Monte Carlo Simulation of an Optical Differential Phase-Shift Keying Communication System with Direct Detection Impaired by In-Band Crosstalk

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Abstract— A Monte Carlo (MC) simulation is used to evaluate the performance of an optical differential phase-shift keying (DPSK) communication system impaired by in-band crosstalk. The MC simulation results are used to study the validity range of an analytical work proposed in the literature addressing the same problem. It is shown that the performance estimates obtained using the analytical work become incorrect, whenever the effect of inter-symbol interference (ISI) is enhanced, for example, for optical filter bandwidths below twice the DPSK signal bandwidth. When ISI is negligible, a slight discrepancy is observed between the performance estimates of both methods, because the two formalisms consider different models for the random phase noise.

Keywords – optical communication systems; Monte Carlo simulation; differential phase-shift keying; in-band crosstalk.

I. INTRODUCTION

In the last decade, differential phase-shift keying (DPSK) modulation [1] has attracted much attention in optical communication systems, mainly due to its ~3 dB advantage in receiver sensitivity when balanced detection is used, in comparison with the conventional on-off keying (OOK) modulation. Additionally, the DPSK outperforms the conventional OOK in its robustness to transmission impairments and tolerance to signal power fluctuations. Regarding optical networking, it has been experimentally found that the DPSK signal with balanced detection has ~6 dB higher tolerance to in-band crosstalk than the OOK signal [2].

Crosstalk due to the imperfect isolation of optical components, such as optical filters, (de)multiplexers and optical switches, used in optical network nodes is considered one of the most important physical layer limitations in optical networks. The crosstalk signals may arise from distinct sources or from the same source of the original signal. When the crosstalk signal arises from distinct sources, it may have the same wavelength as the original signal or have different wavelengths, giving rise to in-band crosstalk and inter-band crosstalk, respectively [3]. In-band crosstalk is the most severe form of crosstalk, because the signal-crosstalk beating terms originated at the receiver cannot be removed by filtering [4].

In this work, the influence of in-band crosstalk on the performance of an optical DPSK receiver is analyzed by Monte Carlo (MC) simulation. One major goal is to study the validity range of an analytical work proposed in the literature [4], [5], that addresses the same issue, keeping in mind that this work was developed considering an isolated DPSK symbol. The MC simulation, on the other way, considers a sequence of DPSK symbols, and takes into account the inter-symbol interference (ISI) effect on the performance. The impact of in-band crosstalk on the receiver performance is investigated for different receiver filters configurations, for an increasing crosstalk level, by taking into account the delay between the DPSK signal and the crosstalk signal, for different sequences of bits on the crosstalk signal, by considering receiver imperfections and for multiple interfering terms.

This work is organized as follows: Section II describes the model used to characterize the optical DPSK receiver and the MC simulation implementation. Numerical results are obtained in Section III. The conclusions are outlined in Section IV.

II. SYSTEM DESCRIPTION

A. Optical DPSK receiver

The structure of a typical direct detection DPSK receiver using balanced detection is shown in Fig. 1. It consists of an optical pre-amplifier with a constant power gain *G* over the amplifier bandwidth; an optical filter with -3 dB bandwidth B_o ; a delay interferometer (DI) with a differential delay equal to the bit period T_b ; a balanced photodetector consisting of two photodetectors with responsivities R_{λ}^+ and R_{λ}^- ; and a postdetection electrical filter with -3 dB bandwidth B_o .



Figure 1. Balanced DPSK receiver schematics.

The electrical fields, represented in Fig. 1, are described with more detail in [4]. The electrical field at the amplifier output, $\vec{E}(t)$, can be expressed as [4]

$$\vec{E}(t) = \sqrt{G} \cdot \vec{E}_s(t) + \sqrt{G} \cdot \sum_{i=1}^{M} \vec{E}_{c,i}(t) + \vec{E}_{ASE}(t), \qquad (1)$$

where the first term, $\vec{E}_{s}(t)$, corresponds to the electrical field of the incoming DPSK signal, described as $\vec{E}_s(t) = \sqrt{P_s} \exp[j\theta_s(t)]\vec{e}_s$, where P_s is the average signal power at the optical pre-amplifier input; $\theta_{i}(t)$ is the signal phase that carries the binary DPSK information; and \vec{e}_s is the signal polarization unit vector. The second term of (1), $\sum_{i=1}^{M} E_{c,i}(t)$, corresponds to the electrical field of the in-band crosstalk, described by M different DPSK signals with the same wavelength and the same bit rate of the original signal. The complex envelope of the *i*-th crosstalk signal field can be represented as $\vec{E}_{c,i}(t) = \sqrt{P_{c,i}} \exp \left[j \left(\theta_{c,i}(t) + \phi_{c,i}(t) \right) \right] \vec{e}_c$, where $P_{c,i}$ is the crosstalk power; $\phi_{c,i}(t)$ is a random phase noise; $\theta_{ci}(t)$ is the crosstalk phase that carries the binary information; and \vec{e}_{c} is the crosstalk signal polarization unit vector. The crosstalk level of the *i*-th interferer is defined as the ratio between the crosstalk power, $P_{c,i}$, and the signal power, P_s . The total crosstalk level is the sum of the crosstalk levels of all the interferers. In this work, we assume that signal and crosstalk fields are co-polarised, $\vec{e}_s = \vec{e}_c$ [5]. Finally, the third term of (1), $\vec{E}_{ASE}(t)$, corresponds to the electrical field of the amplified spontaneous emission (ASE) noise originated at the optical pre-amplifier. The ASE noise is assumed as a zero mean white stationary Gaussian noise, co-polarised with the signal, and with single-sided power spectral density described by $N_{a} = hv_{s}(G-1)F/2$, where hv_{s} is the photon energy at the signal wavelength, and F is the pre-amplifier noise figure. The DI is modeled as in [4] and the photodetectors are assumed as ideal square-law detectors.

Ideally, a DPSK optical receiver should have $R_{\lambda}^{+} = R_{\lambda}^{-}$ and $\theta_{e} = 0$. However, practical optical DPSK receivers may exhibit imperfections, such as: responsivity imbalance, which is quantified by $K = 10 \log_{10} (R_{\lambda}^{+} / R_{\lambda}^{-})$; and an offset between the transmitting laser frequency and the frequency leading to perfect constructive/destructive interference at the DI output. This last effect can be characterized by the phase error θ_{e} and modeled by the DI detuning $\Delta f = \theta_{e} / (2\pi T_{b})$ [6].

B. Implementation of the MC simulation

The main goal of this work is to develop a MC simulation tool capable of evaluating the performance of an optical DPSK receiver impaired by in-band crosstalk. The MC simulation is used to study the validity range of the analytical work proposed in [4], which was developed considering an isolated DPSK symbol. In the simulation, a pseudorandom deBruijn binary sequence with length N_b bits is generated, which allows to study the effect of ISI on the DPSK receiver performance. Then, to obtain the DPSK format, the signal is encoded differentially, considering that for each bit 'one' the optical phase does not change, and for each bit 'zero' a π -phase change is introduced. The bits sequence on the DPSK crosstalk signal is generated randomly and the random phase noise on the crosstalk signal is generated considering a Brownian motion model [7]. Notice that in the analytical work [4], the random phase noise is assumed constant over the bit period and with a uniform distribution. In the simulation, the ASE noise is generated using a random number generator, which follows a Gaussian distribution with zero mean and variance $N_o B_{sim}$, where B_{sim} is the bandwidth used in the MC simulation [8].

At each MC iteration, in accordance with (1), a sample function of the ASE noise and a sample function of the DPSK crosstalk signal with random phase noise are added to the DPSK original signal obtained from the deBruijn sequence. Then, the current at the decision circuit input is computed using the MC simulator, by passing the field described in (1) through the receiver model (using the frequency domain description of the optical and electrical filtering), and it is sampled. After sampling, each received bit, that is corrupted by noise and crosstalk, is compared to the corresponding transmitted bit to find out if an error has occurred. The MC iterations are repeated, until a specified N_e number of counted errors is attained. Then, the bit error probability (BEP) is estimated using BEP = $N_e / [N_u (N_b - 1)]$, where N_u is the number of iterations of the MC simulator.

III. RESULTS

In this section, the results corresponding to the performance of the optical DPSK receiver obtained using MC simulation are presented and compared with analytical results [4]. The performance of the optical DPSK direct detection receiver is assessed in presence of in-band crosstalk, considering: 1) different optical and electrical filters with different -3 dB bandwidths; 2) different crosstalk levels; 3) different sequence of bits on the DPSK crosstalk signal; 4) different delays between the crosstalk signal and the original signal; 5) two types of receiver imperfections, responsivity imbalance and DI detuning; and 6) multiple interfering terms. All results have been obtained considering the parameters shown in Table 1, and unless otherwise stated, these parameters are used throughout this work.

Parameter	Value
Number of bits (N_b)	64
Gain(G)	30 dB
Amplifier noise figure (F)	5 dB
Bit period (T_b)	0.1 ns
Responsivity ($R_{\lambda}^{+}, R_{\lambda}^{-}$)	1 A/W
Phase error of the interferometer (θ_e)	0
Total number of counted errors (N_e)	100
Laser spectral linewidth	10 MHz

TABLE I. SIMULATION PARAMETERS

In our results, we consider two optical receivers filters configurations: 1) ideal receiver configuration based on a rectangular optical filter and an integrate-and-dump electrical filter, which considers $B_e T_b = 1$; and 2) Gaussian receiver

configuration based on a Gaussian optical filter and a Gaussian electrical filter, which considers $B_e T_b = 0.7$.

A. Optical DPSK receiver without in-band crosstalk

In this subsection, the MC simulation is used to evaluate the optical DPSK receiver performance impaired only by ASE noise. The MC simulation results are compared to the results of the analytical formalism [4]. The study is performed for different optical signal powers and for different optical filter bandwidths for the ideal receiver configuration. In-band crosstalk and receiver imperfections are neglected in this study.



Figure 2. BEP as a function of the optical signal power, for the ideal receiver configuration with $B_o T_b = 1$, 10 and 100. Solid lines: MC simulation; dashed lines: analytical results.

Fig. 2 shows the BEP estimates obtained using MC simulation and using the analytical formalism [4] as a function of the optical signal power, P_s , for different optical filter -3 dBbandwidths,. As the MC simulation is obtained with a sequence of bits, it takes into account the ISI effect on the DPSK receiver performance. The analytical formulation neglects this effect, and due to this difference, for smaller normalized optical filter bandwidths $(B_a T_b = 1)$, the BEPs estimated from the MC simulation and the analytical formulation are very discrepant. For larger optical filter bandwidths $(B_{a}T_{b} = 10, 100)$, the MC simulated results are very similar to the results obtained with the analytical formulation, and ensure us that the implementation of the MC simulator is correct. Fig. 2 also shows that, for the same optical signal power, there is a severe increase of the BEP with the optical filter bandwidth enlargement. This occurs because with the increase of the optical filter bandwidth, the filtered ASE noise power is higher, and the DPSK receiver performance becomes degraded.

Fig. 3 depicts the BEPs obtained with the MC simulation and with the analytical formalism [4], for normalized optical filter bandwidths where the ISI effect on the performance is relevant, $B_o T_b \le 2$. As shown in Fig. 3, it can be assumed that the simulated results become sufficiently approximated to the analytical ones for $B_o T_b = 2$. This means that the ISI starts to lose its influence as the dominant source of performance degradation and provides a reference for the optical filter bandwidth above which, the accuracy of the analytical formalism, that neglects ISI, is ensured.

Other receiver filters configurations have been studied and a good agreement was found between the BEPs obtained using MC simulation and analytically. A good agreement with the results presented in [9] was also found, which further ensure us that the MC simulation is well implemented.



Figure 3. BEP as a function of the optical signal power, for the ideal receiver configuration, considering $B_{a}T_{b} \leq 2$. Solid lines: MC simulation; dashed lines: analytical results.

B. Optical DPSK receiver impaired by in-band crosstalk due to a single interferer

In this subsection, the performance of the optical DPSK receiver is analyzed in presence of ASE noise and in-band crosstalk, considering a single interfering term. The estimates obtained through MC simulation are compared to the estimates obtained with the analytical formalism [4]. Firstly, the BEP is evaluated for different crosstalk levels and optical filter bandwidths. Then, the influence of a delay applied on the crosstalk signal in relation to the original signal, of different bits sequences on the interferer and of receiver imperfections on the DPSK receiver performance is studied.

Fig. 4 shows the BEP estimates from the MC simulation and from the analytical formalism [4] as a function of the optical signal power, considering the ideal receiver configuration with $B_o T_b = 1$, 10 and 100, and a crosstalk level equal to -12 dB. The BEPs estimated from the MC simulation, which are represented by the solid green line, are obtained assuming that the random phase noise has a uniform distribution with constant phase along the bit period, similarly to the assumption considered in the analytical work [4]. The other BEPs obtained using MC simulation presented in Fig. 4, are estimated considering that the random phase noise follows the Brownian motion model [7].

Regarding the methods comparison, Fig. 4 shows that for smaller optical filter bandwidths, as the ISI is dominant, the difference between the BEPs obtained using the MC simulation and the analytical formalism is enhanced. For higher normalized optical filter bandwidths ($B_oT_b = 10$ and 100), the BEPs obtained using the MC simulation with the Brownian

motion assumption and in-band crosstalk are slightly discrepant to the analytical BEPs. This difference is attributed to the Brownian motion model assumed for the random phase noise, since when the MC simulation assumes a uniform distribution for the random phase noise, the simulated and analytical curves become very similar. Random phase noise with uniform distribution is a good model to describe the crosstalk that impairs an isolated symbol. However, this model loses reality for a sequence of symbols, due to the phase discontinuity introduced between adjacent symbols. The Brownian motion model provides a physical description that is suitable to describe the influence of random phase noise in a sequence of symbols, due to its temporal continuity.



Figure 4. BEP as a function of the optical signal power, considering a crosstalk level equal to -12 dB, and an ideal receiver configuration with $B_o T_b = 1$, 10 and 100. Solid red, blue and black lines: MC simulation with Brownian motion assumption; solid green lines: MC simulation with uniform assumption; dashed lines: analytical results.



Figure 5. BEP as a function of the crosstalk level, for an optical signal power equal to -46 dBm, and the ideal receiver configuration with different optical filter bandwidths. Solid red, blue and black lines: MC simulation with Brownian motion assumption; solid green lines: MC simulation with uniform assumption; dashed lines: analytical results.

Fig. 5 shows the BEPs obtained by MC simulation and analytically, as a function of the crosstalk level with B_oT_b as a parameter considering the ideal receiver configuration and an optical signal power equal to -46 dBm. Fig. 5 shows that,

whenever the ISI influence on the performance is dominant, the BEPs estimated using MC simulation using both phase noise models are discrepant with the BEPs obtained analytically (see, for example, the curves corresponding to $B_o T_b = 2$ in Fig. 5). When ISI is not relevant ($B_o T_b = 10$) and the crosstalk level is above -18 dB, there is a slight discrepancy between the analytical results and the MC simulation results obtained with the Brownian motion model. When considering the uniform distribution, for a high crosstalk level (above -15 dB), for $B_a T_b = 10$, the analytical results tend to the simulated results.

In the remainder of this subsection, all the MC simulated results are obtained considering the Brownian motion model.

In the following, the performance of the optical DPSK receiver is investigated considering different bits sequences on the DPSK crosstalk signal. Until now, the bits sequence on the DPSK crosstalk signal was always assumed to be random. The next study aims to analyze and compare the performance of the optical DPSK receiver when the bits sequence on the DPSK crosstalk signal is: 1) equal to the sequence of bits on the original DPSK signal; 2) the negation of the sequence of bits on the original DPSK signal; 3) a sequence with only bits '1' and 4) a sequence with only bits '0'.



Figure 6. BEP as a function of the crosstalk level, considering an optical signal power of -46 dBm, for different sequences of bits on the DPSK crosstalk signal and for the ideal receiver configuration with $B_o T_b = 2$ and 10.

Fig. 6 shows the BEP as a function of the crosstalk level, for different bits sequences on the DPSK crosstalk signal considering an optical signal power of -46 dBm, and the ideal receiver configuration with $B_o T_b = 2$ and 10. As shown in Fig. 6, except for the case where the sequence of bits on the DPSK crosstalk signal is equal to the original DPSK signal, the BEPs estimated using MC simulation, when impaired by other DPSK crosstalk signals with different bits sequences are approximated to the ones obtained considering the random bits sequence. When the bits sequence on the DPSK crosstalk signal is equal to the original DPSK crosstalk level results in an improvement of the BEP, because as the DPSK crosstalk signal is added to the original signal, there is a signal power reinforcement.

In order to understand the influence of the bits sequences on the DPSK crosstalk signal, the eye diagrams of the original signal impaired by different bits sequences on the crosstalk signal are shown in Fig. 7, considering: a random bits sequence (left); a bits sequence equal to the original signal (middle) and the negated bits sequence (right). The eye diagrams are obtained for the ideal configuration with $B_oT_b = 2$, for $P_s = -45$ dBm and a crosstalk level of -12 dB.



Figure 7. Eye diagrams at the decision circuit input, for the ideal configuration, $B_o T_b = 2$, an optical signal power equal to -45 dBm, a crosstalk level of -12 dB and considering a random bits sequence on the DPSK crosstalk signal (left), an equal bits sequence (middle) and the negated bits sequence (right).

As can be observed in Fig. 7, it can be concluded that the eye pattern shows a significant enlargement in the eye opening when the bits sequence on the DPSK crosstalk signal is equal to the bits sequence of the original DPSK signal. As a consequence, the BEP achieved through the MC simulation, shown in Fig. 6 is considerably lower.

In the next study, the influence of a delay between the crosstalk signal and the original signal on the optical DPSK receiver performance is analyzed.



Figure 8. BEP as a function of the delay between the original and crosstalk signals, for the ideal receiver configuration with $B_o T_b = 2$, an optical signal power equal to -46 dBm, and a crosstalk level of -12 dB.

Fig. 8 shows the BEP as a function of the delay between the original and crosstalk signals, for the ideal receiver configuration with $B_oT_b = 2$, for $P_s = -46$ dBm, and a crosstalk level of -12 dB. To achieve an improved accuracy, a value of $N_e = 1000$ erroneous bits is assumed in the MC simulation. Fig. 8 shows that, the influence of the delay between the crosstalk signal and the original signal on the performance of the optical DPSK receiver is small, since the BEP variation is not much significant. Nevertheless, Fig. 8 shows that the best BEP is

achieved for a delay of half the bit period, and that the BEP has a symmetric behavior around this point. The worst BEP is obtained when the crosstalk and the original signals are aligned, which is in agreement with the worst-case situation usually assumed in the literature [4].

Afterwards, the performance of the optical DPSK receiver is analyzed considering the effect of two receiver imperfections: the responsivity imbalance and the DI detuning. The MC simulation of an optical DPSK receiver impaired by imperfections without in-band crosstalk has been validated by comparison of its estimates with the analytical results of [6].



Figure 9. Power penalty as a function of the responsivity imbalance with different interferometer detunings, for the Gaussian receiver configuration with $B_o T_b = 5$, with a crosstalk level of -15 dB,. Solid lines: MC simulation; dashed lines: analytical results.

Fig. 9 depicts the power penalty as a function of the responsivity imbalance with different interferometer detunings, considering a Gaussian configuration with $B_{\rho}T_{\mu} = 5$ and a crosstalk level of -15 dB. The reference for the power penalty is obtained with no detuning, K = 0 dB and without crosstalk, which corresponds to a BEP of 10^{-3} obtained for an optical signal power of -46.5 dBm. Accordingly with Fig. 9, it can be concluded that there is a good approximation between the power penalties obtained using the analytical formalism and the MC simulation. The small differences between the performances might be related with the random phase noise models. Without imperfections, the performance degradation due to in-band crosstalk is about 0.9 dB (taken from the MC simulation results). For K = 10 dB and no detuning, the performance degradation is enhanced to about 1.5 dB. Fig. 9 also shows that the performance degradation due to receiver imperfections is not enhanced with the crosstalk influence.

C. Optical DPSK receiver impaired by in-band crosstalk due to multiple interferers

In this subsection, the accuracy of the MC simulation is analyzed in presence of multiple interfering terms in the crosstalk signal.

Fig. 10 shows the BEP as a function of the optical signal power with the number of interferers as a parameter, considering the ideal receiver configuration with $B_o T_b = 2$

(above) and $B_{a}T_{b} = 10$ (below) and a total crosstalk level of -12 dB. Fig. 10 shows that with the increase of the optical signal power, the performance degradation due to a higher number of interfering terms is enhanced. For $B_{a}T_{b} = 2$, the performance degradation is more noticeable due to the lower ASE noise power. In this case, the differences between the analytical estimates and simulated results are attributed to ISI and to the difference between the random phase noise models.



Figure 10. BEP as a function of the optical signal power, considering the ideal receiver configuration with $B_o T_b = 2$ (above) and $B_o T_b = 10$ (below), a total crosstalk level of -12 dB and multiple interfering terms. Solid lines: MC simulation; dashed lines: analytical results.

IV. CONCLUSIONS

In this work, a MC simulator has been developed to evaluate the impact of in-band crosstalk in an optically preamplified DPSK direct detection receiver and to study the validity range and the limitations of the analytical work proposed in [4]. This study has been performed for the scenario without in-band crosstalk and for the scenario with in-band crosstalk originated from multiple interferers.

It was shown that when the ISI plays a key role as the dominant source of performance degradation, for $B_oT_b < 2$, the BEPs predicted by the analytical work became very discrepant from the ones obtained with MC simulation. As a rule of thumb, $B_oT_b = 2$ provides a good bound for the normalized

optical filter bandwidth, where the accuracy of the analytical work is still guaranteed. In presence of in-band crosstalk, it was concluded that even for large optical filter bandwidths, a slight discrepancy is observed between the BEPs obtained using both formalisms due to the fact that the analytical formalism considers that random phase noise has a uniform distribution constant over the bit period, whereas the MC simulation assumes a Brownian motion model for the random phase noise. It was also shown that a delay applied in the crosstalk signal (in relation to the original signal) does not have a relevant influence on the DPSK receiver performance. Furthermore, it was shown that, a DPSK crosstalk bits sequence equal to the original bits sequence leads to a performance improvement due to signal power reinforcement. We have also observed that, the performance degradation due to DPSK receiver imperfections is not enhanced with the presence of crosstalk.

The influence of multiple interfering terms on the DPSK receiver performance was also analyzed, and it was seen that, with the increase of the optical signal power, the performance degradation induced by a higher number of interfering terms is enhanced. The discrepancies between analytical and simulated results, found for multiple interfering terms are similar to the ones obtained with one single interfering term.

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