# Simultaneous Velocity Estimation and Range Compression for High Speed Targets ISAR Imaging Based on the Chirp Fourier Transform

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***Abstract*—High speed targets (e.g., aircraft, satellite) imaging is crucial for air traffic safety and space surveillance. Based on the target motion model, the characteristics of the multi-component linear frequency modulation (LFM) signal caused by the high speed motion are analyzed in detail. Furthermore, a novel method based on the chirp Fourier transform and Shannon minimum entropy principle for high speed targets Inverse Synthetic Aperture Radar (ISAR) imaging is proposed. Firstly, according to the velocity scope, the chirp rate interval is established. Secondly, the corresponding phase term is constructed to compensate the quadratic phase term caused by the high speed motion. After the range compression, the range profile with minimum entropy is regarded as the optimal compensated profile, and finally the required ISAR image is formed by the cross-range compression. The simulation experiments demonstrate that the proposed method can obtain better ISAR images than the classic range doppler algorithm and fractional Fourier transform algorithm.***

***Keywords-high speed targets; Inverse Synthetic Aperture Radar (ISAR); chirp-Fourier transform; minimum entropy.***

## I. INTRODUCTION

Radar can actively transmit electromagnetic waves, and then extract the target information from the received echoes. In this sense, radar can explore targets in any time and under any circumstance. Due to the limitation of resolution, the targets observed by the traditional radars are generally considered as point targets with position and motion parameters. With the development of modern wideband radar, the capability of radar has been expanded from target detection, location, and tracking to target imaging, which greatly improves the ability of radar targets recognition. Therefore, radar imaging has become a research frontier in the field of radar signal processing. Inverse Synthetic Aperture Radar (ISAR), as a representative application in the radar imaging domain, has the ability of acquiring target image with electromagnetic information. In detail, based on the scattering point model, ISAR obtains high range resolution by transmitting wideband signal, and derives high cross-range resolution from the Doppler effect produced by relative rotation between radar and target [1].

As a classical imaging method in the domain of ISAR imaging, the Range-Doppler (RD) algorithm forms a two-dimensional high resolution ISAR image by using a two-dimensional Fourier transform along the range and the cross-range directions respectively [2][3]. However, the RD algorithm is only valid for the low speed moving targets and is not suitable for the high speed targets. The reason lies in the fact that, within a pulse duration time, the moving distance of a high speed target with approximately a few kilometers per second (e.g., satellite, space station) will exceed one range cell, which is in contradiction with the "stop and go" preconditions required by the RD algorithm and may cause the problem of range profile distortion [4][5][6].

Many methods have been proposed to solve the problem of range profile distortion of the high speed targets. Zhang et at. [6] proposed a velocity compensation method， where Wigner Distribution and Radon transform were utilized to estimate the velocity of high speed targets in advance. But the cross terms generated by Wigner Distribution made the velocity difficult to estimate. Fractional Fourier transform (FRFT) and Shannon minimum entropy principle were combined for velocity estimation in [7]. However, the multiple scattering points model has a great impact on the peak search in FRFT, and may lead to pseudo-peak location. A novel high speed motion compensation method based on Keystone transform and FRFT was proposed by Li et al. [8] in 2017. It should be noted that the numerous approximations in the method may lead to a series of biases in the results. In conclusion, most of the existing ISAR imaging methods follow the strategy of 'velocity estimation, velocity compensation, range compression and cross-range compression', which complicates the imaging process and magnifies the error propagation of the velocity estimation.

Since the velocity estimation is only an intermediate step of ISAR imaging, this paper proposes a method to accomplish velocity estimation and range compression simultaneously by using chirp Fourier transform and Shannon minimum entropy principle. After that, the Fourier transform along cross-range direction is implemented to obtain the ISAR image. This method can avoid the peak search biases in FRFT and reduce the complexity of the imaging algorithm, while maintaining the quality of ISAR imaging results.

The rest of this paper is organized as follows: Section II deals with the high speed target echo model and the origin of range distortion. Section III is devoted to the definition of chirp Fourier transform, the concrete process of velocity compensation, and the range compression based on chirp Fourier transform. The simulation results are demonstrated and discussed in Section IV. Section V draws conclusion on the proposed imaging method.

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## II. HIGH SPEED TARGETS ECHO MODEL AND RANGE DISTORTION

The geometry configuration of a monostatic two-dimension ISAR imaging system is shown in Fig. 1. According to the electromagnetic scattering theory, when radar works in the high frequency domain, the total echo received from the target can be modeled as the sum of echoes from different isolate scattering points within the target.

Denoting that the radar transmits LFM signal with the carrier frequency $f_c$, bandwidth *B*, pulse duration time $T_p$, pulse interval $T_r$, and intensity $\sigma_i$ for the *i*th scattering point, then the *m*th echo received from the target can be modeled as follows:

$$S_r(\hat{t}, t_m) = \sum_i \sigma_i rect\left(\frac{\hat{t}-\tau_i}{T_p}\right) \exp\left[-j2\pi\left(f_c\left(\hat{t}-\tau_i\right)+\frac{1}{2}\gamma\left(\hat{t}-\tau_i\right)^2\right)\right] \quad (1)$$

Where $t_m = mT_r$ is the slow-time, $\hat{t} = t - mT_r$ is the fast-time, $\gamma = B/T_p$ is the chirp rate, $\tau_i = \frac{2R_{im}}{c} = \frac{2\left(R_{im0}+v_m\hat{t}\right)}{c}$ is the time delay, $R_{im0}$ is the original distance between the radar and the *i*th scattering point at *m*th pulse, $v_m$ is the velocity of the target component on the radar-target direction at the *m*th pulse.

Denoting the reference distance of *m*th pulse as $R_{ref}$, then the reference time delay is $\tau_{ref} = \frac{2R_{ref}}{c}$. With the same carrier frequency $f_c$ and chirp rate $\gamma$ as the transmitted signal, the reference signal can be denoted as:

$$S_{ref}(\hat{t}, t_m) = \sum_i rect\left(\frac{\hat{t}-\tau_{ref}}{T_{ref}}\right) \exp\left[-j2\pi\left(f_c\left(\hat{t}-\tau_{ref}\right)+\frac{1}{2}\gamma\left(\hat{t}-\tau_{ref}\right)^2\right)\right] \quad (2)$$

The result of the dechirp process, i.e., multiplying the conjugate of (2) with (1), can be written as:

$$S_{if}\left(\hat{t}, t_m\right) = S_r\left(\hat{t}, t_m\right) \times S_{ref}^*\left(\hat{t}, t_m\right) \quad (3)$$

Then the precise echo model after the dechirp process can be modeled as:

$$S_{if}(\hat{t}, t_m) = \sum_i \sigma_i rect\left(\frac{\hat{t}-\tau}{T_p}\right) \exp\left[-j2\pi\left(\varphi_i + f_i\hat{t} + \frac{1}{2}K\hat{t}^2\right)\right] \quad (4)$$

$$\varphi_i = f_c \frac{2\left(R_{im0}-R_{refm}\right)}{c} - \gamma \frac{2\left(R_{im0}^2-R_{refm}^2\right)}{c^2} \quad (5)$$

$$f_i = \frac{2v_m}{c} f_c + \gamma \frac{2\left(R_{im0}-R_{ref}\right)}{c} - \frac{4\gamma R_{im0} v_m}{c^2} \quad (6)$$

$$K = \gamma \frac{4v_m}{c}\left(1-\frac{v_m}{c}\right) \quad (7)$$

As for the low speed targets like airplanes, the moving distance along range direction within the pulse duration time does not exceed one range cell ($v_m T_p << \frac{c}{2B}$). Hence, the velocity item $v_m / c$ can be neglected. Consequently, (4) is simplified to single-frequency signal, which means that implementing Fourier transform along range direction can achieve the high range resolution. Nonetheless, as for the high speed targets like satellites, their velocities no longer meet the above condition, sometimes even $v_m T_p >> \frac{c}{2B}$ will occur. In such cases, the velocity term $v_m / c$ cannot be ignored anymore.

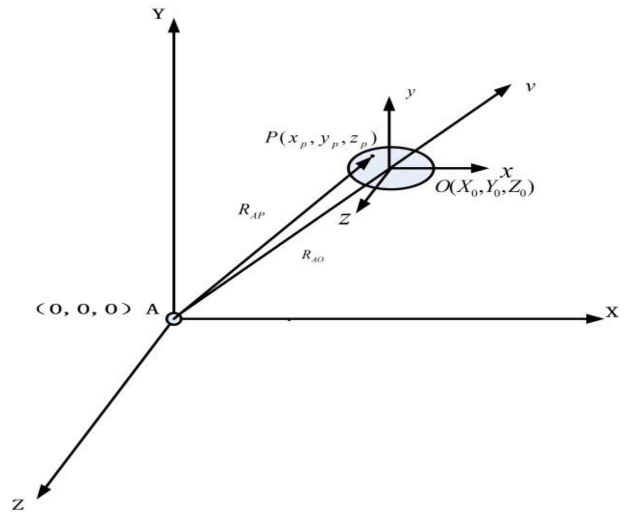


Figure 1. Geometry of imaging system

According to (4) - (7), due to the velocity $v_m$, the echo of the high speed target after dechirp process is a multi-component LFM signal. However, since $R_{im0}$ changes with different scattering point, the carrier $f_i$ of multi-component LFM signal also varies consequently. Furthermore, since the pulse duration time is very short (a fraction of a millisecond), $v_m$ can be regarded as invariant within the pulse duration time. Therefore, the chirp rate *K* in (4) and (7) can also be considered constant. In view of the wide bandwidth of the LFM signal, the direct Fourier transform makes the spectrums of multiple LFM signals overlap, which is so-called range distortion. Consequently, the range resolution will be degraded and the ISAR image will be defocused. For high resolution range profile, multi-component LFM signals should be changed to multiple single-frequency signals in advance. After that, the Fourier transform along cross-range can be taken to form the desired high resolution ISAR image.

## III. VELOCITY COMPENSATION BASED ON CHIRP FOURIER TRANSFORM

Chirp Fourier transform is an effective method to eliminate the quadratic phase term caused by the high speed motion. This section will introduce the definition of chirp Fourier transform and the velocity compensation method based on it.

### A. Chirp Fourier transform

Chirp Fourier transform is a kind of parameter estimation method in signal processing field. Chirp Fourier transform of signal *s*(*t*) is as follows:

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$$F(f,\gamma)=\int s(t)\exp\left[j2\pi\left(ft+\frac{1}{2}\gamma t^2\right)\right]dt \tag{8}$$

Denote $s(t)$ as binomial signal:

$$s(t)=\exp\left[-j2\pi\left(f_1 t+\frac{1}{2}\gamma_1 t^2\right)\right] \tag{9}$$

Where $f_1$, $\gamma_1$ are the carrier and chirp rate of signal $s(t)$ respectively. Plugging (9) into (7), the result of chirp Fourier transform of $s(t)$ is:

$$F(f,\gamma)=\int \exp\left\{j2\pi\left[(f-f_1)t+\frac{1}{2}(\gamma-\gamma_1)t^2\right]\right\}dt \tag{10}$$

When $f_1=f$, $\gamma_1=\gamma$, the integration result of the signal $s(t)$ can accumulate coherently to a peak at $(f_1,\gamma_1)$. If not, the integrated signal will be cancelled out, and the amplitude after accumulation will be much lower. Therefore, after chirp Fourier transform, a peak in the amplitude map will be formed at $(f_1,\gamma_1)$, which are the parameters to be estimated.

### *B. Velocity compensation based on chirp Fourier transform*

As discussed in (4), the received echo from the high speed target is a multi-component binomial signal. The spectrum will be shifted as an entirety, although the carrier frequency $f_i$ varies within the slow-time domain. Range alignment and phase correction can eliminate the effect. It is known that the primary reason for range distortion is the quadratic phase term of the echo. As a result, compensation for the quadratic phase term can avoid range distortion effectively. The quadratic phase term for velocity compensation is constructed as follows:

$$S_{comp}=\exp\left[j2\pi\left(\frac{1}{2}K_1\hat{t}^2\right)\right] \qquad \hat{t}\in\left(-\frac{T_p}{2},\frac{T_p}{2}\right) \tag{11}$$

After multiplying (11) with (4) and taking Fourier transform within the fast-time domain, the range profile is:

$$\begin{aligned}S_r(f,t_m)&=\int\sum_i\sigma_i\,\mathrm{rect}\left(\frac{\hat{t}-\tau}{T_p}\right)\exp\left\{-j2\pi\left[\varphi_i+f_i\hat{t}+f\hat{t}+\frac{1}{2}(K-K_1)\hat{t}^2\right]\right\}d\hat{t}\\&=\sum_i\sigma_i\exp(-j2\pi\varphi_i)\int\mathrm{rect}\left(\frac{\hat{t}-\tau}{T_p}\right)\exp\left\{-j2\pi\left[f_i\hat{t}+f\hat{t}+\frac{1}{2}(K-K_1)\hat{t}^2\right]\right\}d\hat{t}\end{aligned} \tag{12}$$

From (12), when $K_1=K=\dfrac{4v_m}{c}\gamma$, the quadratic phase term will be eliminated efficiently, and LFM signal will be converted to single-frequency signal, which means that the range distortion is avoided.

After chirp Fourier transform, the range profile will focus well. Additionally, for the sake of simplicity, the real chirp rate is changed to the form of digital chirp rate $k=K/f_s^2$. Constructing the compensation phase as follows:

$$S_{comp}=\exp\left[j2\pi\left(\frac{1}{2}kn^2\right)\right] \qquad n\in\left[-\frac{N}{2},\cdots\cdots,\frac{N}{2}-1\right] \tag{13}$$

According to the velocity scope, the digital chirp rate $k=\dfrac{4\gamma v}{cf_s^2}$ interval is determined by:

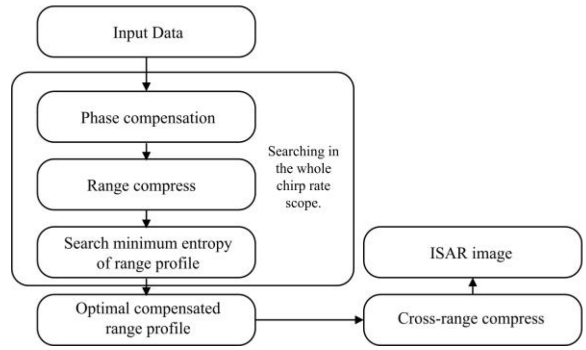


Figure 2. Concrete flowchart of the proposed method

$$k\in\left[\frac{4\gamma v_{\min}}{cf_s^2},\frac{4\gamma v_{\max}}{cf_s^2}\right] \tag{14}$$

As the quadratic phase term is compensated by chirp Fourier transform over the entire range of the digital chirp rate $k$, many range profiles are obtained. The entropy of each range profile is then calculated, and the specified range profile with the smallest entropy is considered as the optimal one. The exact definition of entropy can be written as (15). Through the above process, the high resolution range profile with the optimal velocity compensation is obtained. The concrete flowchart of the proposed method is shown in Fig. 2.

After chirp Fourier transform, the cross-range Fourier transform and the Keystone transform are combined to form the ISAR image of the target. The image entropy and the image contrast are introduced to evaluate the quality of the ISAR image. Larger contrast and smaller entropy indicate that an ISAR image has better quality. Denoting ISAR image as $I(m,n)$, the definition of ISAR image entropy and contrast can be written as [9]:

$$\begin{aligned}H&=-\sum_{m=1}^{M}\sum_{n=1}^{N}P(m,n)\ln P(m,n)\\P(m,n)&=\frac{|I(m,n)|}{\sum_{m=1}^{M}\sum_{n=1}^{N}|I(m,n)|}\end{aligned} \tag{15}$$

$$C=\frac{\sqrt{E\left\{\left[I(m,n)-E(I(m,n))\right]^2\right\}}}{E\left[I(m,n)\right]} \tag{16}$$

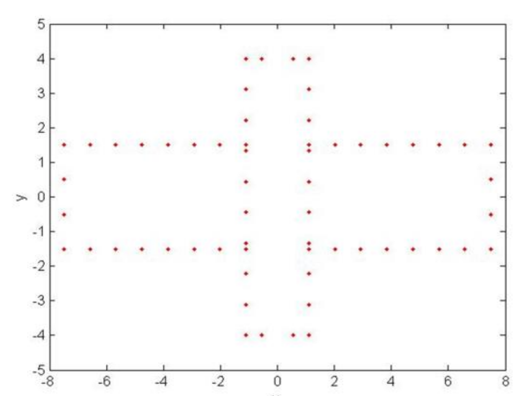


Figure 3. Scattering point model

TABLE I. RADAR AND TARGET PARAMETER CONFIGURATION

| **Target parameter** | **Real size** | | **Flying height** | **Velocity scope** | **Ideal scattering point number** |
|---|---|---|---|---|---|
| | ***15m╳8m*** | | ***100Km*** | ***(6100-6450) Km/s*** | ***64*** |
| **Radar parameter** | $f_c$ | $B$ | $T_p$ | $PRF$ | $f_s$ |
| | ***16GHz*** | ***1GHz*** | ***0.1ms*** | ***256Hz*** | ***5.12MHz*** |

TABLE II. ISAR IMAGE QUALITY

| **Image quality/ algorithm** | **RD algorithm** | **FRFT algorithm** | **The proposed method** |
|---|---|---|---|
| **Entropy** | ***9.7180*** | ***9.5759*** | ***9.0735*** |
| **Contrast** | ***5.4974*** | ***6.9348*** | ***7.7437*** |

## IV. EXPERIMENT RESULT

To demonstrate the compensation performance of the quadratic phase term with chirp Fourier transform, a simple satellite scattering model with 64 scattering points is established, as is shown in Fig. 3. Simulation experiments are taken under Signal Noise Ratio (SNR) of 5dB, and the parameter configurations for the radar and the target are shown in Table I.

The concrete target scattering point model is presented in Fig. 3. Fig. 4 illustrates the range profiles of the 50th pulse obtained by the proposed method and Fast Fourier transform (FFT), wherein the result of FFT is shown in red dotted line, and the result of the proposed method is shown in the blue full line. Owing to the wide bandwidth of LFM signal, direct Fourier transform without velocity compensation will lead to the spectrum overlap between multi-component LFM signals. As a result, the range profile distortion will occur, as shown in red dotted line in Fig. 4. Since chirp Fourier transform can convert LFM signal to single-frequency signal, the range distortion can be avoided efficiently, as is shown in blue full line in Fig. 4. In conclusion, the range profile of the proposed method focuses better than that of FFT.

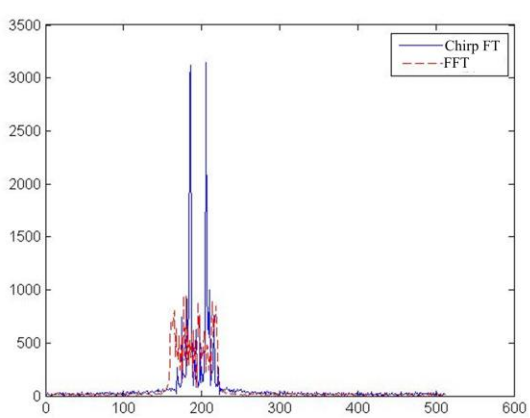


Figure 4. Range profile of 50th echo

Fig. 5, Fig. 6 and Fig. 7 are the ISAR imaging results obtained by RD, FRFT and the proposed algorithm respectively. In addition, the image entropy and image contrast of these three methods are shown in Table II. The image entropy and image contrast of Fig. 5, Fig. 6 and Fig. 7 are (9.7180,5.4974), (9.5759,6.9348), and (9.0735,7.7437) respectively.

Furthermore, as is shown in Fig. 5, the imaging result obtained by the RD algorithm is severely defocused and the scattering points cannot be resolved from each other. Although the ISAR image of the FRFT algorithm shown in Fig. 6 is better focused than that of the RD algorithm, there are still many scattering points difficult to resolve in the target model. Comparing to the above methods, the imaging result obtained by the proposed method is better focused, as is shown in Fig. 7. According to the energy convergence level, the image resolution of the proposed method is higher than that of the RD and FRFT algorithms. Therefore, through qualitative analysis, the ISAR image obtained by the proposed method is much clearer than that of the RD and FRFT algorithms. It can also be noticed that the entropy of the ISAR image obtained by the proposed method is smaller than that of the other two methods, while its contrast is greater than the other two methods. The quantitative comparison on the image entropy and the image contrast demonstrates that the image quality of the proposed method is better than that of RD and FRFT algorithms. In summary, the ISAR image of the proposed method has a higher level of energy convergence.

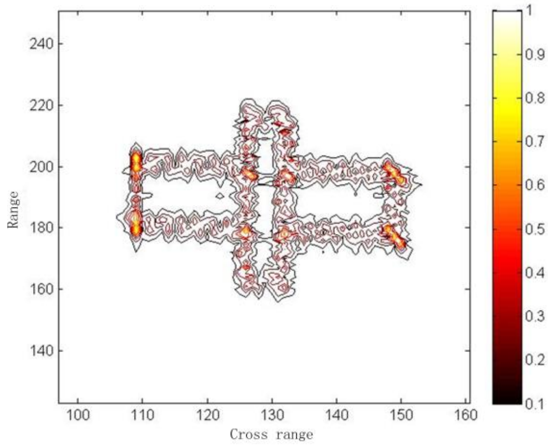


Figure 5. Image of RD algorithm

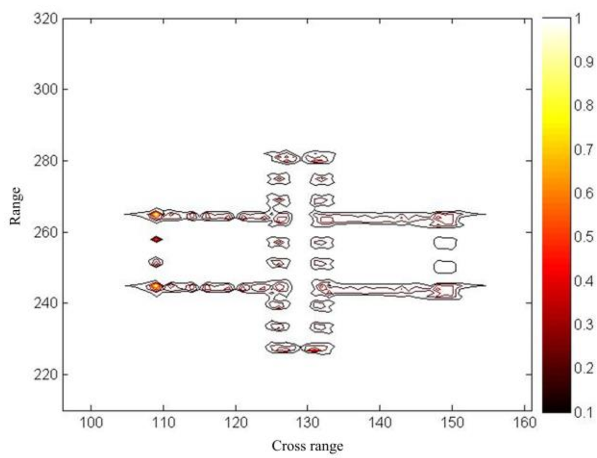


Figure 6. Image of FRFT algorithm

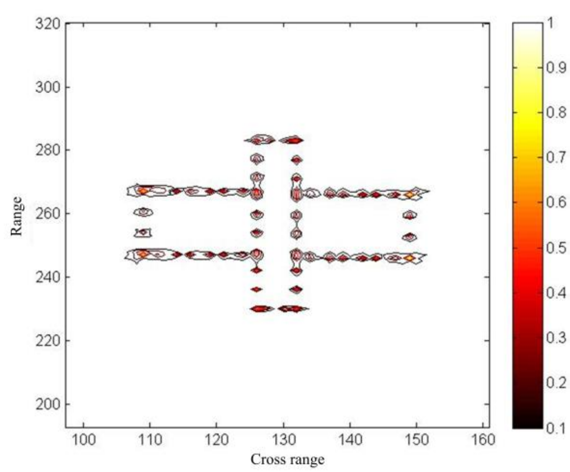


Figure 7. Image of the proposed method

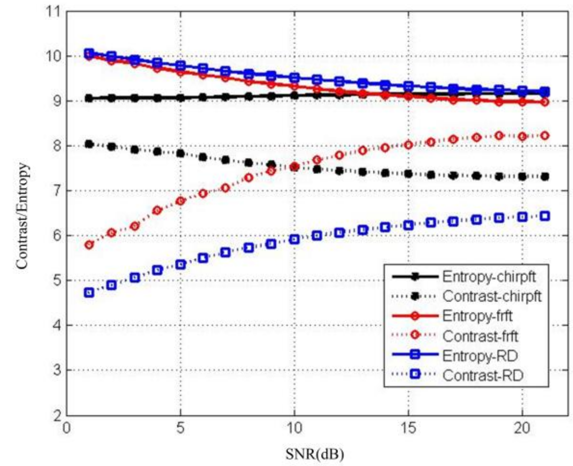


Figure 8. Image quality for different SNR

In conclusion, the simulation experiment results demonstrate that the proposed method can compensate the quadratic phase terms better than the RD algorithm and the FRFT algorithm. Consequently, the proposed method can obtain better ISAR imaging results for the high speed targets

Fig. 8 shows the imaging results of the three methods under different SNR levels (0~20dB). According to the curves in Fig. 8, the image entropy and the image contrast of the proposed method are much smoother than that of the RD algorithm and the FRFT algorithm, which means that the proposed method is more robust to the SNR level. Especially compared with RD and FRFT algorithms, the ISAR image obtained by the proposed method has less entropy and greater contrast at low SNR level, as is shown in Fig. 8. Thus, the proposed method is more suitable to low SNR level situation.

In conclusion, the imaging results presented in this paper are superior to the classic RD and FRFT algorithms. Meanwhile, the proposed algorithm is more robust to the SNR level.

## V. CONCLUSION

The echo of the high speed target is a multi-component LFM signal, therefore, the direct Fourier transform will bring about range distortion, which leads to a blurred ISAR image. This paper proposes a novel imaging method based on chirp Fourier transform and Shannon minimum entropy principle, which can eliminate quadratic phase term caused by high speed motion and achieve range compression simultaneously. The proposed method can simplify the complexity of the algorithm, while maintain the quality of the ISAR imaging result. Additionally, the proposed method is more robust to the SNR level. The simulation experiments show that the imaging results obtained by the proposed method are better than that of RD and FRFT algorithms.

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