		**Operating Condition**						
A	B	1		2		3		
m_A	m_B	$n_A^{(1)}$	$n_B^{(1)}$	$n_A^{(2)}$	$n_B^{(2)}$	$n_A^{(3)}$	$n_B^{(3)}$	
1	2	0	3	0	3	0	3	0.2467
1	3	0	2	0	2	1	2	1.1930
1	3	0	2	0	2	2	1	1.0590
3	1	2	0	2	0	1	2	0.8483
3	1	2	0	2	0	2	1	1.0065
2	2	0	3	0	3	3	0	0.2813
2	2	0	3	1	2	3	0	0.2171
2	2	1	2	0	3	3	0	0.2248
2	2	1	2	1	2	3	0	0.1606
2	3	0	2	0	2	1	2	1.2180
2	3	0	2	0	2	2	1	1.1631
3	2	2	0	2	0	1	2	0.9476
3	2	2	0	2	0	2	1	1.0257
3	2	2	0	0	3	1	2	0.5617
3	2	2	0	0	3	2	1	0.6398
3	2	0	3	2	0	1	2	0.6761
3	2	0	3	2	0	2	1	0.7542
3	2	0	3	0	3	1	2	0.2902
3	2	0	3	0	3	2	1	0.3684
3	3	1	1	1	1	2	0	1.0041
1	4	2	1	2	1	2	1	-0.0604
2	4	1	1	1	1	1	1	0.9000
2	4	1	1	1	1	2	1	1.0713
4	2	1	1	1	1	1	1	0.8570
4	2	1	1	1	1	1	2	1.0158

TABLE V. NUMBER OF REPAIRS AND COST FOR IMMEDIATE REPAIR RULE

Number of Motor A	**Number of Motor B**	**Operating Condition**			**Cost**
		1 (Operating 8217.1 Hours)	**2 (Operating 8122.7 Hours)**	**3 (Operating 9693.6 Hours)**	
		$MRep^{(1)}$	**$MRep^{(2)}$**	**$MRep^{(3)}$**	
0	5	3.3315	3.3048	2.4736	1000
1	4	3.2827	3.2567	2.4469	1020
2	3	3.2336	3.2082	2.4202	1040
3	2	3.1841	3.1594	2.3932	1060
4	1	3.1343	3.1102	2.3660	1080
5	0	3.0841	3.0607	2.3387	1100
0	6	1.6579	1.6647	1.5747	1200
1	5	1.6359	1.6430	1.5629	1220
2	4	1.6138	1.6212	1.5510	1240
3	3	1.5916	1.5993	1.5391	1260
4	2	1.5694	1.5774	1.5271	1280
5	1	1.5471	1.5555	1.5152	1300
6	0	1.5247	1.5335	1.5032	1320

 ISBN: 978-1-61208-698-9