Simple and Precise Analog Arcsine Synthesis Applied to Amplitude to Phase Conversion for Hall Effect Position Sensors

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***Abstract* — Sinusoidal encoders, including Hall effect sensors, are position sensors that provide analog sine and cosine signals of angular position. All schemes used for converting these signals into measure of the angle require either trigonometric or inverse trigonometric function implementation. The proposed converter is based on the use of the alternating pseudo-linear segments of the sensor signals together with a simple and effective linearization technique. The theoretical absolute error of non-linearity of the converter is 0.05 degree over the full 360 degree range. The converter may be implemented numerically or electronically. The paper describes the proposed method, full details of its analog implementation, and experimental results obtained with a Hall effect sensor. Results demonstrate agreement between theory and experimental results.**

Keywords — amplitude-phase conversion, inverse sine synthesis, linearization, position measurement, sinusoidal encoder, Hall effect sensor.

I. INTRODUCTION

Rotational speed and position measurement and control is often required in various applications in industry, military, avionics, communication and other fields. Sinusoidal encoders whether operating on optical, inductive, Hall effect or magnetoelectric principles produce quadrature electrical signals in which the unknown angle is encoded [1]-[10]. In practice, it is common to observe phase and amplitude imbalances in the sensor signals; therefore the sensor outputs may be written as:

$$\begin{cases} U_S(\theta) = A \times \sin(\theta) \\ U_C(\theta) = A(1+\alpha) \times \cos(\theta + \beta) \end{cases} \quad (1)$$

where A is the maximum amplitude of the sinusoidal component of $U_S(\theta)$, θ is the shaft angle of the rotor of the sensor, and α and β are amplitude and phase imbalances respectively. Note that some sensors may produce signals with dc offset components; however, these may easily be removed before further processing. Amplitude and phase balancing may be achieved in a number of ways [11]-[14]. In this work, we propose balancing by generating a perfect cosine signal, from $U_S(\theta)$ and $U_C(\theta)$, with the same amplitude as the sine signal,

$$\begin{cases} \hat{U}_S(\theta) = U_S(\theta) = A \times \sin(\theta) \\ \hat{U}_C(\theta) = \dfrac{U_C(\theta)}{(1+\alpha)\cos\beta} + \tan\beta \times U_S(\theta) = A \times \cos(\theta) \end{cases} \quad (2)$$

The resolver requires a suitable converter in order to determine θ from its signals (2); many open loop and closed loop conversion schemes have been described in literature. Some open loop converters based on the linearization of the pseudo-linear segments of sinusoidal signals have been described [10][15]-[22]. Ratiometric techniques based on arctangent method are also used in open loop converters [23]-[26]; these require the use of look up tables or linearization techniques. Other open loop converters based on the use of reference ac signals and time measurement techniques have also been described [8][9][27]-[29]. Closed loop converters employ the phase-locked loop (PLL) technique [30]–[37].

In the present work, we present an open-loop amplitude-to-phase converter that uses a simple arcsine synthesis technique which can be easily implemented with few standard electronic components. The remainder of the paper is organized as follows. In Section II, the principle and theory of operation of the proposed converter, including a novel linearization technique, are described in details. A dedicated signal shaping technique used for implementing the converter is also described. In Section III, the practical implementation of the converter and experimental results are described. Section IV concludes the paper.

II. PROPOSED CONVERTER

The basic principle of the proposed converter (Figure 1) is based on making use of the alternating pseudo-linear segments of the sensors signals (2) in order to produce a signal $U_0(\theta)$ which is almost proportional to the unknown angle θ in each of its four quadrants (Figure 2), as has been reported in previous works [5]-[19]. The four quadrants are identifiable using two binary outputs whose states depend on the signs of the sum and difference of the transducer signals (i.e., LOW state for negative and HIGH state for positive values):

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$$\begin{cases} Bit1(\theta) = \overline{\text{sign}\left(\hat{U}_{\text{S}}(\theta) + \hat{U}_{\text{C}}(\theta)\right)} \\ Bit0(\theta) = Bit1(\theta) \oplus \text{sign}\left(\hat{U}_{\text{S}}(\theta) - \hat{U}_{\text{C}}(\theta)\right) \end{cases} \tag{3}$$

The signal $Bit0(\theta)$ is used to control the multiplexer that selects the pseudo-linear segments of the sensor signal to produce $U_0(\theta)$,

$$U_0(\theta) = \left(Bit0(\theta) \times \hat{U}_{\text{C}}(\theta)\right) + \left(\overline{Bit0(\theta)} \times \hat{U}_{\text{S}}(\theta)\right) \tag{4}$$

The sign of $\hat{U}_{\text{S}}(\theta)$ - $\hat{U}_{\text{C}}(\theta)$ is used together with a synchronous rectifier to generate a rectified signal $U_{0\text{R}}(\theta)$, a sawtooth-like waveform made up of four identical and positive-slope sections as shown in Figure 1 and Figure 2. The signal $U_{0\text{R}}(\theta)$ requires further linearization in order to obtain a piecewise linear output $U_{0\text{RL}}(\theta)$. The angle may be determined from the linearize signal $V_{0\text{RL}}(\theta)$ using a simple linear equation within each quadrant of input angle with minimal non-linearity error. Within the full 360 degree range, the computed angle noted θ_{C} (in degree) is determined from:

$$\theta_c = 45^{\circ} \times \left[\left(\sqrt{2}/A\right) \times U_{0RL}(\theta) + 2 \times Bit0(\theta) + 4 \times Bit1(\theta)\right] \tag{5}$$

Note that in the range 315 to 360 degree, the angle determined using (5) is negative (i.e., measured clockwise). Evidently, the residual error (θ_{C}-θ) depends on the quality of linearization scheme. Previous works have presented various linearization methods with different degrees of complexity and precision [10][15]-[22]. These schemes that uses multipliers/dividers add complexity and cost, particularly, in analog implementation. In this work, the proposed linearization method is based on signal shaping techniques.

Signal shaping networks are usually associated with trigonometric and inverse trigonometric function synthesis. In this application, the pseudo-linear segments of $V_{0\text{R}}(\theta)$ belong to sinusoidal signals. Hence, linearization requires arcsine function synthesis, or in other words sine-to-triangle conversion. It is well known that signal shaping can be implemented using well established piece-wise linear approximation techniques involving diodes and/or transistors [38]-[40]. Other techniques based on differential pairs of transistors offer attractive and simpler solutions that closely approximate sine function [41]-[47]. In the present application, a signal converter based on the non-linear I-V characteristics of the base-emitter junction of low power bipolar junction transistors. The circuit diagram of the proposed linearization scheme is shown in Figure 4. This is a translinear sine-triangle converter inspired from the triangle-sine scheme presented in [47].

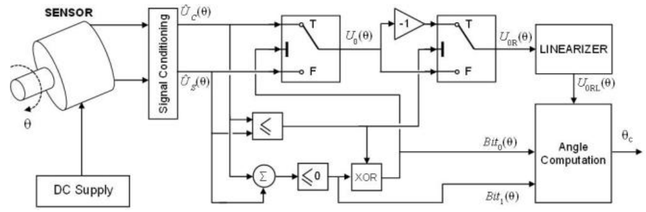

Figure 1. Basic diagram of the proposed converter.

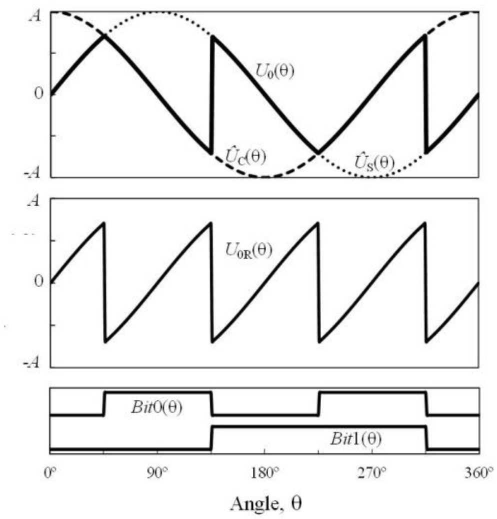

Figure 2. Converter signals: input signals $\hat{U}_{\text{S}}(\theta)$ and $\hat{U}_{\text{C}}(\theta)$, non-linearized signals $U_0(\theta)$ and $U_{0\text{R}}(\theta)$ and binary outputs $Bit0(\theta)$ and $Bit1(\theta)$.

The following analysis assumes that the transistors in the scheme of Figure 4 are matched. Because of symmetry, the current source of magnitude $2I$ ensures that the individual bias currents I in the two trans-diodes are equal. The sinusoidal segments of the rectified voltage $U_{0\text{R}}(\theta)$ applied to the linearization circuit result in a current i= $U_{\text{R}}(\theta)/R$. Note that this current should always be lower than I; the maximum value should correspond to the maximum amplitude A of the sinewaves from which the segments of $U_{0\text{R}}(\theta)$ are extracted and therefore i_{max}= A/R=I. By invoking the Shockley equation for the base-emitter junctions of the transistors, we can write:

$$\begin{aligned} U_{0RL} &= -\left(1+\frac{R_2}{R_1}\right) \times \left[2\left(R_1 // R_2\right) i + V_{BE1} - V_{BE2}\right] \\ &\approx -\frac{2R_2}{R} V_{0\text{R}} - \left(1+\frac{R_2}{R_1}\right) V_T \ln\left[\frac{1+U_{0\text{R}}/(IR)}{1-U_{0\text{R}}/(IR)}\right] \end{aligned} \tag{6}$$

where V_T is the thermal voltage (≈26mV at room temperature). The Maclaurin series expansion of (6) yields:

$$\begin{aligned} U_{0\text{RL}} &= -\frac{2R_2}{R} U_{0\text{R}} - 2\left(1+\frac{R_2}{R_1}\right) V_T \sum_{n=0}^{\infty} \frac{(U_{0\text{R}}/(IR))^{2n+1}}{2n+1} \\ &= -2\frac{U_{0\text{R}}}{(RI)}\left[R_2 I + \left(1+\frac{R_2}{R_1}\right) V_T\right] - 2\left(1+\frac{R_2}{R_1}\right) V_T \left[\frac{1}{3}\left(\frac{U_{0\text{R}}}{RI}\right)^3 + \frac{1}{5}\left(\frac{U_{0\text{R}}}{RI}\right)^5 + \ldots\right] \end{aligned} \tag{7}$$

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The objective is to convert the sinusoidal signal $U_{0R}(\theta)$ into a sinusoidal current which is in turn converted into a triangular output voltage $U_{0RL}(\theta)$; therefore:

$$U_{0RL} = -K_1 \times \arcsin\left(U_{0R}/(RI)\right) = -K_1 \sum_{n=0}^{\infty} \frac{(2n!)}{4^n (n!)^2 (2n+1)} \left(U_{0R}/(RI)\right)^{2n+1} \quad (8)$$
$$= -K_1 \times \left[\left(\frac{U_{0R}}{RI}\right) + \frac{1}{6}\left(\frac{U_{0R}}{RI}\right)^3 + \frac{3}{40}\left(\frac{U_{0R}}{RI}\right)^5 + \ldots \right]$$

By comparing (7) and (8), and considering the first three terms of the expansions, we can deduce the conditions for a successful conversion:

$$\begin{cases} \left(1+\dfrac{R_2}{R_1}\right)V_T + R_2 I = K_1/2 \\ U_{0RL\max} = K_1\pi/4 \\ K_1/6 = 2\left(1+\dfrac{R_2}{R_1}\right)V_T/3 \quad \text{or} \quad 3K_1/40 = 2\left(1+\dfrac{R_2}{R_1}\right)V_T/5 \end{cases} \quad (9)$$

Note that the maximum amplitude of $U_{0RL}(\theta)$ occurs at the peak of $U_{0R}(\theta)$ which is equal to $2^{-½}A$; therefore $U_{0RL\max}=K_1\times\arcsin(2^{½}A/(RI))= K_1\pi/4$. If we keep K_1 within the boundaries defined by the last condition of (9):

$$4\left(1+R_2/R_1\right)V_T \le K_1 \le 80\left(R_2/R_1\right)V_T/15 \quad (10)$$

Straightforward analysis shows that when considering (10), the two conditions in (9) may be re-written as:

$$\begin{cases} \dfrac{V_T}{I} \le R_1 // R_2 \le \dfrac{25V_T}{15I} \\ \dfrac{15U_{RL0\max}}{20\pi V_T} - 1 \le \dfrac{R_2}{R_1} \le \dfrac{U_{RL0\max}}{\pi V_T} - 1 \end{cases} \quad (11)$$

Assuming operation at room temperature (V_T=26mV), a current I= 0.25 mA, sensor signals amplitude A=10V, and choosing a peak output voltage $U_{0RL\max}$=10V, simple analysis yields:

$$\begin{cases} 104\Omega \le R_1 // R_2 \le 173\Omega \\ 90 \le R_2/R_1 \le 126 \end{cases} \quad (12)$$

Since $A=RI$, the required value for R= 10V/0.25mA=40kΩ. Simulation of $U_{0RL}(\theta)$ according to (6) suggest that with R=40.04kΩ, R_1=130Ω and R_2=14130Ω , the residual error in the determination of θ using (5) is within ±0.048° in the full 360° range as shown in Figure 3.

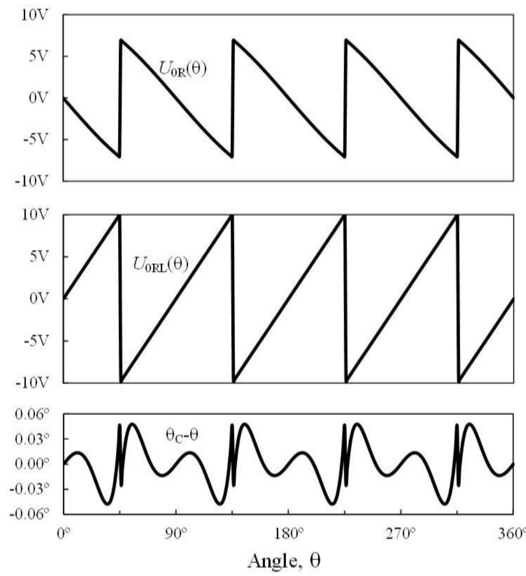

Figure 3. Simulation of the performance of the proposed linearization scheme. $U_{0RL}(\theta)$ is computed according to approximation in (6) with A=10V, I=0.25mA, R=40.04kΩ, R_1=130Ω, R_2=14130Ω, and V_T=26mV.

Note that (i) the inversion of $U_{0RL}(\theta)$ with respect to $U_{0R}(\theta)$ is due to the inverting configuration of the operational amplifier, and (ii) the amplitude A of the sensor signals is assumed to be 20Vpp and therefore $U_{0R}(\theta)$ is $20/2^{½}$ Vpp. It is important to note that the operating temperature usually affects signal shaping schemes; this can be seen in (6) through the thermal voltage term. However, some degree of temperature compensation is achieved in the present scheme by selecting a suitable current source element with appropriate positive temperature coefficient.

It is important to note that since the proposed scheme is open loop, the bandwidth is only limited by the dynamics of the components used, mainly the operational amplifiers and analog switches. In all cases, the bandwidth should be much higher than what is required in practical positioning applications: e.g., for a maximum rotational speed of 24000rpm (i.e., considered to be very high for real applications), the corresponding frequency would be 24000/60=400Hz only which does not pose any problem even for standard components.

III. EXPERIMENT

The practical converter is a straightforward implementation of the scheme of Figure 1 using few standard electronic components. Figure 4 depicts the detailed circuit diagram. Matched dual NPN transistors (MAT01) and adjustable current sources (LM334) have been used in the implementation of the linearizer. The offset cancellation, amplification, amplitude equalization and phase correction of the encoder output signals are not shown. The amplitudes of the converter's input signals $\hat{U}_C(\theta)$ and $\hat{U}_S(\theta)$ are 20Vpp, and the linearized output is 20Vpp.

The encoder used in the present work is a Hall effect sensor (model HSCB22). This sensor produces two quadrature sine and cosine signals with a nominal peak-to-peak amplitude of 4V and an offset of 2.5V. Appropriate analog signal conditioning is applied to the sensor signals in order to (i) remove the offset, (ii) correct the phase between its signals using (2) and (iii) adjust their amplitudes to 20Vpp. The sensor was driven with a dc motor through a 100:1 reduction gear.

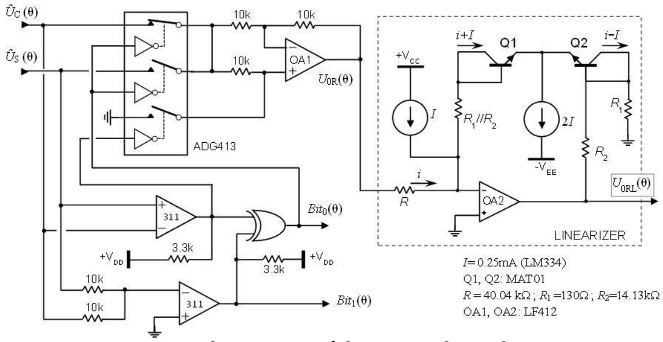

Figure 4. Basic implementation of the proposed open loop converter.

Figure 5 depicts the amplified sensor signals (1) together with the balanced signals (2); the offset in the sensor signals has been removed in the amplification stage generating (1) and the sensor was driven at 37rpm. Figure 6 shows plots of $U_S(\theta)$ versus $U_C(\theta)$ and $\hat{U}_S(\theta)$ versus $\hat{U}_C(\theta)$, and illustrates the effectiveness of phase and amplitude equalization.

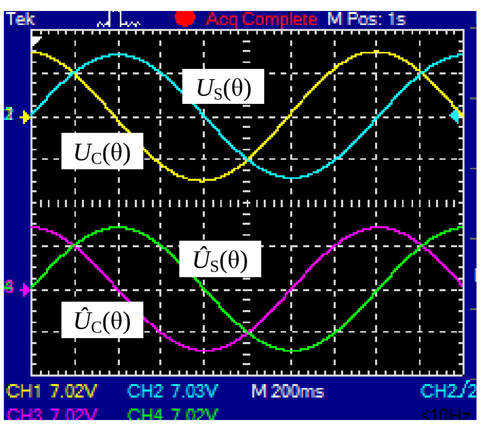

Figure 5. Unbalanced (upper) and balanced (lower) sensor signals at 37rpm.

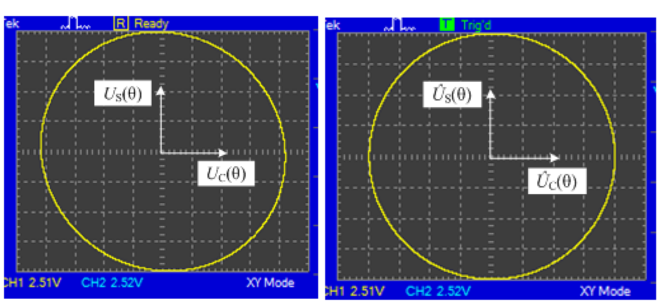

Figure 6. Sine versus cosine sensor signals before (left) and after (right) balancing.

Figure 7 shows the converter input signals $\hat{U}_S(\theta)$ and $\hat{U}_C(\theta)$, the signal $U_0(\theta)$ obtained from the multiplexer, the rectified signal $U_{0R}(\theta)$ and the linearized analog output $U_{0RL}(\theta)$ and binary outputs $Bit0(\theta)$ and $Bit1(\theta)$; the sensor was driven at a fixed rotational speed of 37 rpm. Figure 8. shows the converter signals by driving the sensor at the maximum rotational 240rpm; this is the maximum rotational speed that the setup allowed. It is clear, however, that because of its open loop nature, the converter should work at much higher speeds.

The characterization of the converter is shown in Figure 9 where the sensor was mounted on a miniature rotary table (model A5990TS with an integral 90:1 gear ratio and 100 arc-second accuracy) that enabled precise control of the angle θ.

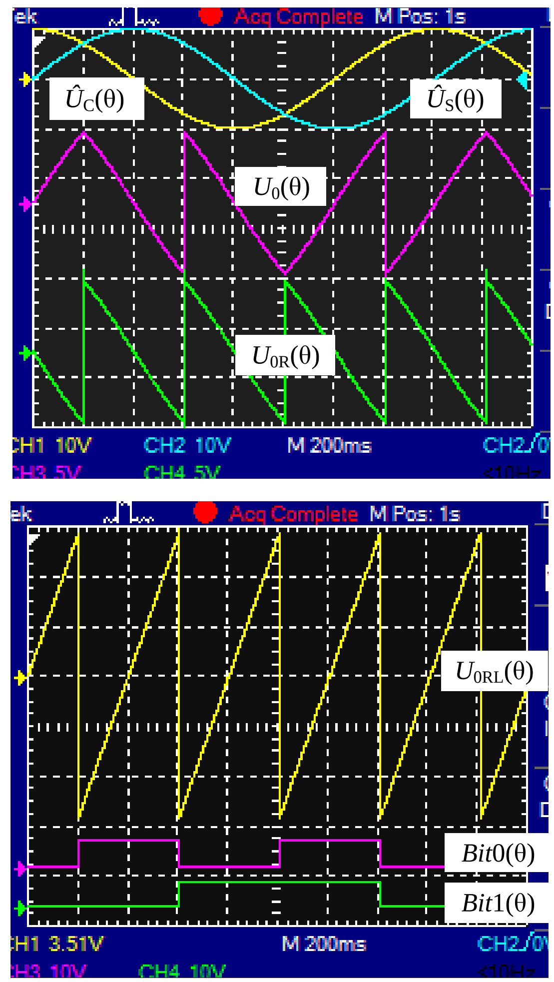

Figure 7. Converter signals at 37rpm.

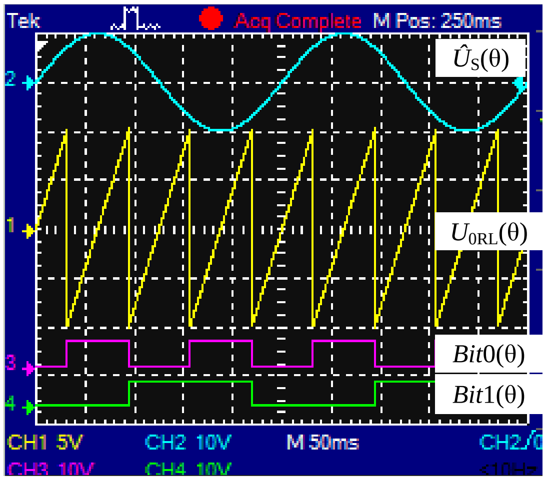

Figure 8. Converter signals at 240rpm.

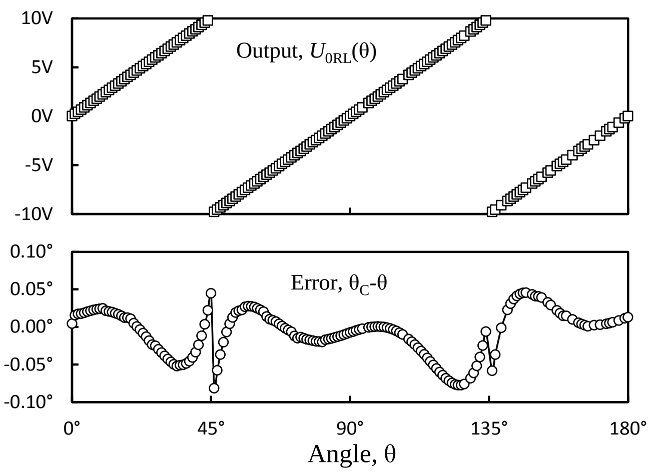

Figure 9. Characterization of the converter by controlling θ using a precision miniature rotary table.

The overall error in the determination of the angle θ_C from the converter signals was lower than 0.09° in the range 0 to 180° of input angle; this compares well with the theoretical error estimated above in Figure 3. Despite the simplicity and low cost of the proposed scheme, its error compares well with those of other more complex closed loop and open loop schemes reported in literature, e.g., 0.01° in [15], 0.10° in [17], 0.04° in [20], 0.18° in [37].

IV. CONCLUSIONS

In this paper, low-cost and simple-to-implement open-loop method for amplitude-to-phase conversion was proposed for use with sinusoidal Hall effect sensors. The conversion was based on a simple and effective linearization technique, full theory of which was given. The theoretical error of non-linearity of the converter is below 0.05°. The scheme was implemented using standard electronic components and was experimentally tested. The results have been excellent and showed that the proposed scheme can be used to measure angles in the full 360° range with an overall error lower than 0.09°.

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