

The Methodology of Text Messaging Quality Assessment

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Abstract— Telecommunication services are expected to be of good quality and offered for reasonable price. Operators, competing strongly for the customers, always present their products in the best light, and underline the highest service quality, which is often measured in incomparable circumstances, using different procedures and measurement methods. The paper presents the efforts of European standards institutions, regulators and operators in scope of improving telecommunication services provisioning and quality ensuring. Authors present the main parameters influencing the quality of Short Message Service which represents a wide range of text messaging services. The authors present methodology and the environment for measuring end-to-end delivery time. The measurement scenarios performed in both, the real network and in a laboratory environment are presented. The results of measurements, performed in real networks of four operators in Poland, show that the message delivery time fluctuates during the course of the day and also depends on the operator, but generally the short text service is of good quality and is highly assessed by the users. Authors present also the quality of experience model for text messaging.

Keywords- text messaging; SMS; QoS; QoE; quality assessment.

I. INTRODUCTION

In just a few decades, mobile telephony has reached a deeper level of penetration worldwide than cars, radio or TV. From over 700 million registered users in 2000, mobile cellular industry has grown widely and exceeded 7 billion subscriptions in 2015. With the increased number of mobile subscribers over the world, Short Message Service (SMS), has also gained a huge popularity [1], i.e., around 8 trillion messages a year [2][3]. Moreover, after voice, messaging is the biggest revenue-generating mobile service on the telecommunication market [4]. Although, in some countries SMS has peaked, and the traffic volumes are in decline, there are more countries where overall SMS traffic and its use-per-subscriber is still growing. A significant growth in mobile subscribers is observed in the Middle East, Asia, Africa and Latin America, thus the dominance of SMS in the immediate future is unthreatened. According to [4] SMS will be one of the major communications tools worldwide for the next decade, despite progressive extension of user equipment utility. The increase in the processing power of mobile devices has made them significantly more multi-functional

and allows Internet browsing, emailing, multimedia and instant messaging. Despite the rapid growth of so called Over-The-Top (OTT) messaging apps and Voice over IP (VoIP) services, SMS is still generating more than half of the total mobile messaging revenue (see Figure 1) [5].

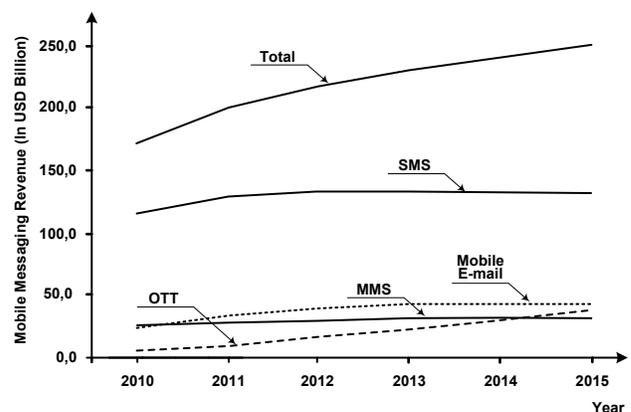


Figure 1. Mobile messaging revenue in recent years

Such large revenues mean that SMS will remain an important service for mobile operators for years. While Instant Messaging (IM) and SMS are both text messaging, there are differences that encourage different user behaviors. IM is rather two-way communication with many quick responses in contrast to SMS, where individual paid-for messages are used just to send information. While there are low to zero costs for the user when using IM, travelers stick to using SMS as it is cheaper than purchasing a mobile data package or subscribing for a data roaming plan in order to send a few messages. SMS is common to all phones and almost all users [5] while IM usually requires smartphones with dedicated apps and specific knowledge how to use it, which can be a barrier for some (older) part of society. Moreover, the market for IM is fragmented by different services which cannot communicate with each other. Besides, users also choose IM apps based on their geographical location. While WeChat is clearly the leader in China and Line for Japan, WhatsApp is also far bigger in East and South-East Asia than the USA. One of the major factors that allows service providers to keep their customers is the price, which is diminishing year by year, while the next ones are common availability, simplicity, and good quality. Service quality is becoming an increasingly more important factor to users at the moment when choosing the network operator or service provider [1][6][7].

Thus, many operators' efforts are concentrated on the efficient mechanisms for handling the message traffic [8]. On the other hand, a service oriented management is focused on service quality rather than network performance [9]. An effective evaluation of service quality can help service provider to increase customer satisfaction. Quality of Experience (QoE) based research mechanism for control and management of resources is getting more attentions in literature [10][11][12]. The SMS quality evaluation is also important in case of professional deployment of the service [13]. It is especially of great importance to many local governments (eg., in Poland) implementing SMS to notify inhabitants and all the guests in the area of the emergency states [14]. On the other hand, growing competition among service providers and network operators forces these entities to provide high quality services. The question of how to describe this quality and what parameters should be used, is asked not only by operators but also by regulators of the telecommunications market in the European Union [15]. One of the factors motivating telecommunications industry operators to act in this direction are regulations undertaken both at the European level [16][17], as well as in individual Member States [18][19]. A particular example of this can be seen in case of Poland, where on the Electronic Communications Office's initiative (2012), a Memorandum on Cooperation for Improving the Quality of Services in the Telecommunications Market has been signed. The first stage of works was finalized in the form of an official report [20], which was published by the Electronic Communications Office in February 2014. Despite long discussions, the current edition of this report does not define any QoS requirements for SMS, but there is a hope for a gradual expansion of the scope of this document. Although, SMS is not a real-time service, it is often perceived as such (a near-real-time) service [21] by a huge amount of users. Therefore, two factors seem to be important from the QoS point of view, these are: delivery rate and time of delivery. Nowadays, the delivery rate is mostly at a high level, reaching, in case of many operators, values around 95% [14]. However, these factors are correlated with each other, because the delivery rate also depends, among other things, on the delivery threshold time after which the message is considered as lost. Therefore, delivery time seems to be a key performance indicator (KPI), which is much more crucial for the service quality perceived by the users. They want the information to be delivered in acceptable time. But what does it mean? In the era of information and communication technologies with more and more bandwidth and rich service offerings, user demands concerning the service are also growing. Today SMS is more often treated as an almost instantaneous communication medium for rapid exchange of information, and even a form of text dialogue between people [21]. It seems that a relatively short time of message delivery is one of the main factors describing SMS quality affecting its popularity among users.

In Section II, Authors present the basics of SMS functionality and the main parameters and statistics describing quality of the service, according to the ETSI standards [22]. Section III presents the methodology and tools used during measurements in the real network. The message delivery time distributions are also presented and discussed. In Section IV, the Authors propose, on the basis of measurements results, the Quality of Experience model for SMS. Section V presents conclusions and the plans for the future work.

II. SHORT MESSAGE SERVICE QUALITY

SMS, belonging to the so-called "non-real-time" class, is a "store-and-forward" type of service [23]. Communication between two users is done via at least one server, acting as an intermediary unit. A user's equipment transmits a message to the server which optionally sends it to the next server and so forth. The end server, after receiving the message, informs a recipient's equipment of receiving a message and, finally, the user can read the message. SMS was originally designed for transmission of text information, where the length of single data unit cannot exceed 140 bytes and, according to ETSI standards [24], remains constant regardless of the number of characters transmitted in a single message. Depending on the alphabet used, the maximum message length may vary between 70 and 160 characters. When the information is longer, then it is divided and encapsulated into several 140-byte data units, and sent as separate messages. The quality study presented in this paper takes into account only the information that does not exceed the size of a single data unit. ETSI standards give a very detailed information regarding SMS quality parameters and their computation. The most important parameters are:

- SMS Successful Ratio - the ratio of correctly sent messages, expressed by the probability of correct message sending and its delivery to a service center,
- Completion Rate for SMS - indicator of properly delivered messages, expressed by the percentage of messages successfully sent and delivered to a recipient,
- End-to-End Delivery Time for SMS - time to deliver a message from end to end, expressed as the time measured from the moment of sending a message by a sender to a service center until it is received by a recipient.

Monitoring of the parameters, mentioned above, is crucial for the operator who has to watch over the process of service delivery at every stage of its implementation. It gives the knowledge of network performance, which in turn impacts the quality of service [25]. Message loss and message integrity are valid concerns, however, they are handled by lower layer network mechanisms and protocol, which are outside the scope of this paper. From the user's point of view, it is very important that messages are delivered to the recipient as soon as possible and in an unchanged form. From this perspective, it can be seen that

the parameter which probably has the strongest impact on the SMS quality, perceived by the user, is the End-to-End Delivery Time. According to the ETSI standard [22], the following statistics should be provided separately:

- the mean value in seconds for sending and receiving short messages,
- the time in seconds within which the fastest 95% of short messages are sent and received,
- the number of observations performed.

It should be noted that, concerning the mobile environment, the values of QoS parameters mentioned above can be affected not only by congestion in the SMS system or signalling channels but also by network or service non-accessibility in the claimed area of coverage. In that case, operators may wish to distinguish the effects of coverage and access congestion, but from the user’s point of view there is no need to do it, because all these phenomena impact on the end-user perception.

III. MEASUREMENTS IN THE REAL NETWORK

This Section presents the methodology, measurement environment, results, and evaluation of SMS quality provided by leading mobile operators functioning in Poland, i.e., Orange, Play, Plus and T-Mobile.

A. Methodology and the Measurement Environment

Data were collected from more than 120 000 tests (individual observations) performed during one week in Wroclaw - one of the biggest Polish cities (650 000 inhabitants). The test environment (see Figure 2) consists of measuring robots (one for the operator), each covering a Personal Computer with a 3G modem and specially designed application, managing the measurement and data collection process. Each robot plays both roles: sender and receiver. Initially, the first one sends a previously prepared text message to the Short Message Service Center (SMSC) located in the operator network, inserting in the destination address field its own number (i.e., both: sender and receiver belong to the same network). The measurements of SMS delivery time for the messages exchanged between users of different networks are not in the scope of this article. The time of message sending is written down into a special record of a log file on the robot’s hard disk. Then the SMSC sends the message further, i.e., to the receiver, which in this situation is the same robot that sent the message before. Next, the receiving part of the robot is informed of the incoming message and then it also records the current time in the log file. In this way, the file collects a number of records with the times of sending and receiving the particular messages. Each robot works independently of the others. However, the individual robot sends the messages sequentially, i.e., sending of the next message is possible only after receiving the previous one. The robot software allows the setting

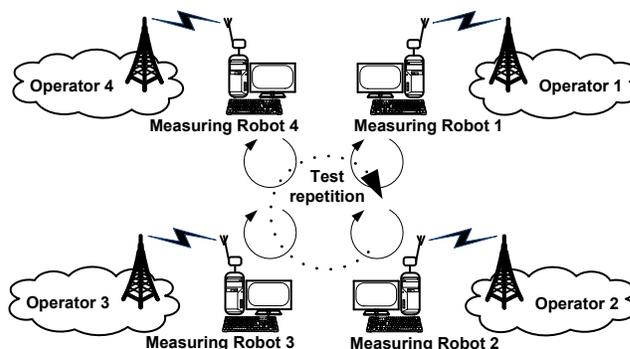


Figure 2. The test environment for the SMS parameter measurement in real mobile networks

of the time interval between the received and sent messages (see Figure 3) in order to control the frequency of message sending. It should be noted that such a solution causes the risk of substantially reducing the number of tests when the delivery time increases enormously. In order to eliminate such phenomenon, it is possible to limit the maximum acceptable delivery time and, if it is overrun, the expected message may be recognized as lost. Then the sending of the next message can start. In this phase of the measurements such a time limit was not used so that the robots were able to capture all delivery time cases.

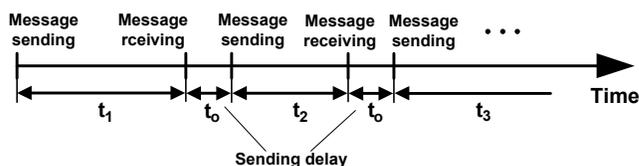


Figure 3. Sequential message sending scenario

B. Message Delivery Time Distributions

The measurements performed in real networks, mentioned before, allowed the message delivery time distributions in each network to be determined (Figures 4-7). After analysis of the distributions it can be stated that although very long delivery times occur in case of some operators (e.g., Play), which may be irritating to somebody who experiences that. From statistical point of view, however, they are not of considerable importance (it concerns about 10^{-4} cases). Moreover, it is negligible especially in the case of discarding 5% of the highest values before further analysis. The majority of the captured message delivery times do not exceed 10 seconds which means that SMS users should be satisfied. Moreover, almost 99% of the messages were delivered, in case of three operators, in time not higher than 6 seconds. Only in the Play network message delivery time distribution was different. The question is: are the captured message delivery times satisfying in case of real time text messaging applications? More detailed analysis will be shown in the Section IV, where the scores of subjective measurements will be taken into account.

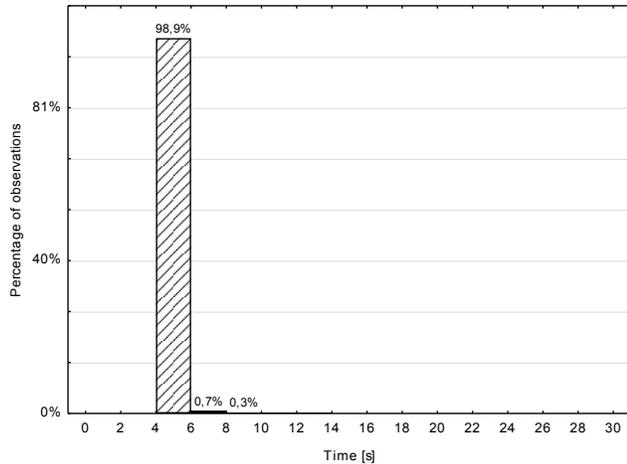


Figure 4. Messae delivery time distribution in the Orange network

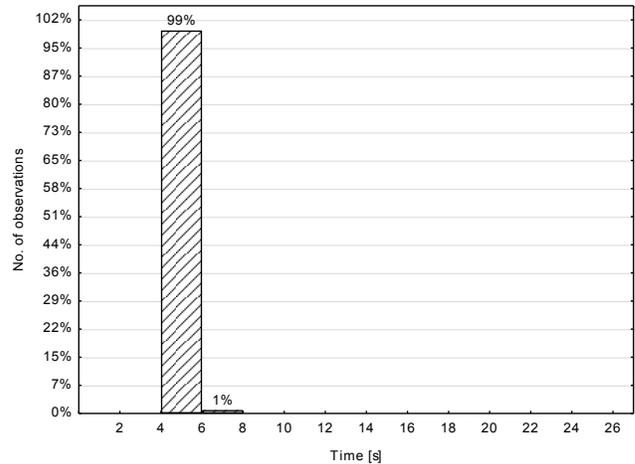


Figure 7. Message delivery time distribution in the T-Mobile network

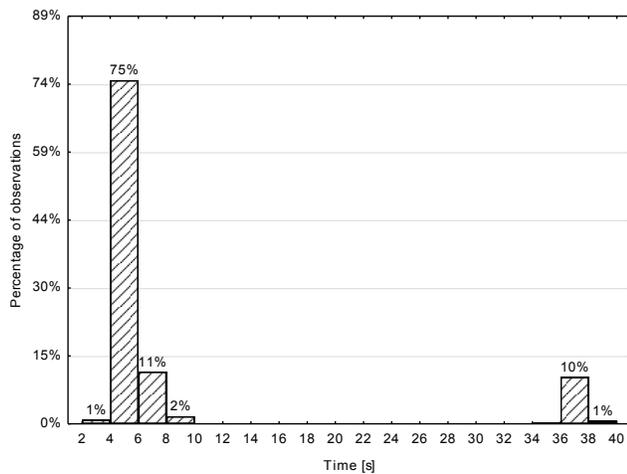


Figure 5. Message delivery time distribution in the Play network

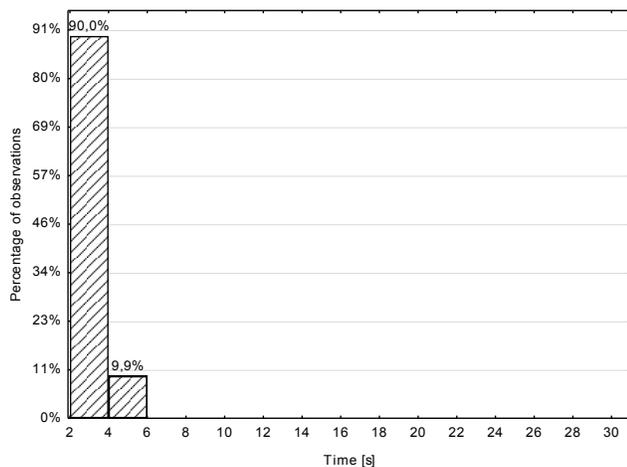


Figure 6. Message delivery time distribution in the Plus network

C. Message Delivery Time as a Function of the Time of Day

Analysis of the results showed that message delivery time, as expected, is not stable and is diversified depending on the operator (see Table I).

It can be noted that the lowest mean value of the message delivery time (3.6 s) occurred in case of operator No. 3.

TABLE I. COMPARISON OF MESSAGE DELIVERY TIMES

Operator (number)	Delivery time [s]				
	Mean	Min.	Max.	Std. dev.	Median
Orange (1)	4.66	2.8	26.1	0.47	4.6
Play (2)	15.79	3.1	10815	256.4	4.5
Plus (3)	3.6	3.3	65.7	0.68	3.5
T-Mobile (4)	4.96	3.1	23.7	0.39	4.9

A slightly worse score can be seen in networks 1 and 4. The longest delivery times, and standard deviation as well, was offered by operator No. 2. Such rough analysis can lead us to the conclusion that the SMS does not work properly and many of users may be dissatisfied with the service. On the other hand, when we take into account the median, which is by definition the value located in the middle of the population, it can be noted that it is comparable with the appropriate parameters of the other operators. Moreover, the median value seems to be a better parameter describing the service quality experienced by the users in the case of high standard deviation of QoS parameters. As mentioned before, ETSI proposes to describe the message delivery time by presenting the time within which the fastest 95% of short messages are sent and received [22]. According to the above, the distributions of message delivery times as a function of the time of day are presented in Figures 8-11. The black points represent median values, whereas the dashed boxes show the ranges of delivery time after discarding 5% of the lowest and highest values, respectively. In another words, they represent 90% percent of the captured samples population. Moreover, the top level of each dashed box

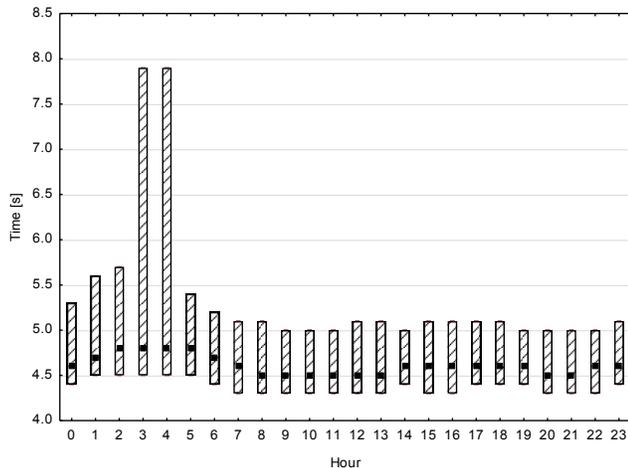


Figure 8. Short message delivery time in Orange network as a function of the time of day

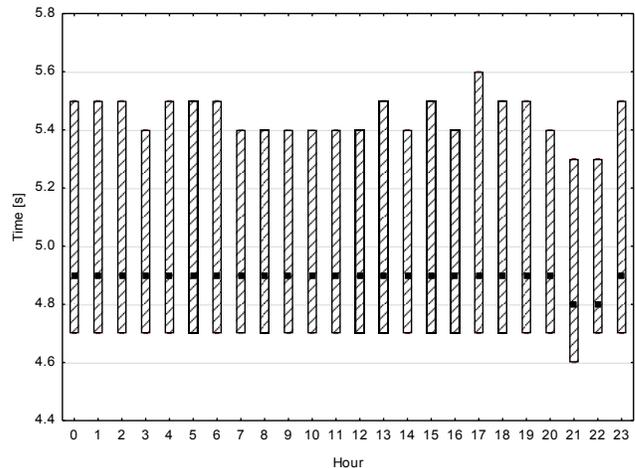


Figure 11. Short message delivery time in T-Mobile network as a function of the time of day

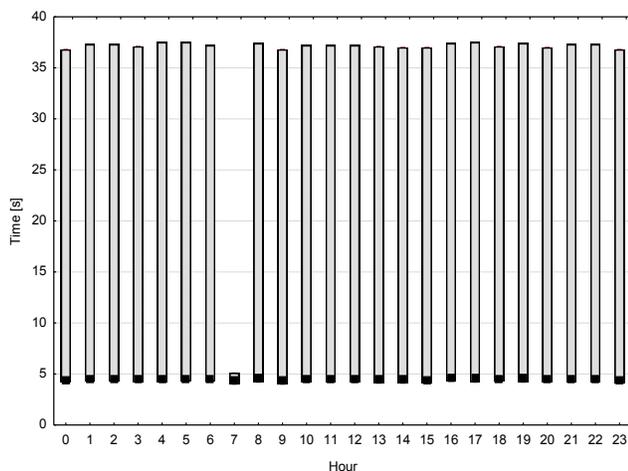


Figure 9. Short message delivery time in Play network as a function of the time of day

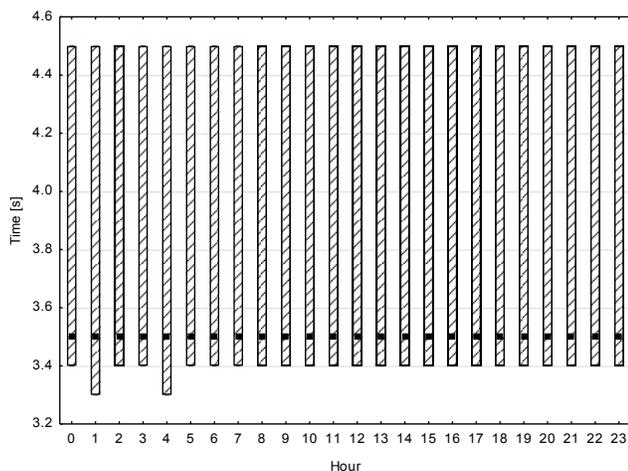


Figure 10. Short message delivery time in Plus network as a function of the time of day

denotes the highest limit of delivery time for 95% of messages sent in the relevant hour [22]. As presented in Figure 4, fluctuations of the message delivery time experienced by almost all SMS users of the Orange network, do not exceed the value of 6 seconds. The highest deviation is observed around 3 and 5 o'clock in the morning. The results of the observations performed in Play network show the values of delivery time deviation which are almost at the same level, except one hour (7 a.m.), during the whole day. However, comparing to the Orange network, the range of deviations, observed in the individual time intervals, is much wider here. It means that users of the Play network still experience relatively high fluctuations of message delivery times previous network, but comparable with them and better than obtained in Play network. It should be noted that the median values and deviations of the message delivery times, presented in Figures 8-11, are valid for 90% of observations and may slightly differ from the values shown in Table I, which takes into account all the captured data. Although the median (or even the mean) values of the message delivery times and their deviations can be used to compare the different operator's network performances or QoS parameters, they do not answer the question concerning the quality assessed by the users. For this reason, the relation between objectively measured QoS parameters and the quality of experience (QoE), which is subjective, should be determined.

IV. QOE MODEL

This paragraph presents the methodology and test-bed (Figure 12) for assessing the quality experienced by users (QoE) of text messaging services. The concept of the measurement environment is based on the server emulating service provider and several test positions representing users of the service. Each position consists of a Personal Computer (PC) with a special application emulating the mobile phone that sends and receives short messages. In the experiment all test messages have the same

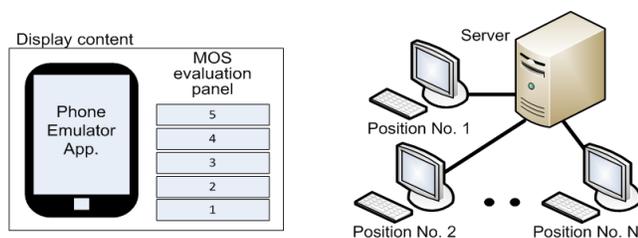


Figure 12. The laboratory test environment for the evaluation of text messaging Quality of Experience

format and content. Users send a number of messages which are passed through the server to the destination addresses after a period of time which may be controlled during the test. After receiving the message, users assess the service quality by evaluation of end-to-end delivery time and choosing the appropriate marks from the MOS evaluation panel, where MOS stands for Mean Opinion Score, expressed in a 5-level scale (0 – the worst case and 5 – the best one, respectively). All the test parameters and user marks are stored on the server and saved for further analysis. Several dozen users took part in the experiment and more than 1.2 thousand tests were performed. The results of the tests allowed building a QoE model (see Formula 1) which indicates a relationship between end-to-end message delivery time and service quality perceived by the users. Statistical analysis shows significant correlation (almost 80%) between message delivery times and the users' evaluation grades. Next, a regression analysis was performed and, using ordinary least squares (OLS) estimation, the approximate relation between message delivery time and users' grades (in MOS scale) was determined:

$$\text{MOS} = -0,1 \cdot T + 4,97 \quad (1)$$

where: T - message delivery time.

Due to the distribution of the data was not normal (checked by Shapiro-Wilk test [26]), authors made a validation of the model, using Mann-Whitney-Wilcoxon (MWW) test [27], which can be applied on even unknown distributions, contrary to t-test which has to be applied only on normal distributions [28]. This test showed a good estimation of the users' quality perception, under assumption of 95% confidence interval (significance level $p < 0.05$).

As it was mentioned before, the mean value of the QoS parameter sometimes might not be the best indicator of the network performance or the quality perceived by users. Therefore, authors presented the SMS QoE model which shows the relation between the message delivery times and the median values of user ratings (Figure 13). Thus the authors proposed a new name for the scale, i.e., Median Opinion Score (MedOS). The black points represent the median value, whereas upper level of the dashed boxes determine the scores given by the 95% of users experiencing the specific message delivery times. Four levels of the quality, acceptable by users, were defined, i.e., excellent quality (EQ), good quality (GQ), fair quality (FQ) and poor quality (PQ), respectively.

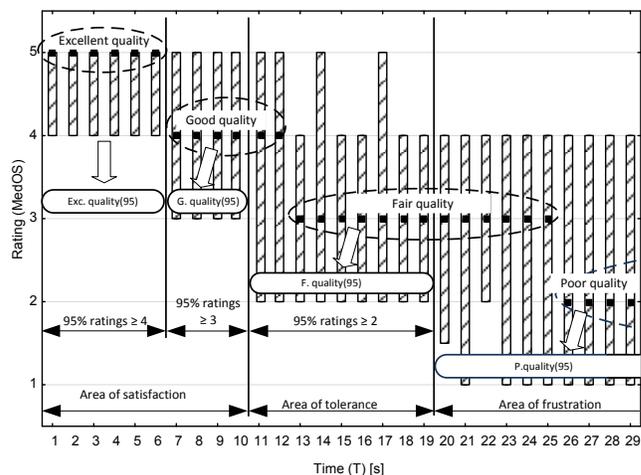


Figure 13. The quality experienced by the users majority (MedOS scale)

of the message delivery times are assigned to the proper quality levels (EQ, GQ, FQ or PQ) on the basis of median values of user scores, given for those times (see the dashed ellipses in Figure 13). According to ETSI [22], the quality levels should take into account the best 95% of samples. Excellent Quality level, denoted by EQ(95), is reached when the median of the user scores is equal to 5, and 95% of the samples have ratings equal or higher than 4.

TABLE II. SHORT MESSAGE DELIVERY TIME DISTRIBUTION FOR 95% CAPTURED SAMPLES

Operator	Percentage (P) [%]			WAQF
	EQ	GQ	FQ	
Orange	100	0	0	5
Play	80	13.7	6.3	4.74
Plus	100	0	0	5
T-Mobile	100	0	0	5

The same procedure is applied to the other quality levels, respectively. In this way, the relations between the message delivery times (QoS) and proper quality levels (QoE) were determined. Table II presents, according to the MedOS scale, the measurement results obtained from the four examined real networks.

$$\text{WAQF} = (P_{EQ} \cdot 5 + P_{GQ} \cdot 4 + P_{FQ} \cdot 3) / 100 \quad (2)$$

Next, the Weighted Average Quality Factor (WAQF) can be calculated using (2). It can be used as a parameter that allows a comparison of the SMS quality provisioned by different operators.

V. CONCLUSION AND FUTURE WORK

Nowadays, the text messaging is one of the most popular means of communication. Therefore, the high quality of the service is crucial in today's competitive market. Operators should continuously monitor network performance parameters in order to detect and isolate the problems and different kinds of threats which can impact on the quality experienced by the end-users. Thus, it is very important

to have not only the knowledge about the values of objectively measured performance parameters, but also about their influence on the service quality subjectively perceived by the users. The results presented in the article show that the SMS provisioned by the operators functioning on the examined area of the Polish telecommunication market is of very good quality and can be used, to some extent, as a medium which supports also other kinds of text communication, especially those that requires short end-to-end delivery times and immediate user-to-user interactions. Obviously, the message delivery time fluctuates during the course of the day and also depends on the operator, but generally brings great satisfaction for the users. It should be also noted that such, a relatively small, amount of collected data does not allow to make a general statement about the whole Polish network. Such generalization would be made after collecting data from a bigger and representative number of selected areas, which is to be done in the next step of the investigations. Authors want to underline that although SMS cannot be treated as a real-time messaging service, in some cases it can be used as an alternative. The main strengths of SMS are world-wide availability and no special requirements for the user equipment or specific software applications. Further work will be devoted to developing the QoE model towards more comprehensive investigation of the quality issues regarding not only intra- but inter-operator communication, as well.

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