Development of Modified Ant Colony Optimization Algorithm for Compliant Mechanisms

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***Abstract*—A compliant mechanism is a mechanism that obtains its mobility or force from the flexibility and elastic deformation of its components. Topology for compliant mechanism is very sensitive and can be obtained very variously according to topology optimization methods and computation conditions. A modified ant colony optimization algorithm is suggested for compliant mechanisms in order to obtain a stable and robust optimal topology. The modified ant colony optimization algorithm was applied for both linear and geometrically nonlinear compliant mechanisms. Using three kinds of objective functions commonly used, optimized topologies were compared for compliant mechanisms. And it was concluded that the modified ant colony optimization algorithm can successfully be applied for linear and geometrically nonlinear compliant mechanisms, and the algorithm provides very stable and robust topologies.**

Keywords-Compliant mechanism; Modified Ant Colony Algorithm (MACO); Topology optimization; Geometrically nonlinear.

I. INTRODUCTION

Topology optimization has been applied for various linear structural problems so far [1][2][3][4][5][6]. However, when a very large load is applied or structural deformation is very large, geometrically nonlinearity may be occurred due to mechanical conditions. In order to obtain more useful and valuable optimal topology of a structure satisfying the given constraints, the above nonlinearity should be considered in analysis and design.

Buhl et al. [7] has performed stiffness designs of geometrically nonlinear structures using topology optimization based on the density method. Pedersen et al. [8] has performed linear and geometrically nonlinear topology optimizations based on the solid isotropic material with penalization (SIMP) method for large displacement compliant mechanisms. Bruns and Tortorelli [9] also carried out linear and geometrically nonlinear topology optimization for nonlinear structures and compliant mechanisms.

Recently, Kaveh [10] suggested ant colony optimization (ACO) algorithm for structural models to find the stiffest structure with a certain amount of material, based on the element's contribution to the strain energy. The results showed that ACO can be a suitable tool to handle the problem as an on-off discrete optimization. However, the field of topology optimization for compliant mechanisms is rarely new in ACO algorithms since researches on compliant mechanisms have not been done so far.

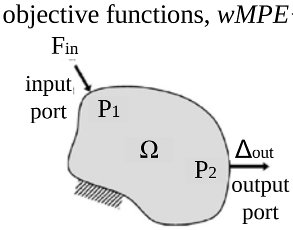
In this study, a new topology optimization algorithm based on ACO algorithm is developed for a compliant mechanism for the first time implemented with a filter scheme [11]. Distribution of material is expressed by density of each element to apply ACO algorithm. Three kinds of objective function are examined to obtain stable and robust topology, it is found that the developed topology scheme is very effective and applicable in a compliant mechanism topology optimization problems and mutual potential energy (*MPE*) / strain energy (*SE*) type of objective function was the best through the comparison of the results of the linear and geometrically nonlinear cases.

This paper is organized as follows. Formulation of a Force Compliant Mechanism in Section II. In Section III, we introduce the proposed method which is called MACO. Section IV contains Numerical Examples, and Section V, concludes the paper.

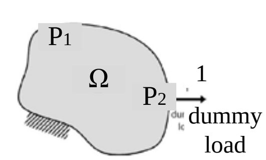
II. FORMULATION OF A FORCE COMPLIANT MECHANISM

A compliant mechanism [12] is a mechanism that obtains its mobility or force from the flexibility and elastic deformation of its components. Fig. 1(a) [13] shows design domain Ω with the input force and the desired output displacement Δ_{out}. P_1 and P_2 represent the input and output ports, respectively, and Δ_{out} represents the desired output displacement at the output port. Fig. 1(b) shows the analytical conditions of *SE* for evaluating the stiffness and *MPE* for evaluating flexibility of a compliant mechanism.

Three kinds of objective functions are usually used for compliant mechanisms. *MPE* [13] is used for maximizing displacement at the output port. One of multicriteria objective functions, *wMPE*+(1-*w*)*SE*, is used for considering



(a) Input force



(b) Dummy load

Figure 1. Analysis load case

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both flexibility and stiffness of compliant mechanisms with weighting factor w. A ratio [14] of MPE to SE is employed for maximizing displacement at the output port considering both flexibility and stiffness of compliant mechanisms. In this paper, the above objective functions were used, and topology optimization based on the MACO method was performed for a force compliant mechanism.

Firstly, we explain how to calculate sensitivity number for the case that *MPE/SE* is employed as the objective function. The sensitivity number can be calculated by adapting the method suggested by Ansola [13]. The finite element equation for *MPE* can be expressed using (2) as follows;

$$MPE = \Delta_{out} = \{D_2\}^T [K]\{D_1\} \tag{2}$$

where $[K]$ is the generalized stiffness matrix, $\{D_1\}$ is the nodal displacement vector due to the input load and $\{D_2\}$ is the displacement vector due to the unit dummy load placed at the output port, respectively. *MPE* means that the displacement at the output port when F_{in} at the input port is applied.

The finite element equation for *SE* can be expressed using (3) as follows. *SE* means the strain energy stored when the unit dummy load applied at the output port.

$$SE = \frac{1}{2}\{D_2\}^T [K]\{D_2\} \tag{3}$$

If the i -th element is added to the previous topology, the change in *MPE* can be calculated as follows (4).

$$\Delta MPE = -\{D_1\}^T [\Delta K]\{D_2\} \tag{4}$$

Similarly, when the i -th element is added to the previous topology, the change in *SE* can be obtained as follows (5).

$$\Delta SE = -\frac{1}{2}\{D_2\}^T [\Delta K]\{D_2\} \tag{5}$$

Element addition affects the generalized stiffness matrix, and we can calculate the change in the generalized stiffness matrix due to the i -th element addition as follows (6);

$$[\Delta K] = [K'] - [K] = [K]_i \tag{6}$$

where $[K']$ is the generalized stiffness matrix after the i -th element is added, $[K]$ is the generalized stiffness matrix before the element is added, and $[K]_i$ is the generalized stiffness matrix of the added element. From (4), (5), and (6), the following relations (7) can be obtained;

$$\begin{aligned} \Delta MPE &= -\{D_1\}_i^T [K]_i \{D_2\}_i \\ \Delta SE &= -\frac{1}{2}\{D_2\}_i^T [K]_i \{D_2\}_i \end{aligned} \tag{7}$$

where $\{D_1\}_i$, and $\{D_2\}_i$ represent the generalized displacement vectors due to the i-th element addition.

In this paper, in the case that *MPE* is employed as an objective function, the sensitivity number is calculated as follows (8).

$$\alpha_i = -\{D_1\}_i^T [K]_i \{D_2\}_i \tag{8}$$

Secondly, in the case that $wMPE+(1-w)SE$ is used as an objective function, the sensitivity number is calculated as follows (9).

$$\alpha_i = w\Delta MPE + (1-w)\Delta SE \tag{9}$$

Finally, the sensitivity number for the i-th element can be obtained by differentiating the objective function (10).

$$\alpha_i = \frac{(\Delta MPE)(SE) - (MPE)(\Delta SE)}{(SE)^2} \tag{10}$$

The above sensitivity numbers can be used for adding, eliminating, or transforming elements in topology optimization for a force compliant mechanism.

III. Modified Ant Colony Optimization Algorithm

If ACO algorithm is applied for topology optimization of geometrically nonlinear compliant mechanisms, a critical problem can be encountered. This method can provides a stable topology in the case of a high target volume in structural topology optimization. However, in the case of a low target volume, the asymmetry of stiffness matrix becomes very severe since the topology consisted of solid elements significantly lose the symmetry of structure. It causes poor accuracy of the solution since ill-condition might be produced. In order to overcome the above weakness, it is necessary to define a design variable such as continuously distributed density in the previous studies [15] for topology optimization. Therefore, a MACO algorithm is suggested in order to remedy the weakness of the ACO algorithm.

The governing equations of the ACO algorithm [10] are briefly described as follows. Contribution of each element i into the overall objective of the problem, which is analog to the pheromone trail of a segment of a route, is here denoted by $\tau_i(t)$. The parameter t represents the time of deployment of ants which is equivalent to the cycles of iteration within the algorithm. Inspired by the procedure employed in TSP [16], and ignoring the effect of the local heuristic values, the ant decision index $a_i(t)$ can be written as (11);

$$a_i(t) = \frac{[\tau_i(t)]^\alpha}{\sum_{j=1}^{M}[\tau_j(t)]^\alpha} \tag{11}$$

where α is a parameter that controls the relative weight of the pheromone trail, M is the number of finite elements and t is an indication of the present cycle which is analogous to the t-th time of deploying our ants. Note that here the probability of an element being chosen by a typical ant is the same as the decision index as defined in (11).

After completion of a cycle of designs by all ants, each ant k deposits a quantity of pheromone $\Delta\tau_i^k$ on each element based on its relative objective function, as shown below, which is an index of the performance of the element, i.e. for a better design a larger amount of pheromone is deposited (12);

$$\Delta\tau_i^k = \frac{\left(U_i^k\right)^\lambda}{\sum_{j=1}^{M}\left(U_j^k\right)^\lambda} \tag{12}$$

where U_i^k is the objective function in each element of design and the exponent λ is a tuning parameter for improvement of performance of the algorithm and its convergence.

The amount of pheromone in each element is due to addition of new pheromone as well as evaporation which is implemented within the algorithm via the following rule (13).

$$\tau_i\left(t+1\right) = \gamma\tau_i\left(t\right) + \Delta\tau_i \tag{13}$$

where $\Delta\tau_i = \sum_{k=1}^{m}\Delta\tau_i^k$ and m is the number of ants used in each cycle. The rate of evaporation coefficient $\gamma \in \left[0,1\right]$ is applied for taking into account the pheromone decay to avoid quick convergence of the algorithm towards a suboptimal solution.

The main difference between ACO and MACO is to use a new continuous variable, which is called "fitness" in finite element analysis (FEA), instead of the positions of the ants. Fitness is defined by the ratio of the summation of the ants number passed each element to the number of inner loop. It can be expressed as (14);

$$fitness_i = \frac{\sum_{iter=1}^{N}(A_{iter})_i}{N}, \quad (A = 1 \; or \; 0) \tag{14}$$

where A represents the existence of ant (existence = 1, no existence = 0). N is the number of iteration, i is the element number. Then, the calculated fitness in iteration process is used to FEA as a design variable in cycle process.

In this study, a new change of objective function α_i^{new} is defined as (15) in order to accelerate the convergence rate of the MACO;

$$\alpha_i^{new} = fitness_i \times \alpha_i \tag{15}$$

where i indicates each element. The resized α_i is applied to the global update equation (12) and the U_i in (12) is replaced by α_i for geometrically nonlinear compliant mechanism (16).

$$\Delta\tau_i^k = \frac{\left(\alpha_i^k\right)^\lambda}{\sum_{j=1}^{M}\left(\alpha_j^k\right)^\lambda} \tag{16}$$

In addition, it is very important to reduce computation time for geometrically nonlinear compliant mechanism. Convergence rate of the ACO is dependent on the ant decision index $a_i\left(t\right)$, so that the parameters of α, λ, γ, affect very much on the convergence rate. It has been suggested that convergence rate can also be accelerated by resizing pheromones newly added to the solution found by rank-based ant system or elite ants [17] [18] [19].

$$\Delta\tau_i = \left(1-\Delta\tau_{\min}^{new}\right)\frac{\Delta\tau_i^{new}}{\Delta\tau_{\max}^{new}} + \Delta\tau_{\min}^{new} \tag{17}$$

where $\Delta\tau_i^{new} = \sum_{k=1}^{m}\Delta\tau_i^k$, $\Delta\tau_{\min}^{new}$ is 0.0001 and $\Delta\tau_{\max}^{new}$ is the maximum value of pheromone trail at each iteration. The resized $\Delta\tau_i$ provides the improved effect of acceleration rate on convergence, and overcome numerical singularity occurred on the low-density region. Also it gives the selection possibility of the elements, which have been removed because of low deposited pheromone in the previous iterations, in the following iterations. The resized $\Delta\tau_i$ is applied to the global update equation (13). The optimization process using MACO can be depicted as a flowchart shown in Fig. 2. The procedure can be outlined as follows:

1. Specify ACO control parameters (α, γ), tune the parameter λ, and assign an initial pheromone trailing value on each element.
2. Create the initial design using a sequence of random selection by spreading pheromone trailing uniformly in the design space
3. Calculate α_i for each element using compliant mechanism finite element analysis with a mesh-independency filter scheme to suppress checker-board pattern.
4. Move ants probabilistically according to (11).
5. Deposit new pheromones at the present locations where ants move using (16).
6. After completion of a cycle, resize pheromones using (17).
7. The amount of pheromone on each element is updated according to (13).
8. Steps 3 through 7 are repeated until convergence criterion is satisfied using (18).

$$error = \frac{\left|\sum_{i=1}^{N'}\left(\alpha_{c-i+1} - \alpha_{c-N'-i+1}\right)\right|}{\sum_{i=1}^{N'}\alpha_{c-i+1}} \leq \varphi \tag{18}$$

where, c is the current iteration number, φ is an allowable convergence error, and N' is the integral number which results in a stable convergence in at least ten successive iterations.

IV. Numerical Examples

A. Force invertng mechanism

A displacement generator in compliant mechanisms having dimensions of 40 µm × 20 µm × 7 µm is subjected to input force F_{input} = 2 mN with the spring constants $k_{input, output}$ = 1 mN/µm at input and output ports as shown in Fig. 3. Design domain is divided into 80 × 40 by four node rectangular element. The material is assumed to have Young's modulus of 100 GPa and Poisson's ratio of 0.3. The coefficients of MACO are defined as α = 1, λ = 2, ρ = 0.8. Allowable convergence error, τ is set to be 0.001. The objective is to obtain a stiffest structure under a volume constraint of 20% of the original volume.

Topologies for linear and geometrically nonlinear cases with *MPE* and *wMPE*+(1-*w*)*SE* where w = 0.8 as objective functions are shown in Fig. 4 and Fig. 5, respectively. And topologies for linear and nonlinear cases with *MPE/SE* are shown in Fig. 6. The displacements of the optimal topologies for three kinds of objective functions are compared in Table 1. It can be found that the displacements of *MPE* type are the largest in linear and nonlinear cases. In other cases of *MPE/SE* type for both linear and nonlinear are the smallest. Also, the joint part is a little more reinforced in the nonlinear case than the linear case.

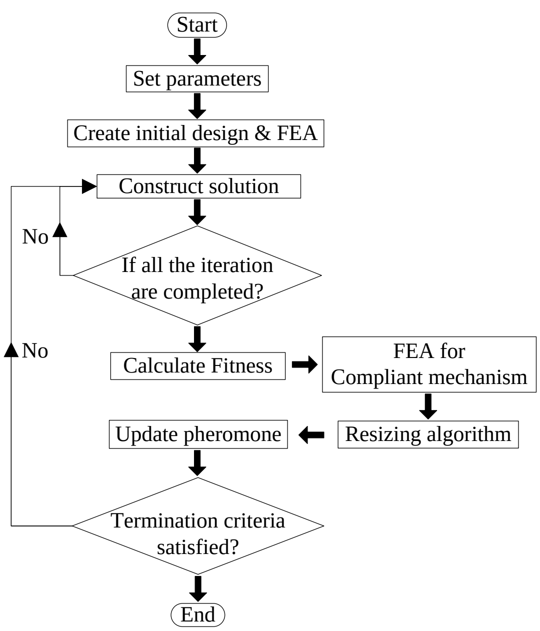

Figure 2. Flowchart of the MACO for compliant mechanisms

From the results of the example, topology at the joint parts is not connected when *MPE* implemented with the MACO is employed as an objective function. Even though topology at the joint part is connected each other when *wMPE*+(1-*w*)*SE* where w = 0.8 is used as objective function, there appears checkerboard pattern, and the topology may change according to weighting factor. As seen here, topology at the joint part is firmly connected each other and a stable topology can be obtained when *MPE/SE* is used as objective function. Therefore, it is concluded that *MPE/SE* is very suitable among three kinds of objective functions for designing compliant mechanisms.

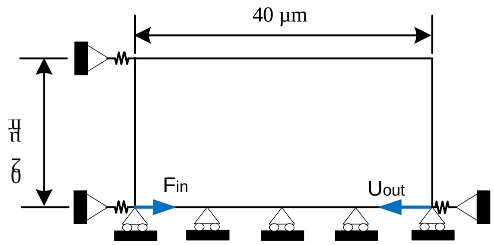

Figure 3. Design domain of a force inverter

(a) Linear case (b) Geometrically nonlinear case

Figure 4. Optimal topology using *MPE*

(a) Linear case (b) Geometrically nonlinear case

Figure 5. Optimal topology using *wMPE*+(1-*w*)*SE* (w = 0.8)

(a) Linear case (b) Geometrically nonlinear case

Figure 6. Optimal topology using *MPE/SE*

TABLE I. Displacements of Linear and Nonlinear Optimal Topologies

Cases	*MPE*	*wMPE*+(1-*w*)*SE*	*MPE/SE*
Linear	–2.972 µm	–1.606 µm	–0.212 µm
Geometrically nonlinear	–3.067 µm	–2.426 µm	–0.587 µm

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B. Gripper mechanism

A gripper mechanism having dimensions of 40 μm × 15 μm × 1 μm is subjected to input force F_{input} = 1 mN with the spring constants $k_{input, output}$ = 0.1 mN/μm at input and output ports as shown in Fig. 7. Design domain is divided into 120 × 45 by four node linear finite elements. The material is assumed to have Young's modulus of 100 GPa and Poisson's ratio of 0.3. The coefficients of MACO are defined as α = 1, λ = 2, ρ = 0.8. Allowable convergence error, τ is set to be 0.001. The objective is to obtain a stiffest structure under a volume constraint of 20% of the original volume.

Topologies for linear and geometrically nonlinear cases with *MPE* and *wMPE*+(1-*w*)*SE* where *w* = 0.8 as objective functions are shown in Fig. 8 and Fig. 9, respectively. And topologies for linear and nonlinear cases with *MPE/SE* are shown in Fig. 10. The displacements of the optimal topologies for three kinds of objective functions are compared in Table 2.

It can be found that the displacements of *MPE* type are the largest in linear and nonlinear cases. In other cases of *MPE/SE* type for both linear and nonlinear are the smallest.

From the results of the example, topology at the joint parts is not connected when *MPE* for nonlinear case implemented with the MACO is employed as an objective function. Even though topology at the joint part is connected each other when *wMPE*+(1-*w*)*SE* where *w* = 0.8 is used as objective function, there appears that topology of output part is unstable compared to the other objective function. Therefore, it is concluded that *MPE/SE* is very suitable among three kinds of objective functions for designing compliant mechanisms.

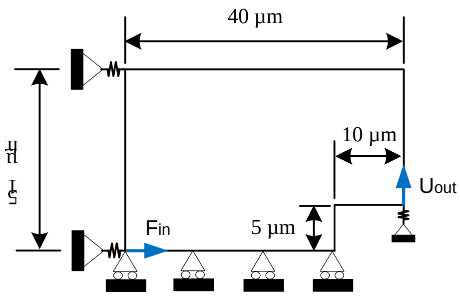

Figure 7. Design domain of a gripper mechanism

(a) Linear case (b) Geometrically nonlinear case

Figure 8. Optimal topology using *MPE*

(a) Linear case (b) Geometrically nonlinear case

Figure 9. Optimal topology using *wMPE*+(1-*w*)*SE* (*w* = 0.8)

(a) Linear case (b) Geometrically nonlinear case

Figure 10. Optimal topology using *MPE/SE*

TABLE II. DISPLACEMENTS OF LINEAR AND NONLINEAR OPTIMAL TOPOLOGIES

Cases	*MPE*	*wMPE*+(1-*w*)*SE*	*MPE/SE*
Linear	−1.922 μm	−1.703 μm	−1.426 μm
Geometrically nonlinear	−2.45 μm	−2.43 μm	−2.28 μm

V. CONCLUSION AND FUTURE WORK

A. Conclusion

In this study, the MACO algorithm has been suggested for compliant mechanism problems and a compliant mechanism using three kinds of objective functions. From the results of examples, the following conclusions are obtained.

(1) It is verified that the MACO algorithm can successfully be applied for a compliant mechanism, and provides stable and robust optimal topology.

(2) MACO algorithm is suggested for applying it for compliant mechanisms in order to obtain a stable topology since ACO algorithm might severely provide asymmetric stiffness matrix due to the characteristics of stochastic methods.

(3) It is found that *MPE/SE* considering flexibility and stiffness together is the most suitable for objective function among three kinds of objective functions for designing compliant mechanisms.

The topology optimization using the ACO could be extended to more complicated thermally actuated compliant mechanism specifications such as electro-thermal actuators subjected to non-uniform temperature fields actuated by Joule heating.

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