Inverse Convolution Method for Periodic Media under Deterministic and Stochastic Condition

Xuefeng Li, Mohamed Ichchou, Abdelmalek Zine, Noureddine Bouhaddi
Ecole Centrale de Lyon, France
Email: xuefeng.li@ec-lyon.fr

Christophe Droz
KU Leuven, Division LMSD, Belgium
Email: christophe.droz@ec-lyon.fr

Abstract—**Wavenumber extraction has attracted widespread attention in periodic structures, sonar, radar and sensor fields. The Inverse Convolution Method (INCOME) is presented and optimized to extract wavenumber of 1D and 2D wave propagating in periodic structures under deterministic and stochastic condition in this paper. This method is one of inverse methods and combines the principle of the Prony series and the Bloch-Floquet theorem based on convolution framework.**

Keywords- Periodic structure; Wavenumber Extraction; Inverse method; Deterministic and Stochastic condition; Wave propagation.

I. INTRODUCTION

Wavenumber extraction has been used in periodic structures field to reveal many unique properties, such as complex band structure which has been applied to many applications, such as damage identification, vibration isolation, unable filters, wave guides and more [1]. The whole frequencies from which wavenumber is extracted can be divided into three regions: low frequency region, medium frequency region and high frequency region. For the low frequency region, traditional model-based analysis methods, such as Transfer Matrix Method (TMM) and Wave Finite Element method (WFEM), have an effective performance to extract wavenumber. But in the medium and high frequency region, these model-based analysis methods generally cause the computational issues and even are invalid. In order to overcome these issues, an amount of inverse methods involving Fourier-based wavenumber estimation methods and improved signal processing methods have been proposed. Fourier-based wavenumber estimation methods contain two-dimensional spatial Discrete Fourier Transform (2D-DFT), McDaniel's method, Inhomogeneous Wave Correlation (IWC) method [2], Inverse Wave Decomposition (IWD) and so on. In particular, IWC method has obvious advantages on identification of guided waves propagating in plates. For example, IWC is smaller sensitive to the boundary condition and the source of force and only requires the sparse signal as input parameters. But as an extension of the Fourier transform, it has itself disadvantages, such as the limitation of wavenumber resolution, which cause the inaccurate results when the number of wavelengths within the signal is few. Additionally, IWC leads to a non-linear optimization problem, maximizing the correlation coefficient between an inhomogeneous wave and the complete wave field. In the past few decades, several improved signal processing methods have been proposed such as Matrix Pencial, ESPRIT, Prony method, MUSIC and HRWA. Prony method and ESPRIT method are limited to 1D application and regard fine periodic measurements as input parameters. Recently, the High Resolution Wavenumber Analysis (HRWA) method proposed by P. Margerit provides an effective way to increase the wavenumber resolution and only requires to solve the linear problem by the combination of ESPRIT algorithm and the ESTER criterion [3]. The signal order is estimated due to the use of ESTER criterion and the complex wavevectors are extracted from the measured frequency response using ESPRIT method. Additionally, from the extracted wavenumber, this method allows the identification of some mechanical properties. But HRWA is implemented without considering the effect of uncertainties factor, such noise and non-periodicity sampling, on the wavenumber extraction. More recently, Boukadia proposed a novel improved signal processing method referred to INCOME as an extension of Prony method [4]. This method can be applied to extract wavenumbers in 1D and process K-space analysis in 2D based on linear algebra. But the periodic measurement is still a strong constraint in this method and the problem of automatic signal order identification needs to be solved. The rest of this paper is organized as follows. Section II introduces the principle of INCOME and uses a beam case to validate the effectiveness of INCOMDE. Section III describes the perspective and conclusion.

II. INCOME PRINCIPLE AND SIMULATION EXAMPLE

A. Principle

The core of the INCOME method is to use the Prony method (commonly used in spectrum estimation) to solve the estimation of the wavenumber in the periodic structure. When only considering the incident wave, the displacement field can be expressed as the sum of complex exponentials including different positive propagation constants.

$$U_n = \sum_{m=1}^{n_w} b_m \lambda_m^{-n} \quad (1)$$

Where b_m is the complex amplitude of propagating wave for m^{th} wave, n_w is the number of waves, λ_m is equal to the propagation constant of m^{th} wave and k_m is equal to wavenumber of m^{th} wave. The corresponding characteristic polynomial with coefficient a_m of the formula (1) is

$$\psi(\lambda) = \sum_{m=0}^{n_w} a_m \lambda^{n_w - m} \quad (2)$$

 ISBN: 978-1-61208-820-4

For INCOME, the input parameter is a series of experimental displacement data. These data can be expressed as an autoregressive matrix with Toeplitz structure and then, convolved with a vector of characteristic polynomial coefficients (product kernel). Finally, the least-squares method is used to estimate the best polynomial coefficients and propagation constant can be obtained by solving the roots of the characteristic polynomial. The out parameter is wavenumber obtained by Bloch-Floquet theorem. The whole process can be regarded as a filtering process and the estimation of the best filter is the key to the method.

B. Simulation example

The effectiveness of INCOME in deterministic condition is validated by the wavenumber extraction of two pairs of bending waves in a Timoshenko beam with resonators. Figure 1 shows a model of Timoshenko beam with resonators and the corresponding results in the positive direction are as follows:

Figure 1. The model of a Timoshenko beam with resonator

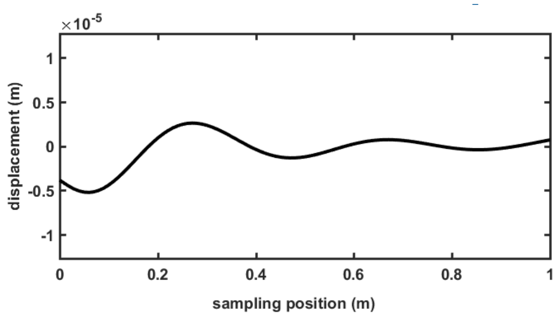

Figure 2. The frequency response curve at 500 Hz

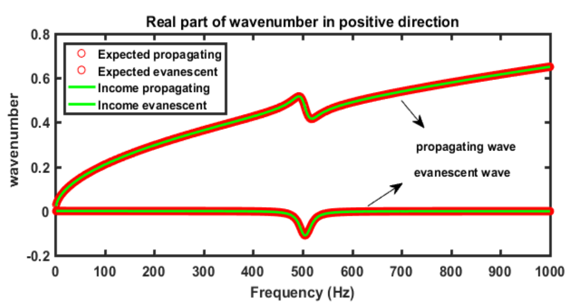

Figure 3. Real part of the complex dispersion curve

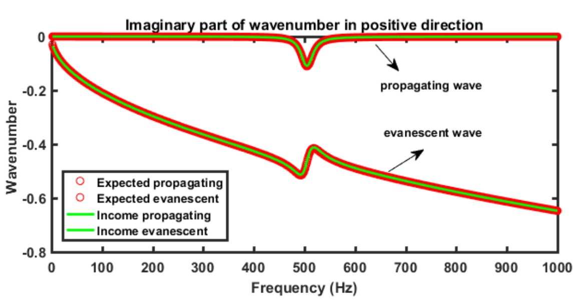

Figure 4. Imaginary part of the complex dispersion curve

The length L of beam is set to 1m and it is divided to 100 elements evenly. The weight and height are set to 2cm and 1cm respectively. The damping, Young's modulus, density and Poisson are set to 0.005, 210 $\times$ (1 + 0.005i) GPa, 7800 kg/m^3 and 0.3 respectively. 30 resonators are distributed evenly through the Timoshenko beam. The mass of resonators is equal to 5% of the overall beam. The Natural frequency of resonator is 500Hz and the damping factor of resonator is 0.05. Figure 2 shows the frequency response curve in 500Hz, which shows the amplitude of displacement is decaying exponentially with wave propagating. Figure 3 and 4 present the complex dispersion curve of waves propagating in positive direction, which can illustrate that the INCOME is effective to extract wavenumbers. On the other hand, the dynamic properties of periodic structure can be explained. When the frequency reaches the natural frequency of the spring oscillator, resonance occurs. The energy is dissipated during the process of resonance, leading to the band gap existing near the natural frequency. In the band gap, the wave can't propagate. This principle is mainly reflected by the imaginary part of the wavenumber. For example, in the imaginary part of the dispersion curve, the wavenumber of the propagating wave changes to negative number near the natural frequency of spring oscillator, which leads to the exponential attenuation of displacement and the appearance of band gap.

III. Perspectives and Conclusion

INCOME overcomes the limitations of Fourier-based methods and other traditional model-based analysis methods. The proposed method only relies on linear prediction theory instead of nonlinear optimization and allows a coherent estimate of the full K-Space in deterministic condition and stochastic condition. The theory of INCOME in deterministic condition is established and its effectiveness is validated by different cases in 1D. For stochastic case, it requires to be explored further, such as considering the effect of different kinds of disturbances on wavenumber extraction and the experimental cases need to be implemented to validate the effectiveness of INCOME further.

References

[1] M. I. Hussein, M. J. Leamy and M. Ruzzene, "Dynamics of Phononic Materials and Structures: Historical Origins, Recent Progress, and Future Outlook," J. Applied Mechanics Reviews, pp: 1380-1419, 2014, doi: 10.1115/1.4026911.

[2] M. N. Ichchou, J. Berthaut and M . Collet, "Multi-mode wave propagation in ribbed plates: Part I, wavenumber-space characteristics," J. International Journal of Solids & Structures, Vol.45, pp: 1179-1195, March 2008. Available from: https://doi.org/10.1016/j.ijsolstr.2007.09.032.

[3] P. Margerit, A. Lebée, J. F Caron, K. Ege and X. Boutillon, "The High-Resolution Wavevector Analysis for the characterization of the dynamic response of composite plates," J. Journal of Sound and Vibration, vol. 458, pp:177-196, 2019, dio: 10.1016/j.jsv.2019.06.026.

[4] R. F. Boukadia, E. Deckers, C. Claeys, M. Ichchou and W. Desmet, "An Inverse Convolution Method for Wavenumber Extraction," J. MSSP, in press.